Decentralized Stabilization of Discrete-Time Networked Strongly Coupled Complex Systems

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Abstract: In this paper, the authors present an approach to decentralized stabilization with delayed feedback for a class of networked discrete-time complex systems. A class of dynamic discrete-time systems with identical linear nominal subsystems, symmetric nominal interconnections, and nonlinear perturbations is considered. The proposed method is based on particular structural properties of these systems which enable to construct a reduced order control design model with equivalent dynamic properties as the original system. Then, the standard method of linear matrix inequalities is used to design the gain matrix for such reduced model. The effect of data-packet dropout and communication delays between the plant and the controller is included in the controller design. It is shown how this methodology can simplify the control design with time-varying delay in the input. For such a purpose, a delay-dependent approach is applied in order to obtain a robustly delay-dependent stable overall closed-loop system with a decentralized controller.

Keywords: Decentralization, large-scale systems, complex systems, discrete-time systems, networked control systems

INTRODUCTION

A control system is called a networked control system (NCS) if its feedback loops are closed-via a shared communication medium. The medium limited capacity in a networked control system must be allocated to all feedback loop components, i.e. the sensors, controller, and actuators. Therefore, various communication constraints such as delays, dropouts, data rate limitations, or quantization effects are potential source of problems influencing on closed-loop system stability and performance. There are two general NCS configurations: Direct structure and Hierarchical structure. The NCS in the direct structure is composed of a controller and a remote system containing a physical plant, sensors, and actuators. The controller and the plant are physically located at different places. They are linked by a data network in order to operate in a remote closed-loop. The basic hierarchical structure consists of a main controller and a remote closed-loop system. The main controller computes and sends the reference signal via a network to the remote controller. The remote system then processes this signal to perform local closed-loop control and sends sensor measurement to the main controller for networked closed-loop control. This paper is focused on the direct structure and deals with the inclusion of delays and dropouts in the feedback loop.

Prior Work

Recent surveys on the emerging area of networked control systems (NCS) can be found in Hristu-Varsakelis and Levine [2005], Antsaklis and Tabuada [2006], Matveev and Savkin [2009], but there is no unified and complete theory in this subject.

Reference Xiong and Lam [2007] is focused on stabilization of NCS from the point of view of zero-order hold. The reference Hu et al. [2007] deals with time-driven digital controller and event-driven holder for NCS. It includes the possibility to deal with time delays and packet dropouts. Stabilization of discrete-time networked control systems is presented in Zhou et al. [2008] and Yu et al. [2004], while the references Stipanović and Šiljak [2001] and Ho and Lu [2003] deal with stabilization by using the LMI approach.

Decentralization generally means that the overall system task can be decomposed into several subsystem tasks so that the solution of subsystem tasks satisfactorily solves the overall system task. Decentralized NCS (DNCS) are the control systems with multiple control stations while transmitting control signals through a network, i.e. date signals are transmitted to multiple controllers in

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the feedback loop. DNCS combine the advantages of the centralized NCS and the decentralized control systems. Such a combination enables to cut unnecessary wiring, reduce the complexity and the overall system cost when designing and implementing control systems. Recently, the results dealing with the DNCS design methods are rare. Relevant problems are introduced in Bakule [2008], Xu and Hespanha [2004]. Decentralized stabilization of NCS using periodically time-varying local controller is presented in Jiang et al. [2008]. References Matveev and Savkin [2009], Nair et al. [2004],Yüksel and T. Başar [2003], Yüksel and T. Başar [2006], Yüksel and Başar [2007] consider the DNCS under date rate constraints, while stability of the DNCS is analyzed in Wei [2008]. Reference Bakule and de la Sen [2009b] deals with continuous-time DNCS for a class of complex composite systems.

Composite systems with a symmetric structure can be found in many different areas of real world systems. It includes for instance industrial manipulators, parallel processes, flexible structure, electric power systems, homogeneous interconnected systems such as seismic cables or the design of reliable control systems. A more complete survey of theoretic and applied results is presented in Bakule [2008], Hovd and Skogestad [1994] with the references therein.

The paper deals with the complexity reduced DNCS design for a class of nonlinear discrete-time symmetric composite systems by using direct configuration in the feedback loop with the focus on network dropouts and communication delays. State space approach in the discrete-time domain with a time-varying delay varying within a given interval is considered in the feedback loop.

