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# TOWARDS STATE ESTIMATION IN THE FACTORIZED FORM

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**Abstract.** *The paper addresses the state estimation in the factorized form. The target application area is the urban traffic control, where the main controlled variables (queues) are not directly observable and have to be estimated. Additional problem is that some state variables are of a discrete-valued nature. Thus, estimation of mixed-type data (continuous and discrete valued) models is highly desirable. Factorized state estimation is a potential solution of this problem. The underlying methodology is Bayesian filtering. Factorized version of the filter is obtained by applying the chain rule to the state-space model. The general solution represents the recursive entry-wise performance of data updating and time updating steps. Application of the solution to linear Gaussian state-space models gives the factorized Kalman filter.*

## 1 INTRODUCTION

The urban traffic systems are overloaded almost everywhere: modern, powerful cars move slowly and inefficiently through towns within permanently extending peak hours. Adequate extension of traffic network is expensive and often impossible, especially in historical towns. It calls for exploiting all available means starting from economical pressure, various regulative measures up to exploitation of modern, ideally adaptive, feedback control. One of the main controlled variables in traffic systems is a queue length, which expresses the optimality of a traffic network most adequately. It is directly unobserved and, therefore, has to be estimated. At the same time, other state variables are of a discrete-valued nature. Thus, estimation of mixed-type data (continuous and discrete valued) models is highly desirable. A potential solution to this problem calls for a factorized version of the state estimator (filter), which allows to model the entries of the state individually. In this way, the task *how to obtain the estimates of the individual time-varying state factors* is addressed in the paper.

In Bayesian methodology, adopted in the paper, the factorized version of the filter is obtained by applying chain rule to the state-space model. The problem was already solved for a degenerate case of time-invariant state, which coincides with parameter estimation [1]. The obtained results indicated a chance to update posterior probability density functions in the entry-wise manner.

The state of the art of the problem includes a series of research in the field. Most works found are devoted to factorization of well-known Kalman filter [2]. Despite the variety of the research at this area, the global aim of majority of these works is reduction of the computational complexity with the help of lesser rank of the covariance matrix, but not the obtaining of the estimates of the individual state entries, which is the aim of the present work. For example, the work [3] deals with factorization of the covariance matrix in Kalman filter, where the covariance matrix was decomposed with the help of square root factorization. The $QR$-factorized filter and smoother algorithms for use on linear time-varying discrete-time problems, that can handle the general case of a singular state transition matrix, are discussed in [4]. The $UD$-factorization of Kalman filter for the multi-sensor data fusion is presented in [5]. Another work, devoted to the $UD$-factorized covariance filter application, is concerned with development of a connected element interferometer [6]. The method for particle filtering, which factorizes the likelihood, was proposed in [7]. It considers the problem, when the state space can be partitioned in groups of random variables, whose likelihood can be independently evaluated.

As regards the nonlinear estimation, the following research works should be noted here. The square root form of unscented Kalman filter (UKF) for the state and parameter estimation, which, in its turn, was proposed as an alternative to the extended Kalman filter, used for nonlinear estimation, is described in [8]. This square-root UKF has better numerical properties and guarantees positive semi-definiteness of the underlying state covariance.

The factorization of the covariance matrices is also used in problems of systems classification, dealing with multivariate Gaussian random field [9].

The problem of filtering with Gaussian models can be also considered with the help of dynamic Bayesian networks [10]. Within this framework the problem of joint state-parameter estimation is often met.

The work [11] was directed exactly at the estimation of the individual state factors, and it proposed the recursive algorithm of factorized filtering, requiring a special, reduced, form of the state-space model. The present paper offers the solution without such a restriction.

Moreover, the overview of the problem showed the results mostly with Gaussian models. The general solution, based on the Bayesian framework, can be potentially helpful in the case of other models.

The layout of the paper includes the following sections. The basic facts of the dynamic Bayesian decision making [1] are given in Section 2. It provides the probabilistic formulation of the model used and basic equations of the Bayesian filtering. Section 3 describes the general solution to the factorized filtering, which represents the recursive entry-wise performance of data-updating and time-updating steps. The example with Gaussian model is provided in Section 4. It demonstrates application of solution to the simple single output system. The remarks in Section 5 close the paper.

