The Proof of Completeness of the Graph Method for Generation of Affine Moment Invariants*

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Abstract. Features for recognition of affinely distorted objects are of great demand. The affine moment invariants can be generated by a few methods, namely the graph method, the tensor method and the direct solution of the Cayley-Aronhold differential equation. The proof of their equivalence is complicated; it can be derived from the Gurevich's proof for affine tensor invariants. The theme of this paper is this derivation.

1 Introduction

Recognition of objects on images is an important part of many image processing applications. The images are often geometrically distorted and derivation of features invariant to such a distortion is of great demand. The invariants with respect to affine transform are often used, mostly as approximation of a projective distortion. The affine invariants can be computed from various measurements of the image, e.g. as point invariants, differential invariants, Fourier descriptors, etc. The invariants computed from moments play important role among them.

The affine moment invariants can be derived by a few ways. Recently, approximately from beginning of 90's, they are generated automatically, by a computer. There are two groups of methods for this generation, the graph method [1] and computationally equivalent tensor method [2] on one hand and direct solution of the Cayley–Aronhold differential equation [3] on the other hand. The graph method is easier to implement, but with worse computing complexity, while the direct solution of the equation is faster, but less numerically stable.

It is natural to ask the question whether or not all the invariants generated by the graph method are equivalent to that found by means of the Cayley-Aronhold equation, and vice versa. We have used invariants from both methods in pattern recognition for many years and have not found any inequivalence between them, so, the positive answer is likely, but precise proof is difficult. Both the book [2] with the tensor method and the book [4] with a survey of both methods reference only to the Gurevich's proof from [5] (Russian edition [6]) that can be used for derivation of our proof. This derivation is theme of this paper.

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2 Basic Terms

Affine transformation can be expressed as

$$\begin{aligned}\hat{x} &= q_1^1 x + q_2^1 y + q_3^1 \\ \hat{y} &= q_1^2 x + q_2^2 y + q_3^2,\end{aligned} \tag{1}$$

its Jacobian is $J = q_1^1 q_2^2 - q_2^1 q_1^2$. The geometric moment of the order $p+q$ of an image $f(x,y)$ is defined

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)\ \mathrm{d}x\ \mathrm{d}y. \tag{2}$$

2.1 Graph Method

The affine moment invariant can be computed as

$$I(f) = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\prod_{k,j=1}^{r} C_{kj}^{n_{kj}} \cdot \prod_{i=1}^{r} f(x_i,y_i)\mathrm{d}x_i\mathrm{d}y_i, \tag{3}$$

where $C_{kj} = x_k y_j - x_j y_k$ is the oriented double area of the triangle, whose vertices are (x_k,y_k), (x_j,y_j), and $(0,0)$ and n_{kj} are some non-negative integers. The number $w = \sum\limits_{k,j} n_{kj}$ is called the weight of the invariant and r is called the degree of the invariant. The maximum order s of moments of which the invariant is composed is called the order of the invariant. Another important characteristic of the invariant is its structure, it is defined by an integer vector $\mathbf{s} = (k_2, k_3, \ldots, k_s)$, where k_j is the total number of moments of the jth order contained in each term of the invariant.

After an affine transform (we consider no translation) it holds $\hat{C}_{kj} = J \cdot C_{kj}$, which means that C_{kj} is a relative affine invariant. The functional (3) can be normalized to translation and scaling to be invariant to the general affine transform. Each such an invariant can be represented by a connected graph, where each point (x_k,y_k) corresponds to one node and each cross-product C_{kj} corresponds to one edge of the graph. If $n_{kj} \geq 1$, the respective term $C_{kj}^{n_{kj}}$ corresponds to n_{kj} edges connecting kth and jth nodes. The problem of derivation of the invariants up to the given weight w is equivalent to generating all connected graphs with at least two nodes and at most w edges.

