Equity home bias in the Czech Republic

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Abstract: Investors reveal a tendency to prefer domestic over foreign equities despite the financial losses. From institutional perspective the factors that cause home biasness are the barriers to entry the foreign markets, transaction costs, illiquidity, asymmetric information and information costs, corporate governance and inflation and exchange rate risks. Behavioral finance argues that irrationality of investors cause the home biasness. Investors tend to be under the influence of psychological biases: optimism, overconfidence, social identity, narrow framing and loss aversion. In this paper we introduce a model of optimal portfolio of Czech investors with three utility functions: Markowitz, exponential and CRRA. The prediction of the model without short selling suggests that Czech investors should have more than 60 % (between 72–83 % for feasible levels of risk aversion) in domestic equities. The OECD data claim that they hold around 87 % in domestic equities.

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1 Introduction

Equity home bias is a situation on a market when investors hold an unreasonably high share of their portfolios in domestic equities. Equity home bias occurs when the proportion of domestic equities in the investment portfolios are higher than the financial theory predicts. According to the theory the investors should diversify into foreign equities to lower their total portfolio risk. The ultimate state in the world without any barriers would be that all investors in the world would hold the exactly same proportion of all international equities. The hypothetical weights would be based on the market capitalizations of the equities.

The question of high domestic equities concentration bothered the economist at least since Levy and Sarnat [5] were one of the first to discover the equity home bias phenomenon on US equities. Since 1970 there has been vast number of studies that confirmed the existence home bias not only in US, but also many other countries in the world. Tesar and Werner [12] presented international investment positions of USA and Canada in the period 1975-1990, pointing out the home biasness of investors in these two countries. Cooper and Kaplanis [3] showed the extent of equity portfolios concentration with domestic equities among 8 world major economies. According to this study, in 1987 the most home biased investors were in Sweden (100 % share of domestic equities), the best situation was in France ("only" 64,4 % share of domestic equities). Further evidence of home bias was provided by Adler and Dumas [1] and Lewis[6], for example. The international capital asset pricing model (ICAPM) based on Sharpe [11] and Lintner [7] predicts that an investor should hold equities from a country as per that country's share of world market capitalization (Mishra [9]). The less the integrated international markets are the higher the benefits from international diversification could be.

The recent papers do not focus mainly on providing only other proofs of the phenomenon, but they try to view the puzzle from different perspectives and value the possible impacts of different factors. From the simplest perspective we can divide these factors into two groups: institutional and behavioral explanations. In the paper we will examine whether there is equity home bias in the Czech Republic. We will compare the actual evidence taken from OECD[10] with the theoretical composition of a investment portfolio model without the possibility of short selling for three utility functions.

2 Methodology

In the literature the recognition of equity home bias has been generally taken as a task to evaluate the optimal investment portfolio and compare it with the actual evidence. The early models were applied from portfolio selection framework of Markowitz [8]. The IAPM based on Sharpe [11] **a**nd Lintner [7] makes a very strong

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conclusion that all investors should in equilibrium hold equities in same proportions: weighted by the market capitalizations. The home bias puzzle was discovered in the papers of international diversification of investment portfolios (Levy and Sarnat [5]). Adler and Dumas [1] proposed an international asset pricing model (CAPM), which resulted in a vector of optimal weights of an investor with a given utility function. This asset pricing approach is based on a mean-variance optimalization. The researchers that try to prove the existence of home bias use concave utility functions and search for their maximum.

The main assumptions of the model are that there are no transaction costs and no barriers to enter on a market. We assume the weak form market efficiency. Furthermore, we assume that the returns are normally distributed with the mean and variance which are constant in time, i.e. same as historical mean and variance during 1998–2008. Therefore, in the model we get N=10 weak form efficient markets with country specific drifts. Investors are assumed to be rational and cannot influence the price. They have free access to all relevant information and evaluate only the relevant information. New events are expected to be random with a zero mean on price change, therefore they form their expectation only based on historical prices and historical variances. Finally, we also assume that all investors are maximizing their utilities. All investors have the same utility function. In the model we assume a risk averse investor with a concave utility function that is increasing in expected profits and decreasing in expected risk. For the sake of simplicity we assume that there is not a risk free investment opportunity other than any investment. This assumption implies that investors will invest into stock all their wealth unless they get less money than their initial wealth at the end of the investment period. We also assume that the investors do not take into account the inflation.

To test the home bias puzzle in Czech Republic we need to simulate a world equity portfolio. In the model we use 10 years of monthly data starting in May 1998 and finishing in May 2008[3]. For Czech investors the world equity market comprises of 9 foreign and one domestic stock index[4]. Foreign equity indices were converted into CZK and we used monthly returns calculated by the formula: $r_t = \ln(P_t) - \ln(P_{t-1)})$ [5].

