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Classification of degraded signals by the method of invariants

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Abstract

A novel approach to the recognition of the signals degraded by a linear time-invariant system with an unknown impulse response is proposed. It consists of describing the signals by the features which are invariant to the degradation and recognizing signals in the feature space. Unlike the blind-deconvolution techniques, neither the impulse response identification nor the signal restoration is performed. Two sets of appropriate blur-invariant features (the first one defined in time domain and the other one in spectral domain) are introduced in this paper and the optimal algorithm for a robust signal classification is proposed. © 1997 Elsevier Science B.V.

Zusammenfassung

Es wird ein neuartiges Verfahren zur Erkennung von Signalen vorgestellt, die durch ein lineares zeitinvariantes System mit unbekannter Impulsantwort gestört wurden. Das Verfahren beinhaltet einerseits die Beschreibung der Signale anhand von Merkmalen, die invariant gegenüber der Störung sind und andererseits die Erkennung der Signale im Merkmalsraum. Im Unterschied zu Techniken der blinden Entfaltung wird weder eine Identifikation der Impulsantwort noch eine Signalrückgewinnung vorgenommen. In diesem Artikel werden zwei Sätze geeigneter verzerrungsinvarianter Merkmale vorgestellt (wobei einer im Zeitbereich und der andere im Spektralbereich definiert wird) und anschließend ein optimaler Algorithmus zur robusten Signalklassifikation vorgeschlagen. © 1997 Elsevier Science B.V.

Résumé

Cet article propose une nouvelle approche de la reconnaissance de signaux dégradés par un système invariant dans le temps linéaire à réponse impulsionelle inconnue. Cette approche consiste à décrire les signaux à l'aide de caractéristiques invariantes à la dégradation et à reconnaître ces signaux dans l'espace des caractéristiques. A la différence des techniques de déconvolution à l'aveugle, ni l'identification de la réponse impulsionelle ni la restauration du signal ne sont effectuées. Deux ensembles de caractéristiques appropriées invariantes au flou (le premier défini dans le domaine temporel, le second dans le domaine spectral) sont introduits, et nous proposons l'algorithme optimal pour une classification robuste des signaux. © 1997 Elsevier Science B.V.

Keywords: Blurred signal; Signal recognition; Moments; Blur invariants

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1. Introduction

A very frequent task in digital signal processing is the classification of 1-D finite (i.e. time-limited) signals (experimental curves) with respect to the template curves stored in a database. This task appears in EEG and ECG processing, speech recognition as well as in many other application areas.

Since the acquisition system and other conditions are usually not ideal, the acquired curve represents only some degraded version of the original signal. The standard model of a time-invariant linear system [3] describes the acquisition process by the convolution of an unknown original signal $f(t)$ with the system impulse response $h(t)$:

$$g(t) = (f * h)(t) + n(t), \qquad (1)$$

where $g(t)$ represents the observed (i.e. blurred) signal and $n(t)$ is an additive random noise. However, the impulse response $h(t)$ is unknown in most cases. Our objective is to analyze the original signal $f(t)$.

There are basically two different approaches to degraded signal analysis: blind restoration and direct analysis.

Blind signal restoration has been discussed extensively in previous works (see [7] for a survey). It is the process of estimating both the original signal and the PSF from the degraded signal using partial information about the acquisition system. However, this is an ill-posed problem, which does not have a unique solution and the computational complexity of which might be extremely high.

There are several groups of blind restoration methods. One of them is based on the modeling of signals by stochastic processes. The original signal is modeled as an autoregressive (AR) process and the blur as a moving average (MA) process. The blurred signal is then modeled as a mixed autoregressive moving average (ARMA) process and the MA process identified by this model is considered as the description of the PSF. In this way the problem of PSF estimation is transformed into the problem of determining the parameters of the ARMA model [8,9,11]. Other authors suggested blind deconvolution methods based on cumulant extrema [2] or various techniques of the impulse response estimation in frequency domain [1,10].

A direct analysis of the degraded signal is based on a different idea: in many cases, one does not need to known the whole original signal, one only needs to recognize, for instance, some part of it (typical examples are the classification of experimental curves against a database of templates or recognition of blurred characters). In such cases, knowledge of only some (partial) representation of the signals is sufficient. However, such a representation should be independent of the acquisition system and should really describe the original signal, not the degraded one. In other words, we are looking for a functional I which is invariant to convolution, i.e.

$$I(f) = I(f * h)$$

for any allowable $h(t)$. The blurred signal is then classified in the Euclidean feature space (by minimum distance rule for instance) without any impulse response identification and signal restoration.