2.2 Tensor Method

The generation of the affine moment invariants can be expressed in terms of tensors. The moments themselves do not behave under affine transform like tensors, but we can define a moment tensor [7]

$$M^{i_1 i_2 \cdots i_r} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^{i_1} x^{i_2} \cdots x^{i_r} f(x^1,x^2)\mathrm{d}x^1\mathrm{d}x^2, \tag{4}$$

where $x^1 = x$ and $x^2 = y$. If p indices equal 1 and q indices equal 2, then $M^{i_1 i_2 \cdots i_r} = m_{pq}$. The behavior of the moment tensor under an affine transform

$$\hat{M}^{i_1 i_2 \cdots i_r} = |J|^{-1} q^{i_1}_{\alpha_1} q^{i_2}_{\alpha_2} \cdots q^{i_r}_{\alpha_r} M^{\alpha_1 \alpha_2 \cdots \alpha_r}, \qquad i_1, i_2, \ldots, i_r, \alpha_1, \alpha_2, \ \ldots, \ \alpha_r = 1, 2; \tag{5}$$

i.e. the moment tensor is a relative contravariant tensor with the weight $g = -1$ (in tensor calculus, the affine transform is understood inversely, p^i_α are the coefficients of the direct transform, q^i_α are that of the inverse transform, $J = p^1_1 p^2_2 - p^1_2 p^2_1$ is its Jacobian).

The covariant unit polyvector $\epsilon_{i_1 i_2 \cdots i_n}$ is a skew-symmetric tensor over all indices and $\epsilon_{12 \cdots n} = 1$. The term *skew-symmetric* means that the tensor component changes its sign and preserves its absolute value when interchanging two indices. In two dimensions, it means that $\epsilon_{12} = 1$, $\epsilon_{21} = -1$, $\epsilon_{11} = 0$ and $\epsilon_{22} = 0$. The contravariant unit polyvector (in two dimensions $\epsilon^{i_1 i_2}$) has similar properties except that it is multiplied as contravariant tensor, e.g.

$$\epsilon_{i_1 i_2} \epsilon^{i_1 i_2} = 2. \tag{6}$$

If we multiply the proper number of moment tensors and unit polyvectors so that the number of upper indices at the moment tensors equals the number of lower indices at polyvectors, we obtain a real-valued relative affine invariant, e.g.

$$M^{ij} M^{klm} M^{nop} \epsilon_{ik} \epsilon_{jn} \epsilon_{lo} \epsilon_{mp} = \\ = 2(m_{20}(m_{21} m_{03} - m_{12}^2) - m_{11}(m_{30} m_{03} - m_{21} m_{12}) + m_{02}(m_{30} m_{12} - m_{21}^2)). \tag{7}$$

This method is analogous to the graph method. Each moment tensor corresponds to a node of the graph and each unit polyvector corresponds to an edge. The indices indicate, which edge connects which nodes. The graph corresponding to the invariant (7) is on Fig. 1.

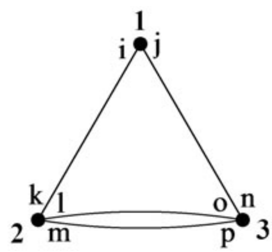

Fig. 1. The graph corresponding to the invariant from (7) and (8)

This invariant can be generated by the graph method as

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} C_{12} C_{13} C_{23}^2 f(x_1, y_1) f(x_2, y_2) f(x_3, y_3) \mathrm{d}x_1 \mathrm{d}y_1 \mathrm{d}x_2 \mathrm{d}y_2 \mathrm{d}x_3 \mathrm{d}y_3. \tag{8}$$

2.3 Solution of the Cayley-Aronhold Equation

The affine transformation (1) can be decomposed into horizontal and vertical translation, scaling, stretching, horizontal and vertical skewing and possible mirror reflection. Each of these transformations imposes one constraint on the invariants.