The paper extends the results from Yu et al. [2004], Zhou et al. [2008], Bakule and de la Sen [2009a] and Bakule and de la Sen [2009b] into the DNCS design for nonlinear discrete-time symmetric composite systems using the reduced-order centralized NCS design when considering the delay-dependent approach within the framework of the LMIs.

Outline of the Paper

The paper presents the method for the decentralized state feedback stabilizing NCS design for a class of nonlinear discrete-time networked symmetric composite systems, when considering a direct configuration in the feedback loop. It means that the plant and the controller are connected through a network. This network is modelled as bounded packet dropouts and communication delays. A single packet transmission approach with the acknowledgements of successfully transmitted sensor signal to the buffer is used. First, the overall systems is transformed into a reduced-order system with equivalent dynamic properties as the original overall system. Then, the gain matrix is designed for this NCS design model by using the LMI based delay-dependent stability approach. The main result is presented in the form of a sufficient condition. It presents the results that when implementing the selected gain matrix as identical local controllers into the overall system, then the overall closed-loop DNCS is robustly delay-dependent stable. The result is proved by using the Liapunov-Krasovski stability approach.

1. PROBLEM FORMULATION

2.1 Structured System Description

Consider a nonlinear symmetric composite system consisting of N subsystems with the ith subsystem described as

$$\begin{aligned} x_i(k+1) = Ax_i(k) + Bu_i(k) + f_i(k, x_i) + s_i(k) \\ x_i(k_o) = x_{io} \\ i = 1, \ldots, N \qquad N > 2 \end{aligned} \tag{1}$$

where $x_i(k) \in \mathbb{R}^n$ is the subsystem state, $u_i(k) \in \mathbb{R}^m$ is the subsystem input, and $s_i(k) \in \mathbb{R}^n$ is the subsystem interconnection input at $k \in \mathbb{Z}_+$ The interconnections are described in the form

$$s_i(k) = \sum_{j=1, i \neq j}^{N} L_{ij} x_j(k) + f_{ij}(k, x_j) \tag{2}$$

Assumption 1. The interconnection matrices L_{ij} have the following structures

$$L_{ii} = L \qquad L_{ij} = L_q \quad (i \neq j) \tag{3}$$

$A, B, L,$ and L_q are constant nominal matrices.

Assumption 2. The nonlinear perturbations $f_i(k, \cdot)$ and $f_{ij}(k, \cdot)$ are uncertain vector-valued functions functions $\mathbf{H}_{(*)}$ as follows

$$\begin{aligned} \mathbf{H}_i &\stackrel{\text{def}}{=} \{f_i(k, x_i) : \mathbb{Z}_+ \times \mathbb{R}^n \to \mathbb{R}^n | f_i(k, x_i)^T f_i(k, x_i) \\ &\quad \leq \alpha^2 x_i^T D^T D x_i, \text{ for all } (k, x_i) \in \mathbb{Z}_+ \times \mathbb{R}^n\} \\ \mathbf{H}_{ij} &\stackrel{\text{def}}{=} \{f_{ij}(k, x_j) : \mathbb{Z}_+ \times \mathbb{R}^n \to \mathbb{R}^n | f_{ij}(k, x_j)^T f_{ij}(k, x_j) \\ &\quad \leq \alpha^2 x_j^T H^T H x_j, \text{ for all } (k, x_j) \in \mathbb{Z}_+ \times \mathbb{R}^n\} \end{aligned} \tag{4}$$

The classes $\mathbf{H}_i$ and $\mathbf{H}_{ij}$ include functions $f_i(k, x_i) = 0$ and $f_{ij}(k, x_j) = 0$, respectively. D, H are given constant matrices and $\alpha > 0$ is a given scalar.

Assumption 3. Suppose that the structure of the unknown nonlinear interconnections has the form for the ith subsystem

$$\begin{aligned} f_i(k, x_i) &= e(k, x_i) D x_i(k) \\ f_{ij}(k, x_j) &= e_q(k, x_j) H x_j(k) \qquad (i \neq j) \end{aligned} \tag{5}$$

where $e(k, x_i)$ and $e_q(k, x_j)$ are arbitrary functions satisfying the constraints $e(k, x_i) : \mathbb{Z}_+ \times \mathbb{R}^n \to [-1,\ 1]$ and $e_q(k, x_j) : \mathbb{Z}_+ \times \mathbb{R}^n \to [-1,\ 1]$ for all i, j.

