Possibilistic Graphical Models and Compositional Models

Jiřina Vejnarová

Institute of Information Theory and Automation of the AS CR,
Pod Vodárenskou věží 4, 182 08 Prague, Czech Republic
&
University of Economics, Prague
vejnar@utia.cas.cz

Abstract. We overview three kinds of possibilistic graphical models (based on directed acyclic graphs) and present, how they can be expressed by means of non-graphical approach to multidimensional models, so-called compositional models. We show that any of these graphical models can be transformed into a compositional model, but not vice versa. The only exception are directed possibilistic graphs, which are as general as so-called prefect sequences of low-dimensional distributions.

Keywords: Possibility distributions, graphical models, triangular norms.

1 Introduction

High dimensionality of problems usually solved in the field of artificial intelligence led in late 1980's to the emergence of new kind of models, usually called *graphical Markov models*. These models, sometimes characterized as a "marriage between probability and graph theories", utilize different types of graphs to express (in)dependences among variables.

Nevertheless, uncertainty can be modeled also by other calculi; among them we concentrated to possibility theory, which has in common with probability theory the advantage, that possibility measure can be expressed by means of possibility distribution. In this contribution we overview three kinds of possibilistic graphical models (based on directed acyclic graphs) and present, how they can be expressed by means of non-graphical approach to multidimensional possibilistic models, so-called *compositional models* — introduced already in [7] and further developed e.g. in [8,11].

The paper is organized as follows. After an overview of necessary notions form possibility theory in Section 2, in Section 3 we will present the most important results on compositional models. Section 4 will be devoted to the graphical models and their relationship to compositional models.

2 Basic Notions

The purpose of this section is to give, as briefly as possible, an overview of basic notions of De Cooman's measure-theoretical approach to possibility theory [3],

E. Hüllermeier, R. Kruse, and F. Hoffmann (Eds.): IPMU 2010, Part I, CCIS 80, pp. 21–30, 2010.

necessary for understanding the paper. Special attention will be paid to conditioning, independence and conditional independence [9]. We will start with the notion of a triangular norm, since most notions in this paper are parameterized by it.

2.1 Triangular Norms

A *triangular norm* (or a *t-norm*) T is a nondecreasing, associative and commutative binary operator on $[0,1]$ satisfying the boundary condition: for any $a \in [0,1]$

$$T(1,a) = a.$$

A t-norm T is called *continuous* if T is a continuous function. Within this paper, we will only deal with continuous t-norms.

Let $x, y \in [0,1]$ and T be a t-norm. We will call an element $z \in [0,1]$ *T-inverse* of x w.r.t. y if

$$T(z,x) = T(x,z) = y. \tag{1}$$

It is obvious that if $x \leq y$ then the equation (1) admits no solution, i.e. there are no T-inverses of x w.r.t. y. On the other hand, if a T-inverse exists, it need not be unique. Nevertheless, we can obtain a unique representative (which is even maximal) using the notion of *T-residual* $y \triangle_T x$ of y by x defined for any $x, y \in [0,1]$ as

$$y \triangle_T x = \sup\{z \in [0,1] : T(z,x) \leq y\}.$$

From the viewpoint of this paper the following lemma proven in [5] is important, as it gives a hint, how to compute with residuals.

Lemma 1. *If $c \geq b$, then $T(a,b) \triangle_T c = T(a, b \triangle_T c)$.*

2.2 Possibility Measures and Distributions

Let $\mathbf{X}$ be a finite set called *universe of discourse* which is supposed to contain at least two elements. A *possibility measure* Π is a mapping from the power set $\mathcal{P}(\mathbf{X})$ of $\mathbf{X}$ to the real unit interval $[0,1]$ satisfying the following two requirements:

(i) $\Pi(\emptyset) = 0$;
(ii) for any family $\{A_j, j \in J\}$ of elements of $\mathcal{P}(\mathbf{X})$

$$\Pi\left(\bigcup_{j \in J} A_j\right) = \max_{j \in J} \Pi(A_j)^1.$$

Within this paper we will always assume that Π is *normal*, i.e. $\Pi(\mathbf{X}) = 1$.