There have been described only few invariants to blurring in the literature. Most of them are related to very special types of the impulse responses in 2-D and are derived in a heuristic manner. The set of invariants to linear motion blur was presented in [5]. Recognition of defocused facial photographs by another set of invariants was described in [6]. All those invariants were constructed in spatial domain only.

The major objective of this paper is to derive blur invariants in the case of symmetric impulse response $h(t)$ both in Fourier as well as time domains, to demonstrate a relationship between them and to introduce a novel algorithm for signal classification.

2. Signal characterization by its moments

In the following text, by *signal* we understand any absolutely integrable function $f(t)$ which is non-zero on bounded support and the integral of which is non-zero:

$$\int_{-\infty}^{\infty} f(t)\,\mathrm{d}t \neq 0.$$

Its pth-order regular moment $m_p^{(f)}$ and central moment $\mu_p^{(f)}$ are defined as

$$m_p^{(f)} = \int_{-\infty}^{\infty} t^p f(t)\,\mathrm{d}t, \tag{2}$$

$$\mu_p^{(f)} = \int_{-\infty}^{\infty} (t - c^{(f)})^p f(t)\,\mathrm{d}t, \tag{3}$$

where $c^{(f)}$ is the mean value (centroid) of $f(t)$.

The moments of a blurred signal $g(t) = (f * h)(t)$ can be expressed in terms of moments of the original signal and the impulse response

$$\mu_p^{(g)} = \sum_{k=0}^{p} \binom{p}{k} \mu_{p-k}^{(f)} \mu_k^{(h)}. \tag{4}$$

3. Invariants in the time domain

In this section, we derive the moment-based signal features which are independent of the degradation, i.e. independent of the type and parameters of $h(t)$.

Let us deal with *symmetric* and *energy-preserving* impulse responses only, i.e. let $h(t) = h(-t)$ and $\mu_0^{(h)} = 1$. Then, due to the symmetry, $\mu_p^{(h)} = 0$ if p is odd.

Theorem 1. *Let $f(t)$ be a signal and p be an odd integer. Let us define the following function $S^{(f)}: \mathrm{N} \to \mathrm{R}$:*

$$S_p^{(f)} = \mu_p^{(f)} - \frac{1}{\mu_0^{(f)}} \sum_{n=1}^{(p-1)/2} \binom{p}{2n} S_{p-2n}^{(f)} \mu_{2n}^{(f)}. \tag{5}$$

Then S_p is a blur invariant for any odd p.

For the proof of this theorem see our recent paper [4].

Evaluating the recursive formula (5) we can derive the invariants in the explicit form. The first four of them are listed below:

$$S_1 = 0,$$

$$S_3 = \mu_3,$$

$$S_5 = \mu_5 - \frac{10\mu_3\mu_2}{\mu_0},$$

$$S_7 = \mu_7 - \frac{21\mu_5\mu_2}{\mu_0} - \frac{35\mu_3\mu_4}{\mu_0} + \frac{210\mu_3\mu_2^2}{\mu_0^2}.$$

If we use the regular moments instead of the central ones, Theorem 1 is still valid but the invariants S_p are no longer invariant to a time-shift.

4. Signal classification

In this section, we introduce an algorithm for robust signal classification by the blur invariants.

Although continuous signals were used to derive the invariants, in practice, we deal with discrete ones. The well-known approximation formula

$$m_p = \sum_{j=1}^{N} j^p f(j)$$

is usually employed to calculate the discrete moments and the moment invariants (N denotes the number of samples of the signal). Moreover, in practical tasks, we use *normalized invariants*

$$S'_p = \frac{S_p}{\mu_0 (N/2)^p},$$

which are invariant to multiplication of the signal by a constant and, due to the factor $(N/2)^p$, the values of which are roughly in the same range regardless of p.

