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Some Averaging Functions in Image Reduction

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Abstract. In this work we propose a definition of weak local reduction operators and an algorithm that uses such operators to reduce images. We show how to construct local reduction operators by means of different aggregation functions. We also present several experimental results obtained using these aggregation functions-based reduction operators.

Keywords: Image reduction, local reduction operators, aggregation functions.

1 Introduction

Image reduction consists in diminishing the resolution of the image while keeping as much information as possible. Image reduction can be used to accelerate computations on an image ([12,8]), or just to reduce its storage cost ([9]).

We know that in the literature, there exist many different image reduction methods. Some of them consider the image to be reduced globally ([9]). Some others divide the image in pieces and act on each of these pieces locally ([7]). In this work we focus on the latter.

The problem we consider is the following: given an image, create a new, smaller image such that the intensity of each of the pixels of the smaller image reflects the interrelation between the elements of a particular region of the original image, in such a way that the new pixel preserves as much information as possible from the considered region in the original image. This problem has led us to axiomatically define the concept of local reduction operator.

In this work, as a particular case, we obtain weak local reduction operators by means of aggregation functions, since these functions have been extensively considered in the literature ([1,4,6]).

We also present an image reduction algorithm. We study experimentally point of view the effect of using some of the most common averaging functions in this algorithm.

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There is no an exact way of determining which is the best reduction method. It depends on a particular application we are considering. In this work, to decide whether a reduction is better than another, we reconstruct the original image from the reduction using the bilinear interpolation of MATLAB. We choose this reconstruction method since we also do the implementation of our methods with MATLAB.

2 Preliminaries

We start by recalling some concepts that will be used along this work.

Definition 1. *An aggregation function of dimension n (n-ary aggregation function) is a non-decreasing mapping $M : [0,1]^n \to [0,1]$ such that $M(0,\ldots,0) = 0$ and $M(1,\ldots,1) = 1$.*

Definition 2. *Let $M : [0,1]^n \to [0,1]$ be a n-ary aggregation function.*

(i) M is said to be idempotent if $M(x,\ldots,x) = x$ for any $x \in [0,1]$.
(ii) M is said to be homogeneous if $M(\lambda x_1,\ldots,\lambda x_n) = \lambda M(x_1,\ldots,x_n)$ for any $\lambda \in [0,1]$ and for any $(x_1,\ldots,x_n) \in [0,1]^n$.
(iii) M is said to be shift-invariant if $M(x_1+r,\ldots,x_n+r) = M(x_1,\ldots,x_n)+r$ for all $r > 0$ such that $0 \leq x_i + r \leq 1$ for any $i = 1,\ldots,n$.

A complete characterization for shift-invariantness and homogeneity of aggregation functions can be seen in [10,11].

We know that a triangular norm (t-norm for short) $T : [0,1]^2 \to [0,1]$ is an associative, commutative, non-decreasing function such that $T(1,x) = x$ for all $x \in [0,1]$. A basic t-norm is the minimum ($T_M(x,y) = \wedge(x,y)$). Analogously, a triangular conorm (t-conorm for short) $S : [0,1]^2 \to [0,1]$ is an associative, commutative, non-decreasing function such that $S(0,x) = x$ for all $x \in [0,1]$. A basic t-conorm is the maximum ($S_M(x,y) = \vee(x,y)$).

3 Local Reduction Operators

In this work, we consider an image of $n \times m$ pixels as a set of $n \times m$ elements arranged in rows and columns. Hence we consider an image as a $n \times m$ matrix. Each element of the matrix has a value in $[0,1]$ that will be calculated by normalizing the intensity of the corresponding pixel in the image. We use the following notation.

- $\mathcal{M}_{n\times m}$ is the set of all matrices of dimension $n \times m$ over $[0,1]$.
- Each element of a matrix $A \in \mathcal{M}_{n\times m}$ is denoted by a_{ij} with $i \in \{1,\ldots,n\}$, $j \in \{1,\ldots,m\}$.
- Let $A, B \in \mathcal{M}_{n\times m}$. We say that $A \leq B$ if for all $i \in \{0,\ldots,n\}, j \in \{0,\ldots,m\}$ the inequality $a_{ij} \leq b_{ij}$ holds.