2.1 Model of optimal equity portfolio with Markowitz utility function

In this section we will introduce a model derived from CAPM model (Lewis[6]) to determine how much foreign equities should have an average Czech investor in his equity portfolio. We will evaluate the more realistic case of risk averse investor who tries to maximize his risk adjusted wealth. He is trading off between risk and returns. In our model are his preferences described by a Markowitz utility function:

$$U = E_t W_{t+1} - \lambda \cdot \mathrm{var}(W_{t+1}) \tag{1}$$

where λ is in this model a proxy of risk aversion. His utility is linearly increasing in the expected wealth: $\frac{\partial U}{\partial E_t W_{t+1}} = 1$ and decreasing in variance of his future wealth: $\frac{\partial U}{\partial\, \mathrm{var}(W_{t+1})} = -\lambda$. The higher is his risk aversion measured by λ, the lower will be his utility from a given level of variance.

We will proceed with the description of the model. In our case, the investor can choose from n stock indices. We assume that investor invests all his wealth into stocks so he gets the maximum utility of the expected wealth at the end of the next period. Let us denote the vector of expected returns as a (n x 1) vector r , the transposed

[3]The monthly data were taken as the closing prices at the end of month, starting at the end of May 1998 and finishing at the end of May 2008. For the indices of New Zealand and South Africa the monthly data were taken as the opening prices at the beginnings of following months. I assume that the one day difference should have only a neglectable impact on the investment decision.

[4]Domestic index: PX. Foreign equities: United States: SP 500 (US), European Union: Dow Jones EUROSTOX 50 (EU) – consists of 12 EU countries excluding Czech Republic , Japan: Nikkei (JA), Russia: RTS $ (RS), China: Schangai composite (CH), India: Bombay Sensex (IN), Brazil: Brazil Bovespa $ (BZ), New Zealand: DJTM NEW ZEALAND $ (NZ), South Africa: DJTM SOUTH AFRICA $ (SA). Sources: PSE, Data Stream

[5] Monthly closing prices of CZK/USD, CZK/EUR and CZK/JPY exchange rates were taken from ČNB ARAD. I calculated the cross exchange rate for CZK/YUYN, CZK/BRL and CZK/INR. We used the monthly opening prices of YUAN/USD, INR/USD and BRL/USD taken from Data Stream. Again we assume that the one day difference should have only a neglectable impact on the investment decision.

vector of returns looks like: $r' = (r_1, r_2, ..., r_n)$[6]. Our investor can sell stock indices even without owning them and buy them with profit at the end of next period. Therefore he can gain even from the downfall of the stock prices.

Let us denote Ω for the (n x n) variance-covariance matrix, ω for a (n x 1) vector of desirable weights of the stock indices in portfolio: $\omega' = (\omega_1, \omega_2, ..., \omega_n)$ and I for a (n x 1) vector: $I' = (1, 1, ..., 1)$. Investor is constrained with an equation: $\omega_1 + \omega_2 + ... + \omega_n = 1$. If we rewrite this condition in matrix algebra we get an optimalization constraint: $\omega' \cdot I = 1$. In this model we will allow for costless short selling so the weights can be also negative.

In this notation the investor utility function of the portfolio at the end of next period:

$$U = W_t(1 + \omega' \cdot r) - \lambda \cdot W_t^2 \cdot \omega' \cdot \Omega \cdot \omega \tag{2}$$

we can simplify the equation by the assumption: $W_t = 1$. To solve the maximization problem we need a Lagrangean function:

$$L = (1 + \omega' \cdot r) - \lambda \cdot \omega' \cdot \Omega \cdot \omega - \varphi \cdot (\omega' \cdot I - 1) \tag{3}$$

where $\varphi \in R$ is the Lagrange multiplier. The first order condition with respect to ω is[7]:

$$\frac{\partial L}{\partial \omega} = r - 2 \cdot \lambda \cdot \Omega \cdot \omega - \varphi \cdot I = 0 \tag{4}$$

solving for ω, $\lambda \neq 0$:

$$\omega = \frac{\Omega^{-1}(r - \varphi \cdot I)}{2 \cdot \lambda} \tag{5}$$

where Ω^{-1} is inverse to Ω. We can rewrite the equation (5):

$$\omega = \frac{1}{2 \cdot \lambda} \cdot \Omega^{-1} \cdot r - \frac{\varphi}{2 \cdot \lambda} \cdot \Omega^{-1} \cdot I \tag{6}$$

which transposed gives:

$$\omega' = \frac{1}{2 \cdot \lambda} \cdot r' \cdot \Omega^{-1'} - \frac{\varphi}{2 \cdot \lambda} \cdot I' \cdot \Omega^{-1'} \tag{7}$$