The goal is to find a stabilizing piecewise constant controller for the system (1)–(5) in the form

$$u_i(k) = K \underline{x}_i(k_l) \qquad k \in [k_l, k_{l+1}) \qquad l = 1, 2, ... \tag{6}$$

where K is a constant gain matrix to be designed and k_l is a sampling instant. Such a structure enables to interpret the connection of the controller with the system via a network channel. The value $\underline{x}_i(k_l)$ is transmitted through a network channel and, if transmitted correctly, it is registered in a buffer. $\underline{x}_i(k_l)$ denotes the output from the buffer. This value is the controller input which generates the control action.

Such an approach requires to respect the basic properties of a network channel when transmitting the signal. In this paper, two essential phenomena appearing in network communication channels are modelled: Data packet dropout and communication delays.

Data packet dropout is a well-known frequent phenomenon appearing in communication networks. The state values of a dropped packet are cumulated from the last update at the instant k_l. Denoting some discrete-time interval-dependent integer $d_l \geq 1$ at the instant k_l, then the output from the buffer yields $\underline{x}_i(k_l) = x_i(k_l - d_l)$.

The resulting input time-varying delay consists of the constant communication delay denoted as d_c and the delay caused by data packet dropout d_l. Therefore, the input of the controller is $\underline{x}_i(k_l) = x_i(k_l - d_l - d_c)$. K is the state gain matrix to be determined. This matrix is considered as identical one for all subsystems, when supposing the symmetric structure of the large scale composite system.

2.2 Overall System Description

The compacted system description of (1)–(5) has the form

$$x(k+1) = \overline{A}x(k) + \overline{B}u(k) + \overline{f}(k,x) \qquad x(k_o) = x_o \tag{7}$$

where $x(k) \in \mathbb{R}^{nN}$ is the system state and $u(k) \in \mathbb{R}^{mN}$ is the control input at $k \in \mathbb{Z}_+$. The nominal matrices are given as

$$\begin{gathered} \overline{A} = (\overline{A}_{ij}) \qquad \overline{A}_{ii} = A + L \qquad \overline{A}_{ij} = L_q \\ \overline{B} = \mathrm{diag}(B, ..., B) \end{gathered} \tag{8}$$

The admissible perturbations $\overline{f}(k,x)$ in (7) are uncertain vector-valued functions satisfying the following inequalities

$$\begin{aligned} \overline{\mathbf{H}} \overset{\mathrm{def}}{=} \{&\overline{f}(t,x) : \mathbb{Z}_+ \times \mathbb{R}^{Nn} \rightarrow \mathbb{R}^{Nn} | \overline{f}(k,x)^T \overline{f}(k,x) \\ &\leq \alpha^2 x^T \overline{D}^T \overline{D} x, \text{ for all } (k,x) \in \mathbb{Z}_+ \times \mathbb{R}^n \} \end{aligned} \tag{9}$$

From Assumptions 2 and 3, the bounding matrix $\overline{D}$ is an N block-partitioned matrix defined as follows

$$\overline{D} = \mathrm{diag}(\overline{D}_1, ..., \overline{D}_N) \quad \overline{D}_i = (H, ..., H, \ D, \ H, ..., H) \tag{10}$$

with D located at the ith position in $\overline{D}_i$. Consider the stabilizing controller for the system (7)–(10) as

$$u(k) = \overline{K}\underline{x}(k_l) = \mathrm{diag}(K, ..., K)\underline{x}(k_l) \qquad k \in [k_l, k_{l+1}) \tag{11}$$

where $\underline{x}(k_l) = x(k_l - d_l - d_c)$. Note that (11) is a compacted equivalent description of (6).

Denote the time-varying delay $d(k) = k - k_l - d_l - d_c$, where $1 \leq d_l \leq (k_{l-1} - d_c)$.

Assumption 4. The number of packet dropouts is bounded so that it satisfies the constraint

$$0 \leq \underline{d} \leq d(k) \leq \overline{d} \tag{12}$$

where $\underline{d}$ and $\overline{d}$ are given positive constant.