– Let $A \in \mathcal{M}_{n\times m}$ and $c \in [0,1]$. $A = c$ denotes that $a_{ij} = c$ for all $i \in \{1,\ldots,n\}, j \in \{1,\ldots,m\}$. In this case, we will say that A is constant matrix or a flat image.

Definition 3. *A weak local reduction operator WO_{RL} is a mapping $WO_{RL} : \mathcal{M}_{n\times m} \to [0,1]$ that satisfies*

(WORL1) For all $A, B \in \mathcal{M}_{n\times m}$, if $A \leq B$, then $WO_{RL}(A) \leq WO_{RL}(B)$.
(WORL2) If $A = c$ then $WO_{RL}(A) = c$.

Remark: We call our operators weak local reduction operators since we demand the minimum number of properties that, in our opinion, a local reduction operator must fulfill.

Definition 4. *We say that a weak reduction operator WO_{RL} is:*

(WORL3) homogeneous if $WO_{RL}(\lambda A) = \lambda \cdot WO_{RL}(A)$ for all $A \in \mathcal{M}_{n\times m}$ and $\lambda \in [0,1]$
(WORL4) stable under translation (shift-invariant) if $WO_{RL}(A + r) = WO_{RL}(A) + r$ for all $A \in \mathcal{M}_{n\times m}$ and $r \in [0,1]$ such that $0 \leq a_{ij} + r \leq 1$ whenever $i \in \{1,\ldots,n\}, j \in \{1,\ldots,m\}$.

4 Image Reduction Algorithm

In this section we present a possible algorithm for image reduction.

Given an image $A \in \mathcal{M}_{n\times m}$ and a reduction block size $n' \times m'$ (with $n' \leq n$ and $m' \leq m$), we propose the following algorithm:

(1) Choose a local reduction operator.
(2) Divide the image A into disjoint blocks of dimension $n' \times m'$.
If n is not a multiple of n' or m is not a multiple of m' we suppress the smallest number of rows and/or columns in A that ensures that these conditions hold.
(3) Apply the local reduction operator to each block.

Example 1. In Figure 1 we show a scheme of this algorithm. On a 9×9 image we take reduction blocks of size given by $n' = m' = 3$. The reduced image will hence have 3×3 dimension.

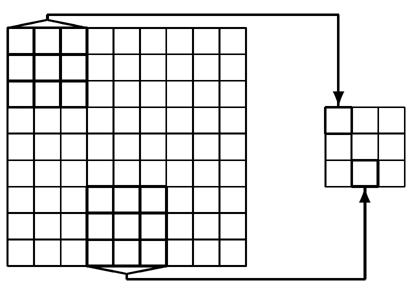

Fig. 1. Example of reduction of a matrix

5 Construction of Weak Local Reduction Operators from Averaging Functions

Proposition 1. *Let M be an idempotent aggregation function. The operator defined by*

$$WO_{RL}(A) = \mathrm{M}(a_{11}, a_{12}, \ldots, a_{1m}, \ldots, a_{n1}, \ldots, a_{nm})$$

for all $A \in \mathcal{M}_{n\times m}$ is a weak local reduction operator.

Example 2. a) Take $\mathrm{M} = T_M$. In Figure 2 we apply the weak local reduction operator obtained from T_M to the image (a) and we obtain image (a1). In image (b) we add some salt and pepper type noise. Applying the same weak local reduction operator as before we obtain image (b1)

b) Take $\mathrm{M} = S_M$. In the same figure, we apply the weak local reduction operator obtained from S_M to image (a) and we obtain image (a2). The same process is done for image (b), getting image (b2).

Observe that these two operators minimum and maximum are not good local reduction operators. If we take the minimum over a block with noise we always obtain the value 0. Analogously, if we consider the maximum and apply it to a block with noise, we always recover the value 1. In this way we lose all information about the elements in the block that have not been altered by noise. This fact leads us to study other aggregation functions.

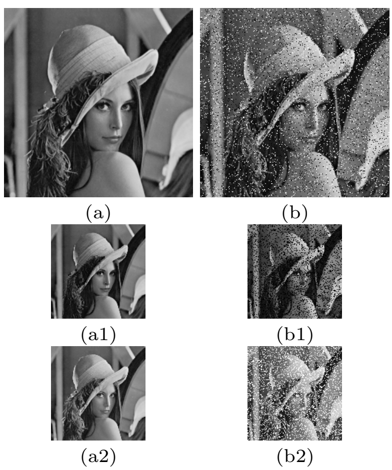

Fig. 2. Reduction of Lena and Lena with noise images using minimum and maximum and block size of 2×2

Proposition 2. *The following items hold:*

(1) $WO_{RL}(A) = T_M(a_{11}, a_{12}, \ldots, a_{1m}, \ldots, a_{n1}, \ldots, a_{nm})$ *is a weak local reduction operator that verifies* $(WORL3)$ *and* $(WORL4)$.