For any Π there exists a mapping $\pi : \mathbf{X} \to [0,1]$, called a *distribution* of Π, such that for any $A \in \mathcal{P}(\mathbf{X})$, $\Pi(A) = \max_{x \in A} \pi(x)$. This function is a

[1] Max must be substituted by sup if $\mathbf{X}$ is not finite.

possibilistic counterpart of a density function in probability theory. It is evident that (in the finite case) Π is normal iff there exists at least one $x \in \mathbf{X}$ such that $\pi(x) = 1$. Throughout this paper we will use possibility distributions instead of possibility measures.

Let $\mathbf{X}_1$ and $\mathbf{X}_2$ denote two finite universes of discourse provided by possibility measures Π_1 and Π_2, respectively. The possibility measure Π on $\mathbf{X}_1 \times \mathbf{X}_2$ is called *T-product possibility measure* of Π_1 and Π_2 (denoted $\Pi_1 \times_T \Pi_2$) if for the corresponding possibility distributions for any $(x_1, x_2) \in \mathbf{X}_1 \times \mathbf{X}_2$

$$\pi(x_1, x_2) = T(\pi_1(x_1), \pi_2(x_2)). \tag{2}$$

Now, let us consider an arbitrary possibility distribution π defined on a product universe of discourse $\mathbf{X}_1 \times \mathbf{X}_2$. The *marginal possibility distribution* $\pi^{\downarrow 1}$ on $\mathbf{X}_1$ is defined by the expression

$$\pi^{\downarrow 1}(x_1) = \max_{x_2 \in \mathbf{X}_2} \pi(x_1, x_2) \tag{3}$$

for any $x_1 \in \mathbf{X}_1$.

2.3 Conditioning, Independence and Conditional Independence

Let T be a continuous t-norm on $[0, 1]$. The *conditional possibility distribution* $\pi_{X|_T Y}$ is defined (in accordance with [3]) as *any* solution of the equation

$$\pi_{XY}(x, y) = T(\pi_Y(y), \pi_{X|_T Y}(x|_T y)) \tag{4}$$

for any $(x, y) \in \mathbf{X} \times \mathbf{Y}$. Continuity of a t-norm T guarantees the existence of a solution of this equation. This solution is not unique (in general), but the ambiguity vanishes when almost-everywhere equality is considered (for more details see [3]). As mentioned in [3,9], this way of conditioning brings a unifying view on several conditioning rules and it also plays an important role in the definition of (conditional) independence, therefore its importance from the theoretical viewpoint is obvious. On the other hand, from the practical point of view, its expression by residual $\pi_{XY}(x, \cdot) \triangle_T \pi_Y(\cdot)$, i.e. the least specific (or maximal) solution of (4), is very useful (for more details see [11]).

Two variables X and Y (taking their values in $\mathbf{X}$ and $\mathbf{Y}$, respectively) are *possibilistically T-independent*[2] [3] if for any $x \in \mathbf{X}$ and $y \in \mathbf{Y}$

$$\pi_{XY}(x, y) = T(\pi_X(x), \pi_Y(y)). \tag{5}$$

In light of these facts, we defined the conditional possibilistic independence in the following way in [8]: Given a possibility measure Π on $\mathbf{X} \times \mathbf{Y} \times \mathbf{Z}$ with the respective distribution $\pi(x, y, z)$, variables X and Y are *possibilistically*

[2] Let us note that the definition presented in [3] is different and (5) is its equivalent characterization. Nevertheless, from the viewpoint of this paper (5) is more convenient.

conditionally T-independent[3] given Z (in symbols $I_T(X, Y|Z)$) if, for any pair $(x, y) \in \mathbf{X} \times \mathbf{Y}$,

$$\pi_{XYZ}(x, y, z) = T(T(\pi_{X|_T Z}(x|_T z), \pi_{Y|_T Z}(y|_T z)), \pi_Z(z)). \tag{6}$$

In [9] we proved its formal properties and studied its relationship with other definitions of conditional possibilistic independence, among others those introduced in [2].