Consider now the closed-loop overall system (7)–(11) in a compacted form as follows

$$\begin{aligned} x(k+1) = \overline{A}x(k) + \overline{BK}x(k - d(k)) + \overline{f}(k,x) \\ x(k_o) = \overline{\Phi}_l(k_o) \qquad k_o \in [-\overline{d}, 0] \end{aligned} \tag{13}$$

where $\overline{K}$ is the gain matrices with the matrices K given in (6). $\overline{\Phi}_l(k_o)$ denotes the function of initial condition of the corresponding instant k_o.

Assumption 5. Acknowledgment ACK about data losses is always available to the sender of the plant.

2.3 The Problem

Consider the system (7)–(10) and the controller (11) satisfying Assumptions 1-5. The goal is to design the gain matrix K of the controller (6) being robustly delay-dependently stabilizing the closed-loop system (13). Employ the structural properties of the system (7) to reduce the DNCS design complexity. Solve the problem by using the LMI approach.

Remark 1. The notion of robust delay-dependent stability means the global asymptotic stability for all admissible nonlinearities and the delay interval of $d(k)$ satisfying Assumption 4.

2. MAIN RESULTS

The specific structure of the system (7)–(10) is used to perform a particular model reduction. Consider the transformation of the states

$$\tilde{x}(k) = Tx(k) \tag{14}$$

where

$$T = \frac{1}{N}\begin{pmatrix} (N-1)I & -I & ... & -I & -I \\ -I & (N-1)I & ... & -I & -I \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -I & -I & ... & (N-1)I & -I \\ I & I & ... & I & I \end{pmatrix} \tag{15}$$

where I denotes the nxn identity matrix.

The transformation of states defined by (14) yields

$$\tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}u(k) + \tilde{f}(k,\tilde{x}) \qquad \tilde{x}(k_o) = \tilde{x}_o \tag{16}$$

where

$$\begin{gathered} \tilde{A} = \mathrm{diag}(A_c, ..., A_c, \ A_o) \\ \tilde{B} = \mathrm{diag}(B, ..., B, \ B) \\ \tilde{f}(k,\tilde{x}) = \mathrm{diag}(f_c(k,\tilde{x}_1), ..., f_c(t,\tilde{x}_{N-1}), \ f_o(t,\tilde{x}_N)) \end{gathered} \tag{17}$$

with

$$\begin{aligned} A_c &= A + L - L_q \\ A_o &= A_c + NL_q \\ f_c(k,\tilde{x}_i) &= (e(k,\tilde{x}_i)D - e_q(k,\tilde{x}_i)H)\tilde{x}_i(k) \\ f_o(k,\tilde{x}_N) &= f_c(k,\tilde{x}_N) + e_q(k,x_N)NH\tilde{x}_N(k) \end{aligned} \tag{18}$$

The state-trajectory solution of the ith subsystem can be described by the system determined by the states $x_i = \tilde{x}_i - \tilde{x}_N$ for $i = 1, ..., N-1$ and $x_N = \tilde{x}_N - \sum_{i=1}^{N-1} \tilde{x}_i$ as

$$\begin{gathered} \hat{x}_i(k+1) = \hat{A}_i\hat{x}_i(k) + \hat{B}\hat{u}(k) + \hat{f}_i(k,\hat{x}_i) \qquad \hat{x}_i(k_o) = \hat{x}_{oi} \\ i = 1, ..., N-1 \end{gathered} \tag{19}$$

with $\hat{x}_i = (\tilde{x}_i^T, \ \tilde{x}_N^T)^T$, $\hat{u}_i = (\tilde{u}_i^T, \ \tilde{u}_N^T)^T$, $\hat{A}_i = \mathrm{diag}(A_c, \ A_o)$, $\hat{B} = \mathrm{diag}(B, \ B)$, and $\hat{f}_i(k,\hat{x}_i) = \mathrm{diag}(f_c(k,\tilde{x}_i), f_o(k,\tilde{x}_N))$. Therefore, the dynamic properties of the original overall system can be described by the subsystem model (19) consisting of two parts operating in parallel. It leads to two systems of order n. Denote $x_c(t)$ a general state for any state $\hat{x}_i(t)$ in (19) and $x_o(t) = x_N(t)$. We get two systems as