(2) $WO_{RL}(A) = S_M(a_{11}, a_{12}, \ldots, a_{1m}, \ldots, a_{n1}, \ldots, a_{nm})$ *is a weak local reduction operator that verifies* $(WORL3)$ *and* $(WORL4)$*.*

Proof: It follows from the fact that both weak local reductions operators constructed from minimum and maximum satisfies $(WORL3)$ and $(WORL4)$.

5.1 Weighted Quasi Arithmetic Means

Definition 5. *Let* $g : [0,1] \to [-\infty, \infty]$ *be a continuous and strictly monotone function and* $w = (w_1, \ldots, w_n)$ *a weighting vector such that* $\sum_{i=1}^{n} w_i = 1$*. A weighted quasi-arithmetic mean is a mapping* $M_g : [0,1]^n \to [0,1]$ *defined as*

$$M_g(x_1, \ldots, x_n) = g^{-1}\left(\sum_{i=1}^{n} w_i g(x_i)\right)$$

Proposition 3. *Let* $M_g : [0,1]^{n \cdot m} \to [0,1]$ *be a weighted quasi-arithmetic mean. The operator defined as*

$$WO_{RL}(A) = g^{-1}\left(\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} g(a_{ij})\right)$$

for all $A \in \mathcal{M}_{n\times m}$ *is a weak local reduction operator.*

Notice that from Definition 5 we can generate well-known aggregation functions as, for instance, the weighted arithmetic mean $(g(x) = x)$ and the weighted geometric mean $(g(x) = \log(x))$.

Proposition 4. *A weak local reduction operator built from a weighted quasi-arithmetic with* $w_{ij} = \frac{1}{n \cdot m}$ *satisfies* $(WORL3)$ *if and only if*

$$WO_{RL}(A) = \left(\prod_{i=1}^{n}\prod_{j=1}^{m} a_{ij}\right)^{\frac{1}{n\cdot m}} \quad \text{or}$$

$$WO_{RL}(A) = \left(\sum_{i=1}^{n}\sum_{j=1}^{m} \frac{a_{ij}^{\alpha}}{n \cdot m}\right)^{\frac{1}{\alpha}} \quad \text{with } \alpha \neq 0$$

for all $A \in \mathcal{M}_{n\times m}$*.*

Proof: See page 118 of [6] □

In Figure 3 we illustrate property $(WORL3)$. Image (a) is Cameraman image with random noise (white pixels). Image (b) has been obtained multiplying the intensity of each of the pixels of (a) by $\lambda = 0.5$. That is, $(b) = 0.5 \cdot (a)$. Under these conditions, we consider the following weak local reduction operators.

- the geometric mean in the second row.
- the following quasi arithmetic mean

$$M_g(x_1,\ldots,x_n) = \begin{cases} \frac{\sqrt[n]{\prod_{i=1}^n x_i}}{\sqrt[n]{\prod_{i=1}^n x_i} + \sqrt[n]{\prod_{i=1}^n (1-x_i)}} & \text{if } \{0,1\} \nsubseteq \{x_1,\ldots,x_n\} \\ 0 & \text{otherwise} \end{cases}$$

  in the third row

We see that $(b1) = 0.5 \cdot (a1)$, so they keep the same proportion than images (a) and (b). However, it is visually clear that $(b2) \neq 0.5 \cdot (a2)$. This is due to the fact that the second aggregation function that we have used does not satisfy $(WORL3)$.

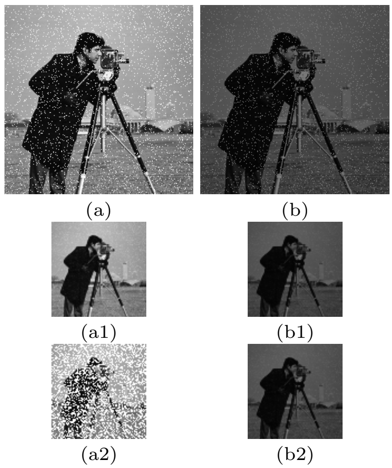

Fig. 3. Test of property $(WORL3)$ of weak local reduction operators. Reduction block size 2×2.