## 2 PRELIMINARIES

The following notations and notions are used in the paper:

$\equiv$ means the equality by definition.

$f(|)$ is the letter reserved for conditional probability density functions (pdf). The meaning of the p(d)f is given through the name of its argument. When the argument $x$ coincides with realization of the corresponding random variable then it is made bold, i.e. $f(\boldsymbol{x})$.

$x^*$ denotes the range of $x$, $x \in x^*$.

$\mathring{x}$ denotes the number of members in the countable set $x^*$ or the number of entries in the vector $x$.

$x_t$ is a quantity $x$ at the discrete time instant labelled by $t \in t^* \equiv \{1, \ldots, \mathring{t}\}$.

$\mathring{t} \leq \infty$ is called (decision, learning, prediction, control, advising) horizon.

$x_t^i$ is an $i$th entry of the array $x$ at time $t$. The subscript symbol is a time index.

$x^{k:l}$ denotes the sequence $x_k, \ldots, x_l$. $x^{l:k}$ is an empty sequence and reflects just the prior information if $l < k$.

The following adopted simplifications are also used: integrals used are always definite ones. The integration domain coincides with support of the pdf in its argument.

Within the considered Bayesian framework, *the most complete description* of the behavior of the closed control loop system is the joint pdf $f(x_t, d^{1:t}|x_0)$, where $x_t$ is the system state, data $d^{1:t} \equiv (y_t, u_t)$ include the measured outputs $y_t$ and the optional inputs $u_t$. Using the chain rule for pdfs, the joint pdf can be decomposed into product of the following models.

<u>The model of observation</u>

$$f\left(y_t \middle| u_t, d^{1:t-1}, x_t\right), \ t \in t^*, \tag{1}$$

which relates outputs $y_t$ to the current inputs $u_t$, the past data $d^{1:t-1}$, and current state $x_t$.

<u>The model of evolution</u>

$$f\left(x_t \middle| u_t, d^{1:t-1}, x_{t-1}\right), \ t \in t^*, \tag{2}$$

which describes time evolution of the state $x_t$.

<u>The model of strategy</u>

$$f\left(u_t \middle| d^{1:t-1}, x_t\right), \ t \in t^*, \tag{3}$$

which describes, generally randomized, generating of inputs $u_t$ based on $d^{1:t-1}, x_t$.

By definition, admissible strategies cannot exploit directly unobserved state, i.e., they have to meet so called *natural conditions of control* [12]

$$f\left(u_t \middle| d^{1:t-1}, x_t\right) = f\left(u_t \middle| d^{1:t-1}\right), \ t \in t^*. \tag{4}$$

**Proposition 2.1 (Bayesian prediction and filtering)** *Under natural conditions of control, the predictor* $f\left(y_t \middle| u_t, d^{1:t-1}\right)$ *is given by the formula*

$$f\left(y_t \middle| u_t, d^{1:t-1}\right) = \int f\left(y_t \middle| u_t, d^{1:t-1}, x_t\right) f\left(x_t \middle| u_t, d^{1:t-1}\right) dx_t. \tag{5}$$

*The pdf* $f\left(x_t \middle| u_t, d^{1:t-1}\right)$*, estimating the state* $x_t$*, evolves according the following coupled formulas.*

*Data updating*

$$f\left(x_t \middle| d^{1:t}\right) = \frac{f\left(y_t \middle| u_t, d^{1:t-1}, x_t\right) f\left(x_t \middle| u_t, d^{1:t-1}\right)}{f\left(y_t \middle| u_t, d^{1:t-1}\right)}, \propto f\left(y_t \middle| u_t, d^{1:t-1}, x_t\right) f\left(x_t \middle| u_t, d^{1:t-1}\right), \quad (6)$$

*that incorporates the experience contained in the data $d_t$ consisting of the output $y_t$ and the input $u_t$.*
*Time updating*

$$f\left(x_{t+1} \middle| u_{t+1}, d^{1:t}\right) \propto \int f\left(x_{t+1} \middle| u_{t+1}, d^{1:t}, x_t\right) f\left(x_t \middle| d^{1:t}\right) \, dx_t. \quad (7)$$

*The recursions start from the prior pdf $f\left(x_1 | u_1\right)$, that expresses the subjective prior knowledge on the initial state $x_1$.*

The application to Gaussian state-space model with Gaussian prior on $x_0$ and Gaussian observations provides Kalman filter [2].