Let $f_1, \ldots, f_K$ be given discrete signals of the same length N and let g be an acquired signal of the same length. In pattern recognition terminology, $f_1, \ldots, f_K$ are the representatives of K individual classes and g is the pattern to be classified. The task is to find such index i_0 so that for some symmetric h it holds

$$g = h * f_{i_0} + n$$

with maximum likelihood.

Let us define for any odd p the distance d_p in the signal space as

$$d_p(f,g) = \varrho_p(\overline{S_p^{(f)}}, \overline{S_p^{(g)}}),$$

where $\overline{S_p}$ is a vector of the normalized blur invariants $\overline{S_p} = (S'_1, S'_3, \ldots, S'_p)$ and ϱ_p is Euclidean metric in $l_2((p+1)/2)$ space. The distance d_p has the properties of a quasi-metric: it is non-negative, symmetric function which satisfies the triangular inequality. However, $d_p(f,g) = 0$ does not imply $f = g$.

Now we classify the signal g by minimum-distance rule:

$$d_p(f_{i_0}, g) = \min_{i=1,\ldots,K} d_p(f_i, g).$$

The only open question is how to choose the appropriate p, i.e. how many invariants to use for classification. The solution depends on the given templates $f_1, \ldots, f_K$ and can be found as follows.

In the noise-free case we define the optimal p_0 as the lowest p for which

$$d_p(f_i, f_j) > 0 \quad \forall (i,j)(i \neq j).$$

The situation is more complicated if the acquired signal g is corrupted by an additive random noise. It is well known that the higher-order moments are less robust to noise than the lower-order ones. It implies that the higher-order blur invariants are less robust too. On the other hand, some signals which are 'similar' to each other can be distinguished only by the higher-order invariants.

Provided that the noise has normal distribution with zero mean and that signal-to-noise ratio (SNR) is known (or that we are able to estimate it somehow) we propose the following algorithm for determining the optimal number of the invariants:

(1) For each f_i generate M its noisy versions $f_i^{(1)}, \ldots, f_i^{(M)}$, where

$$f_i^{(m)} = f_i + n^{(m)},$$

such that the SNR of each $f_i^{(m)}$ is the same as the SNR of the signal g. The $n^{(m)}$ is a realization of a zero-mean Gaussian noise and M is a user-defined parameter.

(2) Choose the upper bound P of the number of the invariants you want to consider.

(3) FOR $p = 1, 3, \ldots, P$ DO

(a) For each $i = 1, \ldots, K$ calculate r_p^i such that the total number of $f_i^{(m)}$ having the distance from f_i less than r_p^i is greater than $0.95M$. Provided that $f_i^{(m)}$ are normally distributed around f_i in the feature space, r_p^i can be estimated as follows:

$$r_p^i = \frac{2.46}{M} \sum_{m=1}^{M} d_p(f_i, f_i^{(m)}).$$

(b) IF

$$2r_p^i < d_p(f_i, f_j) \quad \forall (i,j)(i \neq j),$$

THEN

$$E_p = \frac{\sum_{i=1}^{K} r_p^i}{\sum_{i=1}^{K} \sum_{j=1}^{i-1} d_p(f_i, f_j)},$$

ELSE $E_p = 1$.

ENDFOR

(4) Find p_0 such that

$$E_{p_0} = \min_p E_p.$$

The p_0 obtained by this algorithm ensures the optimal separability of the templates $f_1, \ldots, f_K$ in a noisy environment. If the SNR is high, p_0 also becomes high and vice versa.

5. Invariants in the spectral domain

In Section 3, we introduced the blur invariants working in the time domain. In this section, we show that another set of invariants can be found in the Fourier spectral domain. Moreover, we demonstrate a close relationship between both kinds of the invariants and prove that they are theoretically equivalent.

Theorem 2. *Tangent of the Fourier transform phase is the blur invariant.*

Proof. Due to the well-known convolution theorem, the corresponding relation to Eq. (1) in the spectral domain (provided that no noise is present) has the form

$$G(u) = F(u)H(u), \tag{6}$$

where $G(u), F(u)$ and $H(u)$ are the Fourier transforms of the functions $g(t), f(t)$ and $h(t)$, respectively. Considering only the phase, we get (provided that $G(u) \neq 0$)

$$\operatorname{ph} G(u) = \operatorname{ph} F(u) + \operatorname{ph} H(u). \tag{7}$$

Due to the symmetry of $h(t)$, its Fourier transform $H(u)$ is real (that means $\text{ph}\,H(u) \in \{0;\pi\}$). It follows immediately from the periodicity of tangent that

$$\tan(\text{ph}\,G(u)) = \tan(\text{ph}\,F(u) + \text{ph}\,H(u)) = \tan(\text{ph}\,F(u)). \tag{8}$$

Thus, $\tan(\text{ph}\,G(u))$ is invariant with respect to convolution of the original signal with any symmetric impulse response. □

The following theorem shows the relationship between the time-domain and the spectral-domain blur invariants.