Proposition 5. *A weak local reduction operator built from a weighted quasi-arithmetic mean with* $w_{ij} = \frac{1}{n \cdot m}$ *satisfies* $(WORL4)$ *if and only if*

$$WO_{RL}(A) = \frac{1}{n \cdot m} \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} \qquad \textit{or}$$

$$WO_{RL}(A) = \frac{1}{\alpha} \log \left(\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{e^{\alpha a_{ij}}}{n \cdot m} \right) \qquad \textit{with } \alpha \neq 0$$

for all $A \in \mathcal{M}_{n \times m}$

Proof: See page 118 in [6] □

In Figure 4 we illustrate property $(WORL4)$. The normalized intensity of the pixels in image (a) vary from 0 to 0.5. Image (b) corresponds to add $r = 0.5$ to each of the intensities of the pixels in image (a). That is, $(b) = (a) + 0.5$. We apply the following weak local reduction operators:

- the arithmetic mean in the second row
- the geometric mean in the third row.

Observe that $(b1) = (a1)+0.5$. However, it is visually clear that $(b2) \neq (a2)+0.5$. This is due to the fact that the arithmetic mean satisfies $(WORL4)$ whereas the geometric mean does not.

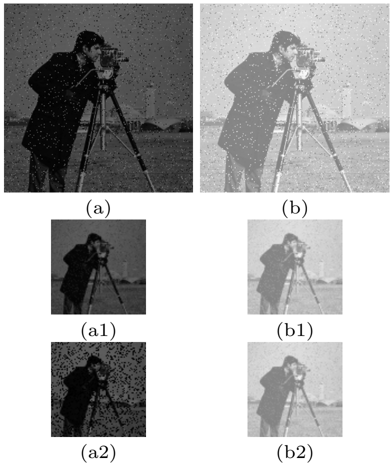

Fig. 4. Test of property $(WORL4)$ of weak local reduction operators. Reduction block size 2×2.

5.2 Median

Proposition 6. *The operator defined as*

$$WO_{RL}(A) = Med(a_{11}, \ldots, a_{1m}, \ldots, a_{n1}, \ldots, a_{nm})$$

for all $A \in \mathcal{M}_{n\times m}$*, where* Med *denotes the median, is a weak local reduction operator verifying* $(WORL3)$ *and* $(WORL4)$*.*

Proof: It is straight forward □

In Figure 5 we show the original House image (image (a)). We take as weak local reduction operator the one defined from the median and obtain image (a1). We add salt and pepper noise to image (a) to get image (b). We apply the same weak local reduction operator and obtain (b1). Observe that for this kind of reduction operators, noise does not have as much influence as for others. The reason for this is that salt and pepper noise adds black and white pixels randomly. As the median gives back the value "in the middle", it is not affected by black and white pixels.

Remark: Observe that we can build weak local reduction operators based in Choquet integral. In particular, if we impose symmetry, we get OWA operators, of which the mean is a prominent case.

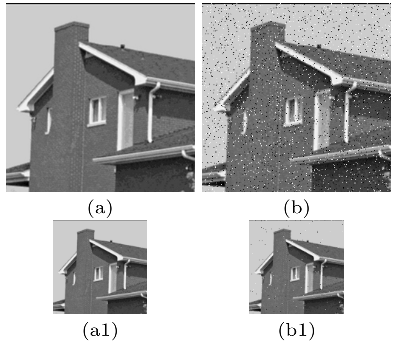

Fig. 5. Reduction of image House using the median operator with block sizes of 2×2

6 Experimental Results

As we have said in the Introduction, there is no exact way of determining which is the best reduction method. It depends on the particular application we are considering ([7,8,9]). In this section we analyze different reductions by reconstructing them to their original size using the two-dimensional bilinear interpolation from MATLAB.