The invariance to translation and scaling is provided by the same way in all the methods. We use central moments for translation invariance

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - x_c)^p (y - y_c)^q f(x, y)\, \mathrm{d}x\, \mathrm{d}y, \qquad (9)$$

where $x_c = m_{10}/m_{00}$, $y_c = m_{01}/m_{00}$ are the coordinates of the object centroid. The normalization to scaling can be assured by using scale-normalized moments

$$\nu_{pq} = \frac{\mu_{pq}}{\mu_{00}^{((p+q)/2+1)}}. \qquad (10)$$

The invariants are supposed to have a form of a linear combination of moment products

$$I = \sum_{j=1}^{n_t} \kappa_j \prod_{k=1}^{r} \nu_{p_{jk},q_{jk}}, \qquad (11)$$

where n_t is the number of terms of the invariant. The invariance to stretching can be achieved by using only that products of moments, where the sum of first indices (labeled as p) equals the sum of second indices (labeled as q)

$$\forall j = 1, \ldots, n_t : \sum_{k=1}^{r} p_{jk} = \sum_{k=1}^{r} q_{jk} = w. \qquad (12)$$

From the horizontal skew constraint we can derive the Cayley–Aronhold differential equation

$$\sum_p \sum_q p\nu_{p-1,q+1} \frac{\partial I}{\partial \nu_{pq}} = 0. \qquad (13)$$

The Cayley–Aronhold differential equation leads to a solution of a system of linear equations for the unknown coefficients κ_j of the invariant.

3 Relation between the Invariants from Both Methods

We have generated two feature sets, one from the graph method, the other as a solution of the Cayley–Aronhold differential equation. The features are homogeneous polynomials of the moments of the same degree, so, the features from one set are linear combinations of the features from the other set and there is the question: are both sets equivalent? If we have some weight limit, then we have finite number of invariants from the graph method, while the number of invariants from the solution of the equations is infinite (each linear combination of the basic solution is also a solution).

All affine moment invariants must satisfy the Cayley–Aronhold differential equation, i.e. if the features generated by the graph method are really affine invariants (and they are), they can be obtained as a solution of the equation. The inverse statement is not so clear, we will formulate it as theorem:

Theorem 1. *All affine moment invariants in the polynomial form (11) can be expressed as linear combinations of some invariants generated by the graph method*

$$I^{(e)} = \sum_{P=1}^{n} c_P I_P^{(g)}. \tag{14}$$

Here $I^{(e)}$ *is a general affine moment invariant, e.g. generated as some solution of the Cayley-Aronhold differential equation, and* $I_P^{(g)}$, $P = 1, \ldots n$ *is a set of invariants generated by the graph method with the same structure as* $I^{(e)}$.

Proof. Without loss of generality, we will work with the moments without normalization to translation and scaling, i.e. the geometric moments, because the main question are coefficients of the invariants, not this normalization. The equation (14) can be understood as a system of liner equations for unknown c_P's and we exert to prove that this system has always a solution.

Decomposition. The invariant $I^{(e)}$ can be decomposed into a part of moments B and a part of coefficients K

$$I^{(e)} = K_{i_1 i_2 \cdots i_{2w}} B^{i_1 i_2 \cdots i_{2w}}, \tag{15}$$

where w is the weight of the invariant. The part B can be expressed as a product of moment tensors

$$B^{i_1 i_2 \cdots i_{2w}} = M^{i_1 i_2 \cdots i_{d_1}} M^{i_{d_1+1} i_{d_1+2} \cdots i_{d_1+d_2}} \cdots M^{i_{2w-d_r+1} i_{2w-d_r+2} \cdots i_{2w}}, \tag{16}$$

where r is the degree of the invariant. If $I^{(e)}$ has a structure $(k_2, k_3, \ldots, k_s)$, then k_2 numbers from d_1, d_2, ... , d_r equals 2, k_3 of them equals 3 up to k_s equals s.

The product of moment tensors in (16) contains all possible products of moments with the given structure, so the decomposition (15) is always possible. If some product of moments occurs several times (e.g. m times) in B, then the corresponding components of K must be multiplied by $1/m$. The invariants $I_P^{(g)}$ for each P can be decomposed to the part of coefficients and the part of moments by the same way, the part B is the same for all $I_P^{(g)}$ and $I^{(e)}$, while the part of coefficients of $I_P^{(g)}$ can be expressed as a product of unit polyvectors

$$I_P^{(g)} = \epsilon_{\{i_1 i_2} \epsilon_{i_3 i_4} \cdots \epsilon_{i_{2w-1} i_{2w}\}_P} B^{i_1 i_2 \cdots i_{2w}}, \tag{17}$$

where $\{i_1 i_2 \cdots i_{2w}\}_P$ means P-th permutation of the indices $i_1, i_2, \ldots, i_{2w}$.