Theorem 3. *Tangent of the Fourier transform phase of any signal $f(t)$ can be expanded into the power series (except the points in which $F(u) = 0$ or $\text{ph}\,F(u) = \pm\pi/2$):*

$$\tan(\text{ph}\,F(u)) = \sum_{n=0}^{\infty} c_n u^n, \tag{9}$$

where

$$c_n = \frac{(-1)^{(n-1)/2}(-2\pi)^n}{n!\,m_0} S_n \tag{10}$$

if n is odd and $c_n = 0$ if n is even.

Proof. The definition of the Fourier transform implies that the spectrum of any signal $f(t)$ can be expressed by means of moments as the power series:

$$F(u) = \int_{-\infty}^{\infty} f(t)e^{-2\pi i u t}\,dt = \int_{-\infty}^{\infty} f(t)\sum_{n=0}^{\infty}\frac{(-2\pi i u)^n}{n!}t^n\,dt = \sum_{n=0}^{\infty}\frac{(-2\pi i)^n}{n!}m_n u^n.$$

Thus,

$$\tan(\text{ph}\,F(u)) = \frac{\text{Im}\,F(u)}{\text{Re}\,F(u)} = \frac{\sum_{n=0}^{\infty}(-1)^n(-2\pi)^{2n+1}m_{2n+1}u^{2n+1}/(2n+1)!}{\sum_{n=0}^{\infty}(-1)^n(-2\pi)^{2n}m_{2n}u^{2n}/(2n)!}.$$

Since the series in the numerator and the denominator are absolutely convergent, their ratio can also be expressed as the power series

$$\tan(\text{ph}\,F(u)) = \sum_{n=0}^{\infty} c_n u^n, \tag{11}$$

which must accomplish the relation

$$\sum_{n=0}^{\infty}\frac{(-1)^n(-2\pi)^{2n+1}}{(2n+1)!}m_{2n+1}u^{2n+1} = \sum_{n=0}^{\infty}c_n u^n \sum_{n=0}^{\infty}\frac{(-1)^n(-2\pi)^{2n}}{(2n)!}m_{2n}u^{2n}. \tag{12}$$

It follows immediately from (12) that $c_n = 0$ for any even n.

Let us the prove by induction that c_n has the form (10).

- $n = 1$

  It follows from (12) that $c_1 = -2\pi m_1/m_0$. On the other hand,

  $$\frac{(-1)^0(-2\pi)^1}{1!\,m_0}S_1 = \frac{-2\pi m_1}{m_0} = c_1.$$

- Let us suppose the assertion has been proven for $c_1, c_3, \ldots, c_{p-2}$. It follows from (12) that

  $$\frac{(-1)^{(p-1)/2}(-2\pi)^p}{p!}m_p = c_p m_0 + \sum_{n=1}^{(p-1)/2}\frac{(-1)^n(-2\pi)^{2n}}{(2n)!}c_{p-2n}m_{2n}.$$

  Substituting (10) into the right-hand side we get

  $$c_p m_0 = \frac{(-1)^{(p-1)/2}(-2\pi)^p}{p!}m_p - \sum_{n=1}^{(p-1)/2}\frac{(-1)^{(p-1)/2}(-2\pi)^p}{(2n)!(p-2n)!\,m_0}S_{p-2n}m_{2n},$$

  $$c_p = \frac{(-1)^{(p-1)/2}(-2\pi)^p}{p!\,m_0}\cdot\left(m_p - \frac{1}{m_0}\sum_{n=1}^{(p-1)/2}\binom{p}{2n}S_{p-2n}m_{2n}\right),$$