3 Compositional Models

From now on, we will deal with joint possibility distributions π on Cartesian product of universes of discourse

$$\mathbf{X}_N = \mathbf{X}_1 \times \mathbf{X}_2 \times \ldots \times \mathbf{X}_n,$$

and their marginals $\pi^{\downarrow K}$ on its subspaces

$$\mathbf{X}_K = \times_{i \in K} \mathbf{X}_i.$$

3.1 Operators of Composition

Operators of composition of possibility distributions introduced in [7] are, in a way, a generalization of T-product possibility distributions defined by (2). Considering a continuous t-norm T, two subsets K_1, K_2 of $\{1, \ldots, n\}$ (not necessarily disjoint) and two normal possibility distributions $\pi_1(x_{K_1})$ and $\pi_2(x_{K_2})$ we define the *operator of right composition* of these possibilistic distributions by the expression

$$\pi_1(x_{K_1}) \triangleright_T \pi_2(x_{K_2}) = T\left(\pi_1(x_{K_1}), \pi_2(x_{K_2}) \triangle_T \pi_2^{\downarrow K_1 \cap K_2}(x_{K_1 \cap K_2})\right),$$

and analogously the *operator of left composition* by the expression

$$\pi_1(x_{K_1}) \triangleleft_T \pi_2(x_{K_2}) = T\left(\pi_1(x_{K_1}) \triangle_T \pi_1^{\downarrow K_1 \cap K_2}(x_{K_1 \cap K_2}), \pi_2(x_{K_2})\right).$$

It is evident that both $\pi_1 \triangleright_T \pi_2$ and $\pi_1 \triangleleft_T \pi_2$ are (generally different) possibility distributions of variables $\{X_i\}_{i \in K_1 \cup K_2}$.

Now, we will present two lemmata proven in [7], expressing basic properties of these operators.

Lemma 2. *Let T be a continuous t-norm and $\pi_1(x_{K_1})$ and $\pi_2(x_{K_2})$ be two distributions. Then*

$$(\pi_1 \triangleright_T \pi_2)^{\downarrow K_1}(x_{K_1}) = \pi_1(x_{K_1})$$

and

$$(\pi_1 \triangleleft_T \pi_2)^{\downarrow K_2}(x_{K_2}) = \pi_2(x_{K_2}).$$

[3] Let us note that a similar definition of conditional independence can be found in [4].

Lemma 3. *Consider two distributions* $\pi_1(x_{K_1})$ *and* $\pi_2(x_{K_2})$*. Then*

$$(\pi_1 \triangleright_T \pi_2)(x_{K_1 \cup K_2}) = (\pi_1 \triangleleft_T \pi_2)(x_{K_1 \cup K_2})$$

for any continuous t-norm T *iff* π_1 *and* π_2 *are projective, i.e.*

$$\pi_1^{\downarrow K_1 \cap K_2}(x_{K_1 \cap K_2}) = \pi_2^{\downarrow K_1 \cap K_2}(x_{K_2 \cap K_1}).$$

The following theorem proven in [8] reveals the relationship between conditional T-independence and operators of composition.

Theorem 1. *Let* T *be a continuous t-norm and* π *be a possibility distribution of* $X_{K_1 \cup K_2}$ *with marginals* π_1 *and* π_2 *of* X_{K_1} *and* X_{K_2}*, respectively. Then*

$$\begin{aligned} \pi(x_{K_1 \cup K_2}) &= (\pi_1 \triangleright_T \pi_2)(x_{K_1 \cup K_2}) \\ &= (\pi_1 \triangleleft_T \pi_2)(x_{K_1 \cup K_2}), \end{aligned} \tag{7}$$

if and only if $X_{K_1 \setminus K_2}$ *and* $X_{K_2 \setminus K_1}$ *are conditionally independent, given* $X_{K_1 \cap K_2}$*.*

3.2 Generating Sequences

In this section we will show how to apply the operators iteratively. Consider a sequence of possibility distributions $\pi_1(x_{K_1}), \pi_2(x_{K_2}), \ldots, \pi_m(x_{K_m})$ and the expression

$$\pi_1 \triangleright_T \pi_2 \triangleright_T \ldots \triangleright_T \pi_m.$$

Before beginning a discussion of its properties, we have to explain how to interpret it. Though we did not mention it explicitly, the operator $\triangleright_T$ (as well as $\triangleleft_T$) is neither commutative nor associative.[4] Therefore, generally

$$(\pi_1 \triangleright_T \pi_2) \triangleright_T \pi_3 \neq \pi_1 \triangleright_T (\pi_2 \triangleright_T \pi_3).$$

Nevertheless, under specific conditions this equality is satisfied. One of these situations, important from the viewpoint of this paper, is described by the following lemma.