## 3 GENERAL SOLUTION OF FACTORIZED FILTERING

The factorized version of the filter is obtained by applying chain rule to the time evolution and observation models, so that individual entries of state are modelled individually. It should be noted, that the proposed factorization can be made even more extensive by full factorizing the observation model; in this case the output entries are modelled individually too. Here, to simplify the presentation, this line will not be followed. But practical algorithms should deal with such a factorization too.

**Proposition 3.1 (Factorized prediction and filtering)** *The factorized prediction, under natural conditions of control and according to (5), is performed in the following way*

$$f\left(y_t \middle| u_t, d^{1:t-1}, x_t^{i+1:\mathring{x}}\right) = \int f\left(y_t \middle| u_t, d^{1:t-1}, x_t^{i:\mathring{x}}\right) f\left(x_t^i \middle| u_t, d^{1:t-1}, x_t^{i+1:\mathring{x}}\right) \, dx_t^i, \quad (8)$$

*where the predictor $f\left(y_t \middle| u_t, d^{1:t-1}\right)$ is the last pdf of the sequence of the partially conditioned predictors.*

*The pdfs $f\left(x_t^i \middle| u_t, d^{1:t-1}, x_t^{i+1:\mathring{x}}\right)$, $i = 1, \ldots, \mathring{x}$, determining the state estimate, are factorized through the chain rule as*

$$f\left(x_t \middle| u_t, d^{1:t-1}\right) = \prod_{i=1}^{\mathring{x}} f\left(x_t^i \middle| u_t, d^{1:t-1}, x_t^{i+1:\mathring{x}}\right). \quad (9)$$

*They evolve according the following coupled formulas.*
Factorized data updating

$$f\left(x_t^i \middle| d^{1:t}, x_t^{i+1:\mathring{x}}\right) = \frac{f\left(y_t \middle| u_t, d^{1:t-1}, x_t^{i:\mathring{x}}\right) f\left(x_t^i \middle| u_t, d^{1:t-1}, x_t^{i+1:\mathring{x}}\right)}{f\left(y_t \middle| u_t, d^{1:t-1}, x_t^{i+1:\mathring{x}}\right)}, \propto f\left(y_t \middle| u_t, d^{1:t-1}, x_t^{i:\mathring{x}}\right) f\left(x_t^i \middle| u_t, d^{1:t-1}, x_t^{i+1:\mathring{x}}\right), \quad (10)$$

*that incorporates the experience contained in the data $d_t$, consisting of the output $y_t$ and the input $u_t$.*
Factorized time updating

*The pdf $f\left(x_{t+1} \middle| u_{t+1}, d^{1:t}\right)$ is the last member of the sequence of the partially conditioned state estimates indexed by $j = 1, \ldots, \mathring{x}$*

$$f\left(x_{t+1} \middle| u_{t+1}, d^{1:t}, x_t^{j+1:\mathring{x}}\right) \propto \int f\left(x_{t+1} \middle| u_{t+1}, d^{1:t}, x_t^{j:\mathring{x}}\right) f\left(x_t^j \middle| d^{1:t}, x_t^{j+1:\mathring{x}}\right) \, dx_t^j. \quad (11)$$

*The recursions start from the prior pdfs $f\left(x_1 | u_1\right)$, that expresses the subjective prior knowledge on the initial sequence of internal quantities.*

*The factorized version of the state estimate after time-updating is obtained by straightforward application of the chain rule for pdfs.*

Proof: See [13].

## 4 APPLICATION TO GAUSSIAN MODELS

Here, Proposition 3.1 is applied to linear Gaussian state space model with Gaussian prior on initial state. This state estimation is known to be described by Kalman filter [14].

Straightforward proofs of the recursions, that will be offered below, are based on the use of operations of square completion and integration of non-normalized Gaussian pdf. Because of the limitation of the place, the proofs will not be given at the present paper, but they can be found in [13]. The proposition of completion of squares follows.