and multiplied by I:

$$\omega' I = \frac{1}{2 \cdot \lambda} \cdot r' \cdot \Omega^{-1'} \cdot I - \frac{\varphi}{2 \cdot \lambda} \cdot I' \cdot \Omega^{-1'} \cdot I = 1 \tag{8}$$

which is the investor's constraint condition. Realizing that $I' \cdot \Omega^{-1'} \cdot I$ is only a number and therefore $I' \cdot \Omega^{-1'} \cdot I = (I' \cdot \Omega^{-1'} \cdot I)' = I' \cdot \Omega^{-1} \cdot I \neq 0$. Also we should note that the variance-covariance matrix Ω and therefore also Ω^{-1} is symmetric which means the transposed matrix is identical to original matrix: $\Omega = \Omega'$ and $\Omega^{-1'} = \Omega^{-1}$. We can now rewrite for φ:

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6 In a model I assume only 1 period investment, therefore I will use henceforth the notation of r instead of $E_t r_{t+1}$. In our case: $r_1 = r_{CR}$, $r_2 = r_{US}$, $r_3 = r_{EU}$, $r_4 = r_{JA}$, $r_5 = r_{RS}$, $r_6 = r_{CH}$, $r_7 = r_{IN}$, $r_8 = r_{BZ}$, $r_9 = r_{SA}$, $r_{10} = r_{NZ}$.

7 This statement is expressed in matrix form, there are in fact n F. O. C.´s.

$$\varphi = \frac{r' \cdot \Omega^{-1} I - 2 \cdot \lambda}{I' \cdot \Omega^{-1} \cdot I} \tag{9}$$

plugging this equation for φ into (6) we finally get the formula of vector of optimal portfolio weights:

$$\omega = \frac{\Omega^{-1} \cdot r}{2 \cdot \lambda} - \frac{\frac{1}{2 \cdot \lambda} r' \cdot \Omega^{-1} \cdot I - 1}{I' \cdot \Omega^{-1} \cdot I} \cdot \Omega^{-1} \cdot I \tag{10}$$

We will solve the optimal portfolio weights for the monthly and quarterly for 5 levels of risk aversion: $\lambda = 3, 2, 1, 1/2$ *and* $1/3$.

However, in this paper we will not present the results of this method, because we focus only on the case, where the short selling is not allowed. Therefore we have to solve the model numerically with a restriction that the individual portfolio weights of a single stock can not be negative. We use the formula (2) and search for its maximum. It is a standard convex problem on polyhedral feasibility set, which assures that the numerical method has a unique solution[8].

2.2 Model of optimal equity portfolio with exponential utility function

In this case we use the CARA (Constant Absolute Risk Aversion Function). Our model can be rewritten as follows:

$$\max_{\omega} U, \textit{so that}: \omega' \cdot I = 1 \tag{11}$$

The CARA utility function:

$$U = -E \exp(-W_t \alpha \omega'(1+r)) \tag{12}$$

where α is the coefficient of absolute risk aversion. For simplicity, we again assume that $W_t = 1$. The assumption of the normally distributed returns leads to a log normal distribution and we search for the expected value:

$$E(LN(\bar{r}, \overline{\Omega})) = \exp(r, \frac{\overline{\Omega}}{2}), \bar{r} = -\alpha \omega' r, \overline{\Omega} = \alpha^2 \omega' \Omega \omega \tag{13}$$

Finally, we can rewrite our utility maximizing problem as:

$$\max_{\omega} U = -\exp(-\alpha \omega' E(r) - \frac{\alpha^2 \omega' \Omega \omega}{2}) \tag{14}$$

2.3 Model of optimal equity portfolio with CRRA utility function

To improve our sensitivity analysis we add the Constant Relative Risk Aversion utility function. In this case, the maximization problem is:

$$\max_{\omega} U = E \frac{(\omega'(1+r)W_t)^{1-\gamma}}{1-\gamma} = \frac{W_t^{1-\gamma}}{1-\gamma} E \frac{(\omega'(1+r)^{1-\gamma}}{1-\gamma} \text{ so that: } \omega' \cdot I = 1 \tag{15}$$

where γ is the coefficient of relative risk aversion. We assume non-negative portfolio weights because the short sell is not allowed. To calculate the portfolio weights we use a numerical approximation of the integral of expected value of the utility function:

$$E(U(x) = \int_{-\infty}^{\infty} U(x) f(x) dx = \frac{W_t^{1-\gamma}}{1-\gamma} \int_{-\infty}^{\infty} \frac{(1+\omega' r)^{1-\gamma}}{1-\gamma} f(x; \omega' r, \omega' \Omega \omega) dr \tag{16}$$

where $f(x; \mu, \Sigma)$ is density of normal distribution with parameters μ a Σ. This utility maximization problem with the CRRA utility is irrelevant on absolute wealth. Portfolio weights will be same with investments 1 CZK or 1 mil CZK.