Lemma 4. *Let* T *be a continuous t-norm and* π_1, π_2 *and* π_3 *be defined on* $\mathbf{X}_{K_1}, \mathbf{X}_{K_2}$ *and* $\mathbf{X}_{K_3}$*, respectively, such that* K_1 *and* K_3 *are disjoint. Then*

$$(\pi_1 \triangleright_T \pi_2) \triangleright_T \pi_3 = \pi_1 \triangleright_T (\pi_2 \triangleright_T \pi_3). \tag{8}$$

Proof. Let $x \in \mathbf{X}_{K_1 \cup K_2 \cup K_3}$ then the right-hand side of (8) is by definition

$$\begin{aligned} &\pi_1 \triangleright_T (\pi_2 \triangleright_T \pi_3)(x) \\ &\quad = T(\pi_1(x_{K_1}), (\pi_2 \triangleright_T \pi_3)(x_{K_2 \cup K_3}) \triangle_T (\pi_2 \triangleright_T \pi_3)(x_{(K_2 \cup K_3) \cap K_1})) \\ &\quad = T(\pi_1(x_{K_1}), T(\pi_2(x_{K_2}), \pi_3(x_{K_3}) \triangle_T \pi_3(x_{K_3 \cap K_2})) \triangle_T \pi_2(x_{K_2 \cap K_1})) \\ &\quad = T(\pi_1(x_{K_1}), T(\pi_2(x_{K_2}) \triangle_T \pi_2(x_{K_2 \cap K_1}), \pi_3(x_{K_3}) \triangle_T \pi_3(x_{K_3 \cap K_2}))) \\ &\quad = T(T(\pi_1(x_{K_1}), \pi_2(x_{K_2}) \triangle_T \pi_2(x_{K_2 \cap K_1})), \pi_3(x_{K_3}) \triangle_T \pi_3(x_{K_3 \cap K_2})) \\ &\quad = (\pi_1 \triangleright_T \pi_2) \triangleright_T \pi_3(x), \end{aligned}$$

[4] Counterexamples can be found in [7].

where we used the fact that $K_1 \cap K_2 = \emptyset$, Lemma 2, Lemma 1 and associativity of a t-norm. $\square$

For the above reason, let us note that in the part that follows, we always apply the operators from left to right, i. e.

$$\pi_1 \triangleright_T \pi_2 \triangleright_T \pi_3 \triangleright_T \ldots \triangleright_T \pi_m = (\ldots((\pi_1 \triangleright_T \pi_2) \triangleright_T \pi_3) \triangleright_T \ldots \triangleright_T \pi_m). \quad (9)$$

This expression defines a multidimensional distribution of $X_{K_1 \cup \ldots \cup K_m}$. Therefore, for any permutation $i_1, i_2, \ldots, i_m$ of indices $1, \ldots, m$ the expression

$$\pi_{i_1} \triangleright_T \pi_{i_2} \triangleright \ldots \triangleright_T \pi_{i_m}$$

determines a distribution of the same family of variables, however, for different permutations these distributions can differ from one another. In the following paragraph we will deal with special generating sequences (or their special permutations), which seem to possess the most advantageous properties.

3.3 *T*-Perfect Sequences

An ordered sequence of possibility distributions $\pi_1, \pi_2, \ldots, \pi_m$ is said to be *T-perfect* if

$$\begin{aligned} \pi_1 \triangleright_T \pi_2 &= \pi_1 \triangleleft_T \pi_2, \\ \pi_1 \triangleright_T \pi_2 \triangleright_T \pi_3 &= \pi_1 \triangleleft_T \pi_2 \triangleleft_T \pi_3, \\ &\vdots \\ \pi_1 \triangleright_T \cdots \triangleright_T \pi_m &= \pi_1 \triangleleft_T \cdots \triangleleft_T \pi_m. \end{aligned}$$

The notion of T-perfectness suggests that a sequence perfect with respect to one t-norm need not be perfect with respect to another t-norm, analogous to (conditional) T-independence. The following lemma, proven in [7], suggests that perfectness is a stronger property than pairwise projectivity (cf. Lemma 3).