Comparison of the decompositions. Equation (14) can then be rewritten as

$$K_{i_1 i_2 \cdots i_{2w}} B^{i_1 i_2 \cdots i_{2w}} = \sum_{P=1}^{(2w)!} c_P \epsilon_{\{i_1 i_2} \epsilon_{i_3 i_4} \cdots \epsilon_{i_{2w-1} i_{2w}\}_P} B^{i_1 i_2 \cdots i_{2w}}. \tag{18}$$

If the moments do not identically equal zero (they identically equal zero for zero image only), then the part B can be omitted

$$K_{i_1 i_2 \cdots i_{2w}} = \sum_{P=1}^{(2w)!} c_P \epsilon_{\{i_1 i_2} \epsilon_{i_3 i_4} \cdots \epsilon_{i_{2w-1} i_{2w}\}_P}. \tag{19}$$

The single equation (18) with variable moments is splitting into system of 2^{2w} linear equations for $(2w)!$ unknown c_P's with constant coefficients. The summation over all permutations of unit polyvector indices is not anything else than summation over all graphs generating invariants with the given structure.

Solvability. Now, we multiply K by the corresponding number of contravariant unit polyvectors. Then we obtain from (19)

$$K_{i_1 i_2 \cdots i_{2w}} \epsilon^{x_1 x_2} \epsilon^{x_3 x_4} \cdots \epsilon^{x_{2w-1} x_{2w}} = \sum_{P=1}^{(2w)!} c_P^* \delta^{x_1}_{\{i_1} \delta^{x_2}_{i_2} \cdots \delta^{x_{2w}}_{i_{2w}\}_P}, \tag{20}$$

where $c_P^* = 2^w c_P$ and $\delta^{i_1}_{i_2}$ is Kronecker delta, $\delta^{i_1}_{i_2} = 1$ if $i_1 = i_2$ and $\delta^{i_1}_{i_2} = 0$ if $i_1 \neq i_2$. The system of equations (20) has 2^{4w} equations for $(2w)!$ unknowns, but many of the equations are linearly dependent, the rank of the system was not increased. Denote it $((2w)! - t)$, where t is some positive integer. Now take the system of equations

$$\sum_{P=1}^{(2w)!} \delta^{x_1}_{\{i_1} \delta^{x_2}_{i_2} \cdots \delta^{x_{2w}}_{i_{2w}\}_P} \lambda_P = 0 \tag{21}$$

with unknowns λ_1, λ_2, ... , $\lambda_{(2w)!}$. The matrices of the systems (20) and (21) are the same, therefore the rank of (21) is also $((2w)! - t)$. That is why the system (21) has t linearly independent solutions

$$\lambda_P = \lambda_P^\sigma, \qquad \sigma = 1, 2, \ldots, t. \tag{22}$$

Now, we can add to the system (21) t equations

$$\sum_{P=1}^{(2w)!} \lambda_P^\sigma \lambda_P = 0 \tag{23}$$

and obtain a system of $2^{4w} + t$ equations. Let the new connected system of equations (21) and (23) has some solution $\lambda_P = \lambda_P^0$, $P = 1, 2, \ldots, (2w)!$. This solution satisfies all the equations (21), therefore it must be a linear combination of the solutions λ_P^σ

$$\lambda_P^0 = \sum_{\sigma=1}^{t} \alpha_\sigma \lambda_P^\sigma, \qquad P = 1, 2, \ldots, (2w)!. \tag{24}$$

The equations (23) must be satisfied for every λ_P, therefore they are satisfied for their arbitrary linear combinations and also for

$$\sum_{\sigma=1}^{t} \alpha_\sigma \sum_{P=1}^{(2w)!} \lambda_P^\sigma \lambda_P = 0. \tag{25}$$