  $$c_p = \frac{(-1)^{(p-1)/2}(-2\pi)^p}{p!\,m_0}S_p. \quad \square$$

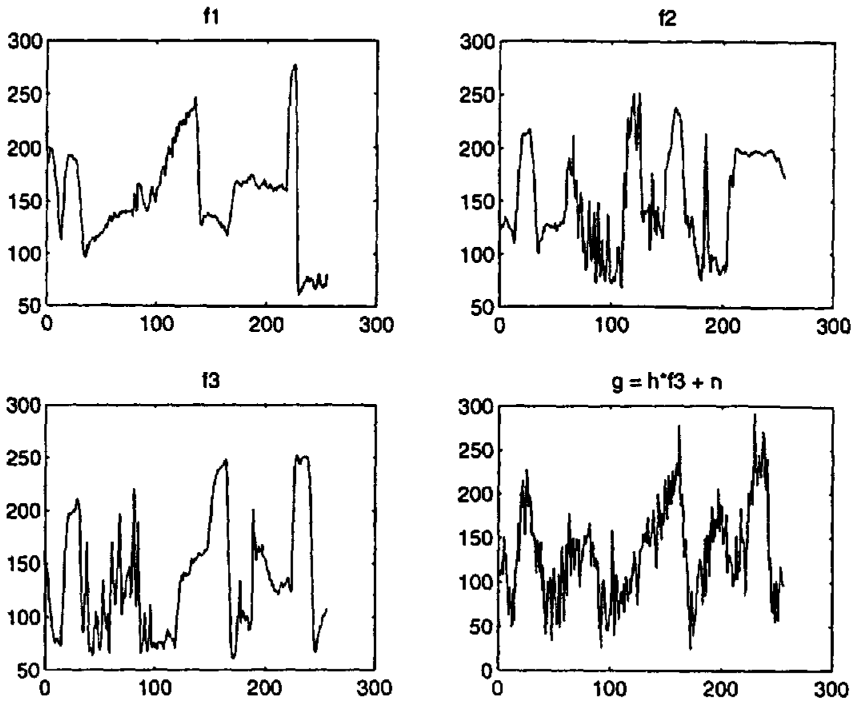

Fig. 1. The signals used in the experiment: the templates f_1, f_2 and f_3 and the blurred and noisy signal g.

6. Numerical experiment

To demonstrate the performance of the above-mentioned technique, the following experiment was carried out.

Fig. 1 shows three template signals f_1, f_2 and f_3 of the length $N = 256$ samples. The signal g (Fig. 1 bottom right) was generated from f_3 by averaging over 9-point neighborhood and adding Gaussian noise (SNR = 10 dB). The question was to assign g to its non-degraded counterpart by means of the time-domain blur invariants.

Table 1
The values of the normalized invariants S'_p (multiplied by 10^2) of the signals from Fig. 1

	f_1	f_2	f_3	g
S'_3	1.55	3.18	5.66	5.67
S'_5	1.85	4.55	7.28	7.19
S'_7	1.69	5.29	7.97	7.77
S'_9	1.39	5.73	8.37	8.03
S'_{11}	0.98	6.04	8.70	8.20

Table 2
The distances (multiplied by 10^2) between the blurred and noisy signal g and the templates f_1, f_2, f_3, respectively

	f_1	f_2	f_3
$d_{11}(f_i, g)$	13.3	5.41	0.64

First, the algorithm described in Section 4 was employed to calculate the optimal number of invariants. The result in this case was $p_0 = 11$. Second, the invariants $S'_3, S'_5, \ldots, S'_{11}$ were calculated for each signal (we do not use S'_1, because it is zero everywhere). Their values are summarized in Table 1. Table 2 shows the distances $d_{11}(f, g)$ between g and each template. Classifying by minimum distance, the signal g was assigned correctly to f_3.

7. Summary

In this paper, the new technique for recognition of signals degraded by a linear time-invariant system is presented. The proposed approach does not require impulse response identification and signal restoration.

Two groups of the invariant features for signal representation were introduced. It has been shown that both groups of features are theoretically equivalent in the following sense: a set $\{S_p\}_{p=1}^{\infty}$ of moment invariants in the time domain is unambiguously determined by phase tangent and vice versa.

The novel algorithm for robust signal classification in the space of the invariants is also presented. To demonstrate practical applicability of the theoretical results of the paper, they were approved by the numerical experiment.

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