Lemma 5. *Let T be a continuous t-norm. The sequence $\pi_1, \pi_2, \ldots, \pi_m$ is T-perfect, if and only if the pairs of distributions $(\pi_1 \triangleright_T \cdots \triangleright_T \pi_{k-1})$ and π_k are projective for all $k = 2, 3, \ldots, m$.*

Although T-perfect sequences may be defined for any continuous t-norm T, their semantics substantially differ from each other. For more details the reader is referred to [10].

The following characterization theorem proven in [11] expresses one of the most important results concerning T-perfect sequences. It says they compose into multidimensional distributions that are extensions of all the distributions from which the joint distribution is composed.

Theorem 2. *The sequence $\pi_1, \pi_2, \ldots, \pi_m$ is T-perfect iff all the distributions $\pi_1, \pi_2, \ldots, \pi_m$ are marginal to distribution $\pi_1 \triangleright_T \pi_2 \triangleright_T \ldots \triangleright \pi_m$.*

If we translate this theorem to the language of artificial intelligence, its meaning is that the global knowledge expressed by the multidimensional distribution keeps all the local knowledge contained in the low-dimensional distributions, i.e. nothing was lost or changed.

4 Graphical Models

Probabilistic graphical models (well-known thanks e.g. to [6]) served as the inspiration for various authors e.g. [1,2] to introduce analogous models also in the framework of possibility theory.

4.1 Possibilistic Trees

Possibilistic trees suggested by de Campos and Huete in [2] for specific conditional independence concepts are based on the following simple idea. If $I_T(X, Y|Z)$, then the joint distribution $\pi(x, y, z)$ of X, Y, Z can be obtained from its marginals $\pi(x, z)$ and $\pi(y, z)$.

This idea can easily be generalized to n-dimensional case. Let us assume variables $X_1, \ldots, X_n$ such that $I_T(\{X_j\}_{j<i}\{X_j\}_{j>i}|i)$, then the joint possibility distribution of these variables can be obtained form the marginals $\pi(x_1, \ldots, x_i)$ and $\pi(x_i, \ldots, x_n)$. This idea can be recursively applied to both subsets of variables. Therefore to obtain the joint possibility distribution, it is enough to store low-dimensional distributions obtained by this process.

Resulting *possibilistic tree* $\mathcal{T}$ consists of two kinds of nodes — *leaf nodes* (which store marginal possibility distributions) and *internal nodes* (storing conditional independence statements).

De Campos and Huete presented two propositions concerning possibilistic trees (induced, in fact, by conditional independence concepts based on Gödel's and product t-norms), which can be generalized as suggested below.

Any possibilistic tree $\mathcal{T}$ can easily be transformed into a generating sequence of its leaves $\pi_{L_1}, \ldots \pi_{L_m}$. The joint possibiility distribution is then obtained in the following way: any fork of $\mathcal{T}$ is substituted by a composition operator connecting marginal distributions of corresponding branches. Let us note, that this transformation keeps according to Theorem 1 all the conditional independences expressed by the possibilistic tree.

Let us also note, that because of rather complicated system of brackets, the resulting model is not generally formed by a T-perfect sequence of possibility distributions. Nevertheless, it must exist, as the following lemma suggests.

Lemma 6. *Any possibilistic tree $\mathcal{T}$ defines a perfect sequence.*

Proof. It follows directly from Theorem 2, as distributions at leaves are marginals of the joint possibility distribution. □

Now, let us preset an example, which is a generalization of examples from [2].

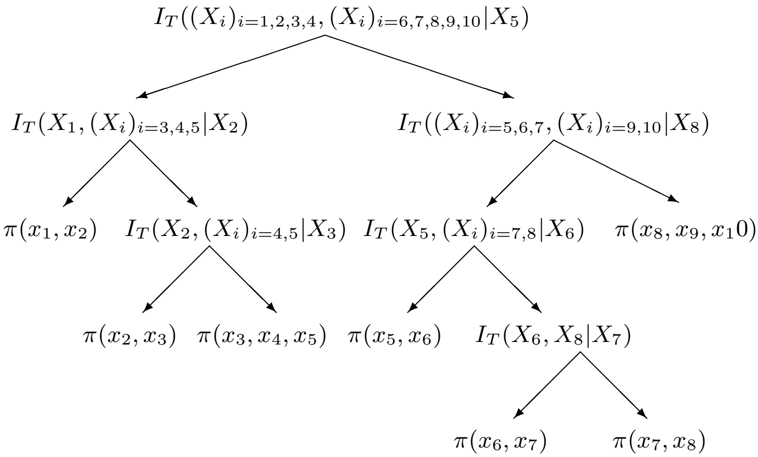

Fig. 1. Possibilistic tree from Example 1

Example 1 Let π be a joint possibility distribution of $X_i, i = 1, \ldots, 10$ with conditional independences (based on continuous t-norm T) expressed by possibilistic tree in Figure 1.