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + Bu_c(k) + f_c(k, x_c) \quad x_c(k_o) = x_{co} \\ x_o(k+1) &= A_o x_o(k) + Bu_o(k) + f_o(k, x_o) \quad x_o(k_o) = x_{oo} \end{aligned} \tag{20}$$

The system (16) has a block diagonal structure where the first $N-1$ blocks are identical ones. We can use only any of the first $N-1$ subsystems in (16) and the last subsystem to get a complete information about the dynamics of overall system (7). The system (20) is directly based on (19). Therefore, the dynamic properties of (7) and (20) are equivalent.

Consider each system in (20) stabilized by the state feedback controller K, i.e. $u_c(k) = Kx_c(k)$ and $u_o(k) = Kx_o(k)$. It leads to the problem of simultaneous stabilization. The relations (18) offer to solve this problem effectively by using the robust stabilization approach with a central plant. The central plant serves as a nominal system, while the uncertainties enable to include both plants in (20) as particular cases. Such an approach results in the following n-dimensional system

$$\begin{aligned} x_r(k+1) &= (A_r + \Delta A_r)x_r(k) + Bu_r(k) + f_o(k, x_r) \\ &\quad + \Delta f_o(k, x_r) \\ &= A_r x_r(k) + Bu_r(k) + f_r(k, x_r) \end{aligned} \tag{21}$$

where the nominal plant is the system (21) with $\Delta A_o = 0$ and $\Delta f_o(k, x_r) = 0$, while the Δ-terms denote the uncertainty. The terms in (21) are constructed to include the systems (20) when using only the structure of (18) as follows

$$\begin{aligned} A_o &= \frac{A_o + A_c}{2} = A + L + (\frac{N}{2} - 1)L_q \\ \Delta A_o &= e_o(k, x_r)\frac{A_o - A_c}{2} = e_o(k, x_r)\frac{N}{2}L_q \\ f_o(k, x_r) &= \frac{f_o(k, x_r) + f_c(k, x_r)}{2} = (e(t, x_m)D \\ &\quad + e_q(k, x_r)(\frac{N}{2} - 1)H)x_r \\ \Delta f_o(k, x_r) &= e_o(k, x_r)\frac{f_o(k, x_r) - f_c(k, x_r)}{2} \\ &= e_o(k, x_r)e_q(k, x_r)\frac{N}{2}Hx_r \\ A_r &= A_o \\ f_r(k, x_r) &= \Delta A_o x_r(k) + f_o(k, x_r) + \Delta f_o(k, x_r) \\ &= (e_o(k, x_r)\frac{N}{2}L_q + e(k, x_r)D \\ &\quad + e_q(k, x_r)(\frac{N}{2} - 1)H \\ &\quad + e_o(k, x_r)e_q(k, x_r)\frac{N}{2}H)x_r \end{aligned} \tag{22}$$

where $e_o(k, x_r) : \mathbb{Z}_+ \times \mathbb{R}^n \to [-1,\ 1]$. Consider $f_r(k, x_r) = F_r(k, x_r)D_r x_r$ and introduce the matrix $F_r(k, x_r) = (\,e_o(k, x_r)\,I,\ e(k, x_r)\,I,\ e_q(k, x_r)\,I,\ e_o(k, x_r)\,e_q(k, x_r)\,I\,)$, where I denotes the n-dimensional identity matrix. The admissible perturbations in (21) are considered as uncertain vector-valued real functions satisfying the inequalities

$$\begin{aligned} \mathbf{H}_r \overset{\text{def}}{=} \{ & f_r(k, x_r) : \mathbb{Z}_+ \times \mathbb{R}^n \to \mathbb{R}^n | f_r(k, x_r)^T f_r(k, x_r) \\ & \leq \alpha^2 x_r^T D_r^T D_r x_r,\ \text{for all } (k, x_r) \in \mathbb{Z}_+ \times \mathbb{R}^n \} \end{aligned} \tag{23}$$

where $D_r = (\frac{N}{2}L_q,\ D,\ (\frac{N}{2} - 1)H,\ \frac{N}{2}H)^T$.