It can be rewritten by (24) in the form

$$\sum_{P=1}^{(2w)!} \lambda_P^0 \lambda_P = 0. \tag{26}$$

It must be satisfied for every λ_P thus also for λ_P^0

$$\sum_{P=1}^{(2w)!} (\lambda_P^0)^2 = 0, \tag{27}$$

i.e. $\lambda_1^0 = \lambda_2^0 = \cdots = \lambda_{(2w)!}^0 = 0$. It means the only solution of the connected system of equations (21) and (23) is zero and therefore its rank is $(2w)!$. A relation of the form

$$\sum_{P=1}^{(2w)!} \lambda_P^\sigma \delta_{\{i_1}^{x_1} \delta_{i_2}^{x_2} \cdots \delta_{i_{2w}\}_P}^{x_{2w}} = 0 \tag{28}$$

corresponds to each of the solutions (22). Let $p_{x_1 x_2 \cdots x_{2w}}$ be an arbitrary tensor of covariance $2w$. Since

$$\delta_{i_1}^{x_1} \delta_{i_2}^{x_2} \cdots \delta_{i_{2w}}^{x_{2w}} p_{x_1 x_2 \cdots x_{2w}} = p_{i_1 i_2 \cdots i_{2w}}, \tag{29}$$

then we obtain from (28)

$$\sum_{P=1}^{(2w)!} \lambda_P^\sigma p_{\{i_1 i_2 \cdots i_{2w}\}_P} = 0. \tag{30}$$

The components of the tensor $p_{x_1 x_2 \cdots x_{2w}}$ can be selected quite arbitrarily and in spite of it each component on the left-hand side of (30) equals zero. From it

$$\sum_{P=1}^{(2w)!} \lambda_P^\sigma c_P^* \delta_{\{i_1}^{x_1} \delta_{i_2}^{x_2} \cdots \delta_{i_{2w}\}_P}^{x_{2w}} = 0, \qquad \sigma = 1, 2, \ldots, t. \tag{31}$$

The equality (31) gives t independent linear relations between the unknown coefficients c_P^*. If we add (31) to the equations (20), we obtain a system (A) of $2^{4w} + t$ equations in the coefficients c_P^*. The matrix of this system coincides with the matrix of the connected system (21) and (23). Consequently, the rank of the system (A) is $(2w)!$ and one may select from it $(2w)!$ equaitons in such a way that the determinant formed by their system (B) is non-zero; the system (B) involves all the t equations (31) and $((2w)! - t)$ equations of the system (20) obtained from certain definite values of the indices $x_1, x_2, \ldots, x_{2w}$. Solving the system (B) we express the left-hand side of (20) in the form of linear combinations of the right-hand sides of the system (B), i.e. again in the form of the right-hand sides of (20). It means (20) has always a solution. □

Notes: The solution of (19) is not unique, since we can add to the right-hand side of (20) any linear combination of the left-hand sides of (28). We supposed two-dimensional space here, but the proof can be generalized for arbitrary number of dimensions.

4 Example

For illustration, we created the simplest affine moment invariants up to the third order by the graph method

$$I_1 = (\mu_{20}\mu_{02} - \mu_{11}^2)/\mu_{00}^4$$

with weight $w = 2$ and structure $\mathbf{s} = (2)$,

$$I_2 = (-\mu_{30}^2\mu_{03}^2 + 6\mu_{30}\mu_{21}\mu_{12}\mu_{03} - 4\mu_{30}\mu_{12}^3 - 4\mu_{21}^3\mu_{03} + 3\mu_{21}^2\mu_{12}^2)/\mu_{00}^{10}$$

with weight $w = 6$ and structure $\mathbf{s} = (0, 4)$,

$$I_3 = (\mu_{20}\mu_{21}\mu_{03} - \mu_{20}\mu_{12}^2 - \mu_{11}\mu_{30}\mu_{03} + \mu_{11}\mu_{21}\mu_{12} + \mu_{02}\mu_{30}\mu_{12} - \mu_{02}\mu_{21}^2)/\mu_{00}^7$$