**Proposition 4.1 (Square completion and integration of Gaussian pdf)** *For real scalars $x, \alpha, \beta, \gamma$ and positive scalars $r, p$, it holds*

$$\begin{aligned} h(x) &\equiv \exp\left\{-\frac{(\beta-\gamma x)^2}{2r} - \frac{(x-\alpha)^2}{2p}\right\} = \exp\left\{-\frac{(x-\hat{x})^2}{2R} - \frac{\lambda}{2}\right\} \textit{with} \\ R &= \frac{rp}{r+\gamma^2 p},\ \hat{x} = \frac{\alpha r + \beta\gamma p}{r+\gamma^2 p},\ \lambda = \frac{(\beta-\alpha\gamma)^2}{r+\gamma^2 p}, \\ \int h(x)\,dx &= \sqrt{2\pi R}\exp\left\{-\frac{\lambda}{2}\right\}. \end{aligned} \tag{12}$$

Proof: See [13].

The state estimate is assumed in the form

$$f\left(x_t^i \middle| u_t, d^{1:t-1}, x_t^{i+1:\mathring{x}}\right) = \mathcal{N}_{x_t^i}\left(\hat{\mu}_{t|t-1;i} + \sum_{k=i+1}^{\mathring{x}} g_{t|t-1;ik} x_{t;k}, p_{t|t-1;i}\right), \tag{13}$$

where $\hat{\mu}_{t|t-1;i}$ is the term, depending on the data $u_t, d^{1:t-1}$ only, $g_{t|t-1;ik}$ are coefficients, which are data and state independent similarly as the variance $p_{t|t-1;i} > 0$.

For presentation simplicity, the system with single output, $\mathring{y} = 1$, is considered. This helps us to avoid consequences of the incomplete factorization, mentioned in Section 3. Thus, we assume

$$f\left(y_t \middle| u_t, d^{1:t-1}, x_t\right) = \mathcal{N}_{y_t}\left(\rho_t + \sum_{k=1}^{\mathring{x}} c_{t;k} x_{t;k}, r_t\right), \tag{14}$$

where the offset $\rho_t$, coefficients $c_{t;k}$, $k = 1, \ldots, \mathring{x}$, and variance $r_t$ are assumed to be known functions of $u_t, d^{1:t-1}$.

**Proposition 4.2 (Partially conditioned Gaussian observation models)** *For the Gaussian factors of the state estimate (13) and the observation model (14), it holds*

$$f\left(y_t \middle| u_t, d^{1:t-1}, x_t^{i:\mathring{x}}\right) = \mathcal{N}_{y_t}\left(\rho_{t;i} + \sum_{k=i}^{\mathring{x}} c_{t;ik} x_{t;k}, r_{t;i}\right), \tag{15}$$

*where the state independent offsets $\rho_{t;i}$, coefficients $c_{t;ik}$, $k = i, \ldots, \mathring{x}$, evolve according to the following recursions, for $i = 1, \ldots, \mathring{x}$*

$$\begin{aligned} \rho_{t;i+1} &= \rho_{t;i} + c_{t;ii}\hat{\mu}_{t|t-1;i} \\ c_{t;(i+1)k} &= c_{t;ik} + c_{t;ii} g_{t|t-1;ik}, \text{ for } k > i \\ r_{t;i+1} &= r_{t;i} + p_{t|t-1;i} c_{t;ii}^2. \end{aligned} \tag{16}$$

*The recursions start from $\rho_{t;1} = \rho_t$, $c_{t;1k} = c_{t;k}$ and $r_{t;1} = r_t$.*

Proof: See [13].

The recursions contain no numerically dangerous operation.