The generating sequence $\pi(x_1, x_2), \pi(x_2, x_3), \pi(x_3, x_4, x_5), \pi(x_5, x_6)$, $\pi(x_6, x_7)$, $\pi(x_7, x_8), \pi(x_8, x_9, x_{10})$ forms a joint distribution

$$\begin{aligned}(\pi(x_1,x_2) \triangleright_T (\pi(x_2,x_3) \triangleright_T \pi(x_3,x_4,x_5)))& \\ \triangleright_T((\pi(x_5,x_6) \triangleright_T (\pi(x_6,x_7) \triangleright_T \pi(x_7,x_8))) \triangleright_T \pi(x_8,x_9,x_{10})).&\end{aligned} \tag{10}$$

Although it is not obvious at the first sight it is also perfect as (10) can be transformed into

$$\begin{aligned}\pi(x_1,x_2) \triangleright_T \pi(x_2,x_3) \triangleright_T \pi(x_3,x_4,x_5)& \\ \triangleright_T \pi(x_5,x_6) \triangleright_T \pi(x_6,x_7) \triangleright_T \pi(x_7,x_8) \triangleright_T \pi(x_8,x_9,x_{10}).&\end{aligned}$$

due to Lemma 4 and convention (9). Therefore $\pi(x_1, x_2), \pi(x_2, x_3)$, $\pi(x_3, x_4, x_5)$, $\pi(x_5, x_6)$, $\pi(x_6, x_7)$, $\pi(x_7, x_8), \pi(x_8, x_9, x_{10})$ is a perfect sequence.

Let us note, that another ordering of the marginal possibility distributions, e.g. $\pi(x_1, x_2), \pi(x_3, x_4, x_5), \pi(x_2, x_3), \pi(x_5, x_6)$, $\pi(x_6, x_7)$, $\pi(x_7, x_8), \pi(x_8, x_9, x_{10})$, may lead to a different model than that expressed by a possibilistic tree $\mathcal{T}$, as the resulting model does not keep $\pi(x_2, x_3)$ unless $\pi(x_2, x_3) = \pi(x_2) \cdot \pi(x_3)$.

Let us also note that not every perfect sequence can be transformed into a possibilistic tree, e.g. if one variable appears in three (or more) marginals.

4.2 Dependence Trees

In dependence trees [2] nodes represent variables (or groups of variables) and edges represent direct dependence relationship among variables (or groups). Conditional independence statements can be obtained from the graph in an analogous way to Bayesian networks.

In [2] a simple algorithm for the construction of dependence tree of a possibility distribution in question is presented. In that paper it is also shown (by examples) how to a transform dependence tree to a possibilistic tree and vice versa. It is also mentioned that possibilistic tree is a more general structure than dependence tree. As we have shown that any possibilistic tree can be transformed to perfect sequence, it is obvious, that the same holds for dependence trees.

Nevertheless, we present a direct procedure of the transformation of dependence tree to a perfect sequence, which is extremely simple.

For each dependence tree one can construct a perfect sequence $\pi_1, \ldots, \pi_m$ of distributions of variables $X_{K_1}, X_{K_2}, \ldots, X_{K_m}$, respectively. These distributions are such that each $\{X_i\}_{i \in K_k}$ equals some $cl(X_j) = \{X_j\} \cup pa(X_j)$ and $\pi_1 \triangleright \ldots \triangleright \pi_m$ equals the distribution represented by the dependence tree.

This approach can be applied also to more general directed possibilistic graphs [1], which will be in the center of our attention in the next part.