with weight $w = 4$ and structure $\mathbf{s} = (1, 2)$,

$$\begin{aligned}I_4 = (&-\mu_{20}^3\mu_{03}^2 + 6\mu_{20}^2\mu_{11}\mu_{12}\mu_{03} - 3\mu_{20}^2\mu_{02}\mu_{12}^2 - 6\mu_{20}\mu_{11}^2\mu_{21}\mu_{03} - 6\mu_{20}\mu_{11}^2\mu_{12}^2 \\ &+12\mu_{20}\mu_{11}\mu_{02}\mu_{21}\mu_{12} - 3\mu_{20}\mu_{02}^2\mu_{21}^2 + 2\mu_{11}^3\mu_{30}\mu_{03} + 6\mu_{11}^3\mu_{21}\mu_{12} \\ &-6\mu_{11}^2\mu_{02}\mu_{30}\mu_{12} - 6\mu_{11}^2\mu_{02}\mu_{21}^2 + 6\mu_{11}\mu_{02}^2\mu_{30}\mu_{21} - \mu_{02}^3\mu_{30}^2)/\mu_{00}^{11}\end{aligned}$$

with weight $w = 6$ and structure $\mathbf{s} = (3, 2)$,

$$\begin{aligned}I_5 = (&\mu_{20}^3\mu_{30}\mu_{03}^3 - 3\mu_{20}^3\mu_{21}\mu_{12}\mu_{03}^2 + 2\mu_{20}^3\mu_{12}^3\mu_{03} - 6\mu_{20}^2\mu_{11}\mu_{30}\mu_{12}\mu_{03}^2 \\ &+6\mu_{20}^2\mu_{11}\mu_{21}^2\mu_{03}^2 + 6\mu_{20}^2\mu_{11}\mu_{21}\mu_{12}^2\mu_{03} - 6\mu_{20}^2\mu_{11}\mu_{12}^4 + 3\mu_{20}^2\mu_{02}\mu_{30}\mu_{12}^2\mu_{03} \\ &-6\mu_{20}^2\mu_{02}\mu_{21}^2\mu_{12}\mu_{03} + 3\mu_{20}^2\mu_{02}\mu_{21}\mu_{12}^3 + 12\mu_{20}\mu_{11}^2\mu_{30}\mu_{12}^2\mu_{03} \\ &-24\mu_{20}\mu_{11}^2\mu_{21}^2\mu_{12}\mu_{03} + 12\mu_{20}\mu_{11}^2\mu_{21}\mu_{12}^3 - 12\mu_{20}\mu_{11}\mu_{02}\mu_{30}\mu_{12}^3 \\ &+12\mu_{20}\mu_{11}\mu_{02}\mu_{21}^3\mu_{03} - 3\mu_{20}\mu_{02}^2\mu_{30}\mu_{21}^2\mu_{03} + 6\mu_{20}\mu_{02}^2\mu_{30}\mu_{21}\mu_{12}^2 \\ &-3\mu_{20}\mu_{02}^2\mu_{21}^3\mu_{12} - 8\mu_{11}^3\mu_{30}\mu_{12}^3 + 8\mu_{11}^3\mu_{21}^3\mu_{03} - 12\mu_{11}^2\mu_{02}\mu_{30}\mu_{21}^2\mu_{03} \\ &+24\mu_{11}^2\mu_{02}\mu_{30}\mu_{21}\mu_{12}^2 - 12\mu_{11}^2\mu_{02}\mu_{21}^3\mu_{12} + 6\mu_{11}\mu_{02}^2\mu_{30}^2\mu_{21}\mu_{03} \\ &-6\mu_{11}\mu_{02}^2\mu_{30}^2\mu_{12}^2 - 6\mu_{11}\mu_{02}^2\mu_{30}\mu_{21}^2\mu_{12} + 6\mu_{11}\mu_{02}^2\mu_{21}^4 - \mu_{02}^3\mu_{30}^3\mu_{03} \\ &+3\mu_{02}^3\mu_{30}^2\mu_{21}\mu_{12} - 2\mu_{02}^3\mu_{30}\mu_{21}^3)/\mu_{00}^{16}\end{aligned}$$

with weight $w = 9$ and structure $\mathbf{s} = (3, 4)$, The corresponding graphs are on Fig. 2. All other invariants were eliminated as linearly dependent. The invariant I_5 has dependent absolute value

$$|I_5| = \sqrt{-4I_1^3I_2^2 + 12I_1^2I_2I_3^2 - 12I_1I_3^4 - I_2I_4^2 + 4I_3^3I_4},$$

therefore it was omitted from the set. Its sign can be used for recognition of an object from its mirror reflection.