Consider a stabilizing controller for the system (21) as follows

$$u_r(k) = K\underline{x}_r(k_l) \tag{24}$$

Note that $\underline{x}_r(k_l) = x(k_l - d_l - d_c)$ and the time-varying delay $d(k) = k - k_l - d_l - d_c$, where $1 \leq d_l \leq (k_{l-1} - d_c)$. The closed-loop system (21)–(24) for $k \in [k_l, k_{l+1})$ has the form

$$\begin{aligned} x_r(k+1) &= A_r x_r(k) + BKx_r(k - d(k)) + f_r(k, x_r) \\ x_r(k_o) &= \Phi_{rl}(k_o) \qquad k_o \in [-\overline{d}, 0] \end{aligned} \tag{25}$$

where

$$0 \leq \underline{d} \leq d(k) \leq \overline{d} \tag{26}$$

$\Phi_{rl}(k_o)$ denotes the function of initial condition for the corresponding instant k_o. The gain matrix K can be determined by the procedure resulting in the robust delay-dependent stable overall closed-loop system.

The controller design proceeds in the following way of reasoning. The robust delay-dependent stability is evaluated as a test based on the linear matrix inequalities. This concept has been derived by means of the S-procedure and the Liapunov-Krasovski functional as presented in Zhou et al. [2008]. Consider the descriptor form equivalent to the system (21) as

$$\begin{aligned} x_r(k+1) &= y_r(k) \\ 0 &= -y_r(k) + A_r x_r(k) + BKx_r(k - d(k)) \\ &\quad + f_r(k, x_r) \end{aligned} \tag{27}$$

Consider the Liapunov-Krasovskii functional candidate for the system (27) in the form

$$\begin{aligned} V(k, z_r) &= z_r(k)^T EPz_r(k) + \sum_{i=k-d(k)}^{k-1} x_r(i)^T Qx_r(i) \\ &\quad + \sum_{j=-\overline{d}+2}^{-\underline{d}-1} \sum_{i=k+j-1}^{k-1} x_r(i)^T Rx_r(i) \end{aligned} \tag{28}$$

where

$$z_r(k) = \begin{pmatrix} x_r(k) \\ y_r(k) \\ x_r(k-d(k)) \\ f_r(k,x_r) \end{pmatrix} \quad E = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad P = \begin{pmatrix} P_1 & 0 & 0 & 0 \\ P_2 & P_3 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$$
$$R > 0 \quad Q > 0 \tag{29}$$

It can be easily verified that $V(k, z_k)$ satisfies the inequalities $a|x_r(k)|^2 \leq V(k, z_r) \leq \sup_{s \in [-\overline{d}, 0]} b|z_r(t+s)|^2$ with positive constants a and b.

The standard Liapunov stability method, when applying (28) on the system (27), confirms that (28) is a Liapunov-Krasovskii functional for the system (21). It results in the following theorem.

Theorem 1. Given the system (25) and the bounds $\underline{d}, \overline{d}$ satisfying Assumptions 4,5. Then, the system (25) is robustly delay-dependent stable for all $d(k)$ if there exist matrices $P_1 > 0, P_2, P_3, Q > 0, R > 0$ and a constant $\alpha > 0$ satisfying the inequalities

$$M1(A_r) < 0 \qquad M2(A_r) \leq 0 \tag{30}$$

where

$$\begin{aligned} M1(A_r) &= \begin{pmatrix} \Gamma 1 & -A_r^T P_3 + P_2^T & \Gamma 2 & -P2^T \\ \bullet & P_1 + P_2 + P3^T & -P_3^T BK & -P_3^T \\ \bullet & \bullet & \Gamma 3 & 0 \\ \bullet & \bullet & \bullet & -\alpha I \end{pmatrix} \\ M2(A_r) &= Q - R \end{aligned} \tag{31}$$

and

$$\begin{aligned} \Gamma 1 &= Q + (\overline{d} - \underline{d})R - P_1 - A_r^T P_2 - P_2^T A_r + \alpha D_r \\ \Gamma 2 &= -P_2^T BK \\ \Gamma 3 &= -Q \end{aligned} \tag{32}$$

Note only that the matrices P_1, P_2, P_3 compose the matrix P given by (29).

Theorem 1 serves as a test of the robust delay-dependent stability for the closed-loop system. Its extension for the design of the state gain matrix K is based on the use of the Schur complement as introduced in Zhou et al. [2008]. It results in the following theorem.