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[8] As it is explained in textbook of Chong and Zak[4] .

3 Results of the optimal portfolio model

We tested the optimal portfolio model for three different utility functions. In this paper we present only the results for the special case, when the short selling is not allowed. This restriction is not in contrast with the reality, because in practice the short selling is costly and almost impossible for small investors.

3.1 Results of the model with Markowitz utility

In the **Table 1** below, the results for the Markowitz utility function can be found. We numerically solved the optimization problem with restriction on short sale with several levels of risk aversion.

Investor's risk aversion:	3	2	1	1/2	1/3
Czech Republic	0,71	0,84	0,86	0,75	0,63
United States	0,01	0,00	0,00	0,00	0,00
Japan	0,00	0,00	0,00	0,00	0,00
Russia	0,00	0,05	0,14	0,25	0,37
South Africa	0,07	0,08	0,00	0,00	0,00
New Zealand	0,21	0,03	0,00	0,00	0,00
Utility	**-0,002**	**0,002**	**0,007**	**0,009**	**0,010**

Table 1: Optimal monthly portfolio weights (Markowitz utility, no short selling)

For the coefficient of risk aversion $\lambda = 1/2$, the Czech investor would choose about 75 % of Czech equities and 25 % Russian equities. For $\lambda = 1$, the Czech investor would choose about 86 % of Czech equities and 14 % Russian equities. As we can conclude from the **Table 1** the happiest investor would be the investor with the lowest risk aversion that is able to sell the stock indices short. Investor with $\lambda = 1/3$ would gain 4,7 % increase in his utility if he is allowed to short sell free of costs, but gains only 1 % if he is not.

3.2 Results of the model with exponential utility (CARA) and CRRA

We use the estimations of the coefficient of risk aversion from the paper of (Bliss and Panigirtzoglou[2]): $\alpha = 0{,}91$. for CARA and $\gamma = 4{,}05$ for CRRA[9]. The vector of optimal share of equity indices was obtained as a result of numerical solution with restriction on short sales.

	CARA	**CRRA**
Risk aversion:	***0,91***	***4,05***
Czech Republic	0,72	0,83
Russia	0,28	0,05
South Africa	0	0,08
New Zealand	0	0,04

Table 2: Optimal monthly portfolio weights (no short selling)

As we can see in **Table 2** the investors would invest no less than 72 % in Czech equities for the chosen level of risk aversion. Furthermore, the CRRA function, that provides solution independent on absolute investment wealth, implies that the Czech investors should hold 83 % of Czech equity index in their equity portfolios.

4 Evidence

The most valuable evidence of the portfolio allocation of Czech investors can be obtained from the OECD data. If we include the mutual funds[10] into equity, the ratio of domestic/total equities in Czech portfolio was 87,2 %.

[9] This is an option-implied coefficient for 4 weeks (1 month) period on 95 % level of significance.
[10] We assume that mutual funds are perfect substitutes to foreign shares.

This high figure can be little bit biased, because it includes the government, which is not a typical investor. However, if the government is excluded, the ratio is still quite high: 83,3 %.

If we compare these figures with the results of our model without short selling possibility that are shown in **Tables 1 and 2,** we can conclude that the model is not in favor of the equity home bias hypothesis in the Czech Republic. Results of utility maximization problem with three different utility functions indicate that the optimal weight for Czech equities should be above 60 % and for reasonable levels of risk aversions between 72-83%.

5 Conclusion

Based on our model and actual evidence from Czech Republic we could not prove the home biasness of Czech investors. This conclusion is a result of the comparison of the evidence of international portfolio allocation and the model of optimal portfolio allocation. Czech investors hold 87 % domestic equities out of total equity holdings. This figure includes the ownership of Czech government. The model without short selling for three different utility functions suggests that there should be more than 60 % of domestic equities in the portfolio (and 72–83 % for the feasible levels of risk aversion).

The weakness of this result can be found in the strong assumptions behind the optimal portfolio model. The strong assumption of markets without imperfections and rational investors can make these results non-corresponding to the real world portfolio selections. The extension of the equity home bias issues would be to develop a model that incorporates the information and transaction costs. Behavioral finance teaches that the investors make their investment decisions in a process that cannot be simplified by the selected utility functions. The further extension would therefore be a different utility function (for example: utility function that is convex and steeper over the losses and concave and flatter over gains) or different behavioral approach.

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