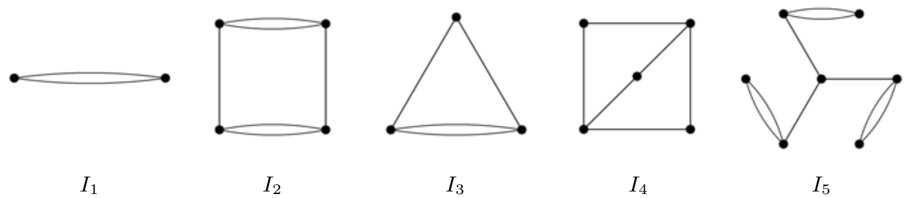

Fig. 2. The generating graphs of the invariants I_1, I_2, I_3, I_4 and I_5

The same invariants were also generated as a solution of the Cayley-Aronhold differential equation except $I_4' = -(I_4 + 6I_1I_3)$ instead of I_4. So, we have two sets of features for recognition of affinely distorted objects, $\{I_1, I_2, I_3, I_4\}$ and $\{I_1, I_2, I_3, I_4'\}$, there is one-to-one mapping between them and features from one set are linear combinations of that from the other set. The presented proof means the same situation is in all higher orders.

5 Numerical Experiment

The goal of this experiment is to show the behavior of the affine moment invariants. We have photographed a series of cards used in a game called mastercards

Fig. 3. The mastercards: Girl, Old scratch, Tyre-ride, Room-bell, Fireplace, Winter cottage, Spring cottage, Summer cottage, Bell and Star

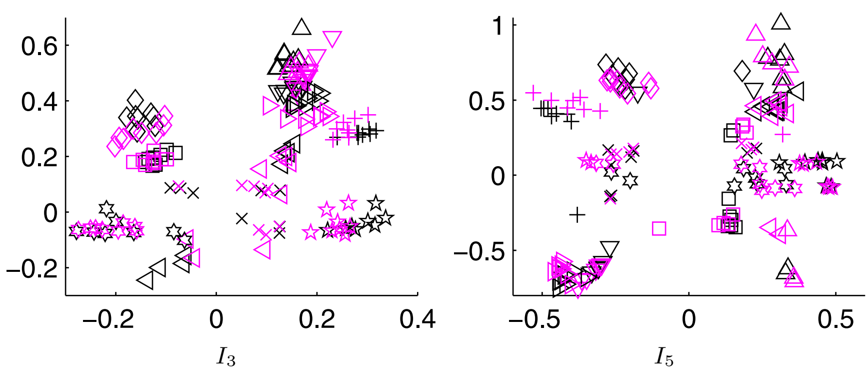

Fig. 4. The feature space of I_3 and I_5 of red (horizontal axis) and blue (vertical axis) channels. Legend: ▽ Girl, △ Old scratch, □ Tyre-ride, ◇ Room-bell, ▷ Fireplace, × Winter cottage, ◁ Spring cottage, + Summer cottage, ☆ Bell and ✡ Star. A card from each pair is expressed by the black symbol while the other card is expressed by the magenta (gray) symbol.

(also pexeso), where the objective is to find the same pairs of cards turned face-down. Cards from each of the ten pairs are shown on Fig. 3.

First, we used the feature set $\{I_1, I_2, I_3, I_4\}$ for each color channel separately, i.e. 12 features. The result was 3 errors from 140 cases, i.e. error rate 2.1%. Then we added I_5 to the feature set and the result was 17 errors, i.e. the error rate worsened to 12.1%. It is an illustration, how a dependent invariant can worsen the recognition. The feature space of the independent invariant I_3 is on Fig. 4a, that of the dependent invariant I_5 is on Fig. 4b.

6 Conclusion

The proved theorem means the features from all the methods mentioned in the paper are equivalent. The suitable method for the generation of the affine moment invariants can be chosen freely, only on the base of the computational aspects as complexity of programming, computing complexity, memory demands and numerical precision.

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