**Proposition 4.3 (Factorized data updating)** *The functional form of the state estimate (13) preserves in data updating. Specifically, after data updating it is obtained*

$$f\left(x_t^i \middle| d^{1:t}, x_t^{i+1:\mathring{x}}\right) = \mathcal{N}_{x_{t;i}}\left(\hat{\mu}_{t|t;i} + \sum_{k=i+1}^{\mathring{x}} g_{t|t;ik} x_{t;k}, p_{t|t;i}\right), \tag{17}$$

*with*

$$\begin{aligned} K_{t|t;i} &\equiv \frac{c_{t;ii}p_{t;i}}{r_{t;i+1}} \\ \hat{\mu}_{t|t;i} &= \hat{\mu}_{t|t-1;i} + K_{t|t;i}(y_t - \rho_{t;i} - c_{t;ii}\hat{\mu}_{t|t-1;i}) \\ g_{t|t;ik} &= g_{t|t-1;ik} - K_{t|t;i}(c_{t;ik} + c_{t;ii}g_{t|t-1;ik}) \text{ for } k > i \\ p_{t|t;i} &= \frac{r_{t;i}}{r_{t;i+1}} p_{t|t-1;i} \end{aligned} \tag{18}$$

Proof: See [13].

For time updating, we have to evaluate partially conditioned linear Gaussian time evolution model. The chain rule implies, that the fully conditioned model can be always given by the form

$$\begin{aligned} f\left(x_{t+1} \middle| u_{t+1}, d^{1:t}, x_t\right) &= \prod_{i=1}^{\mathring{x}} f\left(x_{t+1}^i \middle| x_{t+1}^{i+1:\mathring{x}}, u_{t+1}, d^{1:t}, x_t\right) \\ &= \prod_{i=1}^{\mathring{x}} \mathcal{N}_{x_{t+1}^i}\left(\zeta_{t+1;i} + \sum_{k=i+1}^{\mathring{x}} \alpha_{t+1;ik}x_{t+1;ik} + \sum_{k=1}^{\mathring{x}} \beta_{t+1;ik}x_{t;k}, R_{t+1;i}\right), \end{aligned} \tag{19}$$

where for all $i \in \{1, \ldots, \mathring{x}\}$ the offset $\zeta_{t+1;i}$, coefficients $\alpha_{t+1;ik}$ with $k = i+1, \ldots, \mathring{x}$, $\beta_{t+1;ik}$ with $k = 1, \ldots, \mathring{x}$ and variances $R_{t+1;i}$ are assumed to be known functions of $u_{t+1}, d^{1:t}$.

The scalar variable $x_t^j$ to be integrated out occurs now in all factors in (19). Thus, for proofs of the recursions of the factorized time updating the modified version of Proposition 4.1 is to be used.

**Proposition 4.4 (Integration of a product of Gaussian pdfs)** *For real scalar $x$, vectors $\beta \equiv [\beta_1, \ldots, \beta_{\mathring{\beta}}]'$, $\gamma \equiv [\gamma_1, \ldots, \gamma_{\mathring{\beta}}]'$ and diagonal precision matrix $\omega \equiv$ diag $\left[r_1^{-1}, \ldots, r_{\mathring{\beta}}^{-1}\right]$, it holds*

$$\begin{aligned} \int h(x)\, dx &\equiv \int \exp\left\{-\sum_{i=1}^{\mathring{\beta}} \frac{(\beta_i - \gamma_i x)^2}{2r_i}\right\} dx \propto \exp\left\{-\frac{\lambda}{2}\right\} \textit{with} \\ \lambda &\equiv \beta'\left(\omega - \frac{\omega\gamma\gamma'\omega}{\gamma'\omega\gamma}\right)\beta \equiv \sum_{l=1}^{\mathring{\beta}-1} \frac{\left(\sum_{i=l}^{\mathring{\beta}} U_{li}\beta_i\right)^2}{p_l}, \textit{ where} \end{aligned} \tag{20}$$

*the upper triangular $(\mathring{\beta} - 1, \mathring{\beta})$ matrix $U$ with unit diagonal is found via $U'DU$ decomposition*

$$U' \mathrm{diag}\left[p_1^{-1}, \ldots, p_{\mathring{\beta}}^{-1}\right] U = \omega - \frac{\omega\gamma\gamma'\omega}{\gamma'\omega\gamma}. \tag{21}$$

The relation is obtained by completion of squares with respect to the scalar $x$ and integration of univariate Gaussian pdf. The mentioned matrix decomposition can be performed similarly to the algorithm REFIL [12].
Proof: See [13].