Theorem 2. Given the system (21) and the bounds $\overline{d}, \underline{d}$ satisfying Assumptions 4,5. The system (21) is robustly delay-dependently stabilized for all $d(k)$ by the controller (24) if there exist positive definite matrices $X_1 > 0, S_1, Z_1$, matrices X_2, X_3, Y_1, and a constant $\alpha > 0$ satisfying the inequalities

$$\begin{aligned} \begin{pmatrix} \Lambda 1 & \Lambda 2 & 0 & 0 & X_2^T & \alpha X_1 D_r^T \\ \bullet & \Lambda 3 & -BY_1 & I & X_3^T & 0 \\ \bullet & \bullet & -S_1 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -\alpha I & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -X_1 & I \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\alpha I \end{pmatrix} &< 0 \\ S_1 - Z_1 &\leq 0 \end{aligned} \tag{33}$$

where $\Lambda 1 = -X_1 + S_1^T + Z_1(\overline{d} - \underline{d})$, $\Lambda 2 = -X_1 A_r^T + X_2^T$, $\Lambda 3 = X_3^T + X_3$. The controller gain K in (24) is given by

$$K = Y_1 X_1^{-1} \tag{34}$$

Consider the closed-loop overall system (13) with the gain matrix K determined by (34).

The following theorem states the main result.

Theorem 3. Given the discrete-time symmetric composite system (1)–(6) satisfying Assumptions 1-5. Proceed as follows:

1) Construct the discrete-time reduced-order design system (21)–(23).

2) Select the controller matrix K given by (34) in the controller (24) for the reduced-order system (21)–(23) satisfying the inequalities (33) within a given delay interval $d(k) \in [\overline{d} - \underline{d}]$.

4) Implement the matrices K into the decentralized controller (7).

Then, the overall closed-loop system (13) is robustly delay-dependent stable for all $d(k)$ within the specified delay interval.

The proof of Theorem 3 is in the Appendix.

3. CONCLUSION

We have studied the problem of decentralized NCS design with delayed feedback for linear discrete-time systems under nonlinear time-varying perturbations, where the systems posses the features of symmetric composite systems. A network channel is modelled as a time-varying delay within a given interval. It has been shown how to reduce the original overall system to a low order control design model with the equivalent dynamic properties. The design model serves as a system for the state feedback control design, when a delay-dependent approach is used to choose the gain matrix by using the LMI constraints. A sufficient condition guaranteeing the robust delay-dependent stability of the overall closed-loop system with implemented identical local controllers is proved. Those controllers are synthesized by the state feedback gain matrix selected for the control design model.

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Appendix A. PROOF OF THEOREM 3

Consider the description of the overall closed-loop system (13) in an equivalent descriptor form as follows

$$\begin{aligned} x(k) &= y(k) \\ 0 &= -y(k) + \overline{A}x(k) + \overline{BK}x(k-d(k)) + \overline{f}(k,x) \end{aligned} \tag{A.1}$$

Denote $z(t) = \begin{pmatrix} x(k) \\ y(k) \\ x(k-d(k)) \\ f(k,x) \end{pmatrix}$. Consider the Liapunov-Krasovskii functional candidate for the system (A.1) in the form

$$\begin{aligned} \overline{V}(k,z) &= z(k)^T\overline{EP}z(k) + \sum_{i=k-d(k)}^{k-1} x(i)^T\overline{Q}x(i) \\ &+ \sum_{j=-\overline{d}+2}^{-\underline{d}-1} \sum_{i=k+j-1}^{k-1} x(i)^T\overline{R}x(i) \end{aligned} \tag{A.2}$$

where

$$\overline{E} = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \overline{P} = \begin{pmatrix} \overline{P}_1 & 0 & 0 & 0 \\ \overline{P}_2 & \overline{P}_3 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix} \quad \overline{R} > 0 \quad \overline{Q} > 0 \tag{A.3}$$

The matrices in (A.3) are supposed in the block diagonal form corresponding with the subsystem structure given in (1) as follows

$$\begin{aligned} &\overline{E} = \mathrm{diag}(E, ..., \ E) && \overline{P} = \mathrm{diag}(P, ..., \ P) \\ &\overline{P}_1 = \mathrm{diag}(P_1, ..., \ P_1) && \overline{P}_2 = \mathrm{diag}(P_2, ..., \ P_2) \\ &\overline{P}_3 = \mathrm{diag}(P_3, ..., \ P_3) && \overline{R} = \mathrm{diag}(R, ..., \ R) \\ &\overline{Q} = \mathrm{diag}(Q, ..., \ Q) \end{aligned} \tag{A.4}$$

The Liapunov stability method when applied on the system (A.2) leads to the following result.