There exist many methods to calculate the similarity between the original image and the reconstructed one. In fact, we know that there is a relation between the different types of errors (absolute mean error, quadratic mean error, etc.) and the aggregation function to be considered in each case ([4]). We will study in the future the relationship between the error measure and the aggregation function in image comparison. On the other hand, we can consider an image as a fuzzy set ([3]). For this reason, we are going to use fuzzy image comparison indexes. In [2] an in depth study of such indexes is carried out. In our work we are going to consider the Similarity measure based on contrast de-enhancement. This index has been used, for instance, in [5] and it satisfies the six properties demanded in [2] to similarity indexes. With the notations we are using, given $A, B \in \mathcal{M}_{n\times m}$, this index is given by:

$$S(A,B) = \frac{1}{n \cdot m} \sum_{i=1}^{n} \sum_{j=1}^{m} 1 - |a_{ij} - b_{ij}|.$$

In the experiments we are going to reduce images in Figure 6 using as weak local reduction operators those based on: minimum, geometric mean, arithmetic mean, median and maximum. In Table 1 we show the results of the comparison by means of the S index between the original images in Figure 6 and the reconstructed ones.

Notice that, in general, the results are very good. In particular, for all the images we obtain the best result with the median. We also obtain similar results with the geometric and the arithmetic means. On the contrary, we get the worst results with the minimum and the maximum. This is due to the fact that with

Table 1. Comparison between reconstructed images and images in Figure 6

	Minimum	Geom. Mean	Arith. Mean	Median	Maximum
Image (a)	0.9586	0.9748	0.9750	0.9759	0.9582
Image (b)	0.9832	0.9854	0.9848	0.9866	0.9803
Image (c)	0.9429	0.9650	0.9651	0.9673	0.9393
Image (d)	0.9509	0.9708	0.9706	0.9719	0.9482

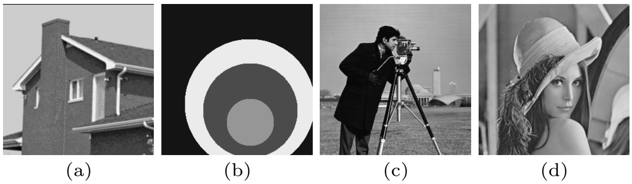

Fig. 6. Original images of size 256 × 256

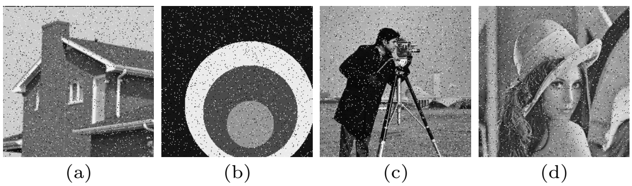

Fig. 7. Images with salt and pepper noise of size 256 × 256

Table 2. Comparison between reconstructed images and images in Figure 7

	Minimum	Geom. Mean	Arith. Mean	Median	Maximum
Image (a)	0.8517	0.8696	0.9621	0.9757	0.8729
Image (b)	0.9116	0.9129	0.9639	0.9865	0.8509
Image (c)	0.8586	0.8819	0.8536	0.9671	0.8457
Image (d)	0.6824	0.8851	0.9607	0.9717	0.8569

these two operators we only take into account a single value for each block, which needs not to be representative of the rest of values in that block.

In Table 2 we show the comparison of the images in Figure 7 and the reconstructed ones. Images in Figure 7 have been obtained by addition of salt and pepper noise to images in Figure 6. In these conditions, the best result is also obtained using the median as weak local reduction operator. This is due to the fact that the value provided by the median is not affected by salt and pepper noise. Moreover, we observe that the operators given by the minimum, the geometric mean and the maximum are very sensitive to this noise. For the first two ones, a single pixel of 0 intensity determines that the value for the corresponding block is also 0. For the maximum, if there is a pixel with intensity equal to 1, then the result is also equal to 1.

So, from the analysis of Tables 1 and 2 and with the considered reconstruction method, we can say that the best weak local reduction operator is that based on the median. Moreover, we must remark that whenever we study noisy images, the best comparison measures are those given in [2] since one of the axioms demanded by the authors is precisely that of reaction in the presence of noise.

7 Conclusions

In this work we have axiomatically defined local reduction operators. We have studied how to construct these operators by means of averaging functions. We have analyzed which properties are satisfied by some of these aggregation-based reduction operators.

From our operators, we have proposed an image reduction algorithm. To settle which is the best local reduction operator, we have proposed an application based on reconstructing the original images from the reduced ones. To compare images we have used a fuzzy similarity index. We have seen that, in all of the cases, the best weak local reduction operator is provided by the median. Moreover, this operator is not affected by salt and pepper noise.

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