**Proposition 4.5 (Partially conditioned Gaussian time-evolution models, Factorized time-updating)** *For the Gaussian factors of the state estimate (17) and the time evolution model (19), it holds*

$$\begin{aligned} & f\left(x_{t+1;i} \middle| x_{t+1}^{i+1:\mathring{x}}, u_{t+1}, d^{1:t}, x_t^{j+1:\mathring{x}}\right) \\ = \;& \mathcal{N}_{x_{t+1;i}}\left(\hat{\mu}_{t+1|t;ij} + \sum_{k=i+1}^{\mathring{x}} g_{t+1|t;ikj}x_{t+1;k} + \sum_{k=j+1}^{\mathring{x}} \beta_{ikj}x_{t;k}, p_{t+1|t;ij}\right), \end{aligned} \tag{22}$$

*where for $i \in \{1, \ldots, \mathring{x}\}$ offsets $\hat{\mu}_{t+1|t;ij}$, coefficients $g_{t+1|t;ikj}$ with $k = i+1, \ldots, \mathring{x}$, $\beta_{ikj}$ with $k = j+1, \ldots, \mathring{x}$ and variances $p_{t+1|t;ij} > 0$ are the state independent. The following recursions over $j = 1, \ldots, \mathring{x}$ hold*

$$\begin{aligned} \hat{\mu}_{t+1|t;i(j+1)} &= \hat{\mu}_{t+1|t;ij} + \sum_{l=i+1}^{\mathring{x}} U_{il}\hat{\mu}_{t+1|t;lj} - U_{i(\mathring{x}+1)}\hat{\mu}_{t|t;j} \\ g_{t+1|t;ik(j+1)} &= g_{t+1|t;ikj} + \sum_{l=i-1}^{\mathring{x}} U_{il}g_{t+1|t;lkj} - \sum_{l=i+1}^{\mathring{x}} U_{il} \end{aligned} \tag{23}$$

$$\beta_{ik(j+1)} = \beta_{ikj} + \sum_{l=i+1}^{\mathring{x}} U_{il}\beta_{lkj} - U_{i(\mathring{x}+1)}g_{t|t;jk}, \; k > j. \tag{24}$$

*The upper triangular* $(\mathring{x}, \mathring{x}+1)$ *matrix* $U$ *with unit diagonal as well as the positive scalars* $p_{t+1|t;i(j+1)}$ *are obtained via the decomposition*

$$\begin{aligned} \omega - \frac{\omega\gamma\gamma'\omega}{\gamma'\omega\gamma} &\equiv U'\mathrm{diag}\left[p^{-1}_{t+1|t;1(j+1)}, \ldots, p^{-1}_{t+1|t;\mathring{x}(j+1)}\right] U \textit{ with} \\ \gamma' &\equiv [\beta_{1jj}, \ldots, \beta_{\mathring{x}jj}, 1] \\ \omega &\equiv \mathrm{diag}\left[p^{-1}_{t+1|t;1j}, \ldots, p^{-1}_{t+1|t;\mathring{x}j}, p^{-1}_{t|tj}\right]. \end{aligned} \tag{25}$$

*The recursions start from* $\hat{\mu}_{t+1|t;i1} = \zeta_{t+1;i}$, *and* $p_{t+1|t;i1} = R_{t+1;i}$, $g_{t+1|t;ik1} = \alpha_{t+1;ik}$ *and* $\beta_{ik1} = \beta_{t+1;ik}$ .

*The result, obtained after the step* $\mathring{x}$, *provides parameters of the time-updated factors.*

Proof: See [13].

## 5 CONCLUSION

The paper proposes solution to the factorized filtering, obtained by applying the chain rule to the state-space models. Factorized filter provides the update of posterior probability density functions for the individual state entries, that can be helpful for solution to the task of the joint modelling of the mixed-type data. The recursions for calculating the factorized data updating and time updating are offered. The application of the solution is shown at the example of the linear Gaussian single output system, which gives the factorized Kalman filtering. For the factorized Kalman filtering the operations of completion of square and integration of non-normalized Gaussian probability density functions are used.

Among the advantages of the proposed approach one may note, unlike the previous solution of the factorized filtering, offered in [11], the present work does not require any restrictions for the state-space models. It should be also noted, that the proposed recursions do not contain any numerically dangerous operations.

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