Theorem 4. Given the system (A.1) and the bounds $\overline{d}, \underline{d}$ satisfying Assumptions 1-5. Then, the system (A.1) is robustly delay-dependent stable for all $d(k)$ if there exist matrices $\overline{P}_1 > 0, \overline{P}_2, \overline{P}_3, \overline{Q} > 0, \overline{R} > 0$ and a constant $\alpha > 0$ satisfying the inequalities

$$\overline{M}1(\overline{A}) < 0 \qquad \overline{M}2(\overline{A}) \leq 0 \tag{A.5}$$

where

$$\begin{aligned} \overline{M}1(\overline{A}) &= \begin{pmatrix} \overline{\Gamma}1 & -\overline{A}^T\overline{P}_3+\overline{P}_2^T & \overline{\Gamma}2 & -\overline{P}2^T \\ \bullet & \overline{P}_1+\overline{P}_2+\overline{P}3^T & -\overline{P}_3^T\overline{BK} & -\overline{P}_3^T \\ \bullet & \bullet & \overline{\Gamma}3 & 0 \\ \bullet & \bullet & \bullet & -\alpha I \end{pmatrix} \\ \overline{M}2(\overline{A}) &= \overline{Q} - \overline{R} \end{aligned} \tag{A.6}$$

and

$$\begin{aligned} \overline{\Gamma}1 &= \overline{Q} + (\overline{d} - \underline{d})\overline{R} - \overline{P}_1 - \overline{A}^T\overline{P}_2 - \overline{P}_2^T\overline{A} + \alpha\overline{D} \\ \overline{\Gamma}2 &= -\overline{P}_2^T\overline{BK} \\ \overline{\Gamma}3 &= -\overline{Q} \end{aligned} \tag{A.7}$$

Consider the closed-loop systems with the matrices K given by (34)

$$\begin{aligned} x_c(k+1) &= A_cx_c(k) + BKx_c(k-d(k)) + f_c(k,x_c) \\ x_c(k_o) &= \Phi_{ck}(k_o) \qquad k_o \in [-\overline{d}, 0] \\ x_o(k+1) &= A_ox_o(k) + BKx_o(t-d(o)) + f_o(k,x_o) \\ x_o(k_o) &= \Phi_{ok}(k_o) \qquad k_o \in [-\overline{d}, 0] \end{aligned} \tag{A.8}$$

Proof of Theorem 3.
Consider only the matrix $\overline{M}1(\overline{A})$ in Theorem 4. Let $M1(A_c)$ and $M1(A_o)$ be matrix structures defined in a similar way as $M1(A_r)$ in (30) when considering the systems (A.8) instead of the system (25). Then, when applying the transformation of states T by (14) on the equation $\overline{M}1(\overline{A})$ in (A.6), we get the following relation

$$T1^{-1}\overline{M}1(\overline{A})T1 = \mathrm{diag}(M1(A_c), ..., \ M1(A_c), \ M1(A_o)) \tag{A.9}$$

where $T1 = \mathrm{diag}(T, \ ... \ , \ T)$ has the corresponding number of repeated blocks of the matrix T. Note that T is a non-singular matrix. If $M1(A_r) < 0$ from Theorem 1, then $M1(A_c) < 0$, $M1(A_o) < 0$ because the system (25) includes both systems (A.8) by construction as its special cases.

An analogous way of reasoning leads to the same conclusions for $\overline{M}2(\overline{A})$ because both Q and R are block diagonal matrices.

Thereby, the closed-loop system (1)–(7) with the gain matrix K determined according to Theorem 2 by (34) is robustly delay-dependent stable for all $d(t)$ taking values from the interval $[\underline{d}, \overline{d}]$. *Q.E.D.*