4.3 Directed Possibilistic Graphs

Directed possibilistic graph (or *possibilistic belief network*) is a possibilistic counterpart of Bayesian network (and a generalization of dependence trees) and can be defined in the following way:

Relationships among variables in directed possibilistic graph are determined in two ways. Structural information describing the existence of a “direct” dependence of variables is given by a graph, while the quantitative information is given by a system of conditional possibility distributions. Thus, a possibilistic belief network is a couple: an *acyclic directed graph* and a *system of conditional probability distributions*. In this system there are as many distributions as variables, i.e. nodes of the graph (in contrary to dependence trees). For each variable there is a conditional distribution given all *parent* variables in the condition. Some of nodes (at least one because of acyclicity) are parentless and their distributions are in fact unconditional.

To transform a possibilistic belief network into a a perfect sequence the procedure described in the preceding section can be used. Here we present a reverse procedure for transformation of a perfect sequence into a possibilistic belief network.

Having a perfect sequence $\pi_1, \pi_2, \ldots, \pi_m$ (π_k being the distribution of X_{K_k}), we first order (in an arbitrary way) all the variables for which at least one of the distributions π_k is defined, i.e.

$$\{X_1, X_2, X_3, \ldots, X_n\} = \{X_i\}_{i \in K_1 \cup \ldots \cup K_m}.$$

Then we get a graph of the constructed possibilistic belief network in the following way:

1. the nodes are all the variables $X_1, X_2, X_3, \ldots, X_n$;
2. there is an edge $(X_i \rightarrow X_j)$ if there exists a distribution π_k such that both $i, j \in K_k$, $j \notin K_1 \cup \ldots \cup K_{k-1}$ and either $i \in K_1 \cup \ldots \cup K_{k-1}$ or $i < j$.

5 Conclusions

We overviewed the non-graphical approach to multidimensional possibilistic models based on operators of composition — so-called compositional models. We presented three types of graphical models and showed, how these models can be expressed by means of compositional models. Furthermore, we showed that any of these three models can be expressed by a perfect sequence of low-dimensional distributions. Finally, we presented a procedure by which any perfect sequence of low-dimensional distributions can be transformed into directed possibilistic graph (or possibilistic belief network).

Acknowledgments. The work of the author was supported by the grant GA ČR 201/09/1891, by the grant GA AV ČR A100750603 and by the grant MŠMT 2C06019.

References

1. Benferhat, S., Dubois, D., Gracia, L., Prade, H.: Directed possibilistic graphs and possibilistic logic. In: Bouchon-Meunier, B., Yager, R.R. (eds.) Proc. of the 7th Int. Conf. on Information Processing and Management of Uncertainty in Knowledge-based Systems IPMU 1998, Editions E.D.K. Paris, pp. 1470–1477 (1998)
2. de Campos, L.M., Huete, J.F.: Independence concepts in possibility theory: Part 1 and Part 2. Fuzzy sets and systems 103, 127–152, 487–505 (1999)
3. de Cooman, G.: Possibility theory I – III. Int. J. General Systems 25, 291–371 (1997)
4. Janssen, H., de Cooman, G., Kerre, E.E.: First results for a mathematical theory of possibilistic Markov processes. In: Proceedings of IPMU 1996. Information Processing and Management of Uncertainty in Knowledge-Based Systems, Granada, Spain, vol. III, pp. 1425–1431 (1996)
5. Jiroušek, R., Vejnarová, J.: General framework for multidimensional models. International Journal of Intelligent Systems 18, 107–127 (2003)
6. Pearl, J.: Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference. Morgan Kaufmann, San Francisco (1988)
7. Vejnarová, J.: Composition of possibility measures on finite spaces: preliminary results. In: Proceedings of 7th International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems IPMU 1998, Paris, pp. 25–30 (1998)
8. Vejnarová, J.: Possibilistic independence and operators of composition of possibility measures. In: Hušková, M., Víšek, J.Á., Lachout, P. (eds.) Prague Stochastics 1998, JČMF, pp. 575–580 (1998)
9. Vejnarová, J.: Conditional independence relations in possibility theory. Int. J. Uncertainty, Fuzziness and Knowledge-Based Systems 8, 253–269 (2000)
10. Vejnarová, J.: On interpretation of T-product possibility distributions. In: Bouchon-Meunier, B., Foulloy, L., Yager, R.R. (eds.) 9th International Conference on Information Processing and Management of Uncertainty IPMU 2002, ESIA - Université de Savoie, France, pp. 1053–1060 (2002)
11. Vejnarová, J.: On possibilistic marfginal problem. Kybernetika 43, 657–674 (2007)