

# Decentralized $H_\infty$ Quantizers Design for Networked Complex Composite Systems

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Abstract: The objective of the paper is to propose an approach to decentralized networked quantized state feedback  $H_\infty$  controller design for a class of complex symmetric composite continuous-time systems. The effect of data-packet dropouts and communication delays between the plant and the local controller as well as quantizers design are included in the design of asymptotically stabilizing controller and quantizer parameters. Local quantizers are located between the subsystem and the local controller. It is shown how the reduced-order control design of sampled quantized delayed feedback combined with the multiple packet transmissions approach can lead to an effective decentralized control design of the overall closed-loop system.

Keywords: Large-scale complex systems, decentralization, quantizers, networked systems, interconnected systems

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## 1. INTRODUCTION

Networked control systems (NCS) are feedback control systems with network channels in the feedback loop. Time delays, packet losses, and quantization are essential issues that require careful treatment in the NCS design. Two main changes in the control system research directions are the explicit considerations of the interconnections and a renewed emphasis on distributed control systems which is closely related to the notion of decentralized control of complex large scale systems. Recently, new methods and algorithms have been proposed to include communication issues into the decentralized control design framework. Though a variety of structures and models in this framework have been analyzed, a gap remains between decentralized control and control over networks.

### 1.1 Prior Work

There is no systematic theory in the emerging area of NCS up to now, but the literature in this field is vast. Recent surveys on this topic can be found in Hristu-Varsakelis and Levine [2005], Antsaklis and Tabuada [2006], Matveev and Savkin [2009].

Delays and packet dropouts have been incorporated in the centralized NCS design by several approaches. Wang and Yang [2009] deals with a combined switching and parameter uncertainty-based method to treat with time-varying delays and packet dropouts to design the  $H_\infty$  controller. An active varying sampling period is proposed to employ fully the network bandwidth. Ogori et al. [2007] presents a stabilization method of NCS with bounded disturbances and unreliable links by using a kind of randomly-switched system under the disturbances. A common Liapunov function approach is used. Xiong and Lam [2007] is focused on stabilization of NCS from the point of view of zero-order hold. Xiong and Lam [2009] presents stability conditions of NCS to solve the stabilization problem with both arbitrary and Markovian packet losses via a packet-loss Liapunov approach. One of the promising approaches considers a continuous-time system and a sampled-data controller in the feedback network channel. Hu et al. [2007] deals with time-driven digital controller and event-driven holder for NCS. It includes the possibility to deal with time delays and packet dropouts.

Decentralized NCS (DNCS) are control systems with multiple control stations while transmitting control signals through a network, i.e. data signals are transmitted to multiple controllers in the feedback loop. DNCS combine the advantages of the centralized NCS as well as the decentralized control systems. Such a combination enables to cut unnecessary wiring, reduce the complexity and the overall system cost when designing and implementing

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control systems. Recently, the results dealing with the DNCS design methods are rare. Relevant problems are introduced in Bakule [2008], Bakule and de la Sen [2009], Bakule and de la Sen [2010], and Xu and Hespanha [2004]. Decentralized stabilization of NCS using periodically time-varying local controller is presented in Jiang et al. [2008], while Zhong et al. [2009] deals with the synchronization within the DNCS design. Matveev and Savkin [2009], Yüksel and Başar [2007] consider the DNCS under data rate constraints, while stability of the DNCS is analyzed in Wei [2008]. General form of a quantizer is defined in Liberzon [2003]. This approach has been extended into the design of quantizers for the DNCS in Zhai et al. [2009], Chen et al. [2010], and Bakule [2010].

Symmetric composite systems appear in very different real world systems. A more complete survey of theoretic and applied results is presented in Bakule [2008], Hovd and Skogestad [1994] with the references therein.

This paper deals with the effective DNCS  $H_\infty$  quantizer design for a class of symmetric composite systems when taking into account structural properties of these systems as well as network dropouts and communication delays. The motivation for the usage of the  $H_\infty$ -norm approach is explained for instance in Hovd and Skogestad [1994] and Lam and Huang [2007], while its extension to the NCS setting is presented in Yue et al. [2005].

The paper extends the results by Yu et al. [2005], Bakule and de la Sen [2009], Chen et al. [2010], and Zhang and Meng [2009] into the DNCS design for symmetric composite systems using the reduced-order centralized NCS design when considering the delay-dependent approach within the framework of the Linear Matrix Inequalities (LMI).

To the authors best knowledge, the problem of decentralized  $H_\infty$  quantizer design with time-varying delayed feedback has not been solved up to now for this class of complex composite systems.

## 1.2 Outline of the Paper

A sufficient condition for the decentralized  $H_\infty$  quantizer design is derived by using the reduced-order DNCS approach with a state stabilizing controller for a class of complex systems. The plant and the controller are connected through a network channel. A network channel is modelled using bounded packet dropouts and communication delays. First, the gain matrix is designed for a low order NCS design model without quantizer by using the LMI. Then, quantizer parameter  $\mu$  is designed. Finally, the gain matrix and the quantizer are synthesized into local controllers of the original system so that such closed-loop system is asymptotically stable with the prescribed disturbance attenuation level.

## 2. PROBLEM FORMULATION

### 2.1 Structured System Description

Consider a nonlinear symmetric composite system consisting of  $N$  subsystems, where the  $i$ th subsystem is described as follows

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B_1 u_i(t) + B_2 w_i(t) + s_i(t) \\ z_i(t) &= Cx_i(t) \quad i = 1, \dots, N \quad N > 2 \end{aligned} \quad (1)$$

where  $x_i \in \mathbb{R}^n$ ,  $u_i \in \mathbb{R}^m$ , and  $z_i \in \mathbb{R}^p$  are the subsystem states, the control inputs, and the controlled outputs, respectively.  $A$ ,  $B_1$ ,  $B_2$ , and  $C$  are constant matrices of appropriate dimensions.  $w_i \in \mathcal{L}_2[0, \infty)$  denotes  $h$ -dimensional vector of exogenous disturbance signal, where  $\mathcal{L}_2[0, \infty)$  is the Lebesgue space of time domain square-integrable functions on  $[0, \infty)$ . It also implies that  $w_i$  is essentially bounded and it converges to zero as time tends to infinity. The interconnections  $s_i$  are defined as

$$s_i(t) = \sum_{j=1}^N L_{ij} x_j(t) \quad (2)$$

*Assumption 1.* The interconnection matrices  $L_{ij}$  have the following particular structure

$$L_{ii} = L_p \quad L_{ij} = L_q \quad (i \neq j) \quad (3)$$

$L_p$ , and  $L_q$  are constant nominal matrices.

$$u_i(t) = K \underline{x}_i(t_k) \quad t \in [t_k, t_{k+1}) \quad k = 1, 2, \dots \quad (4)$$

where  $K$  is a constant gain matrix to be designed, while  $t_k$  is a sampling instant. It is supposed that  $t_k = kT$  with  $k$  being any positive integer, a sampler with the sampling period  $T$ , and a standard zero order hold in the feedback loop. The sampled value  $\underline{x}_i(t_k)$  is transmitted through a network channel. If transmitted correctly, it is registered in a buffer, where  $\underline{x}_i(t_k)$  denotes the output from the buffer. This value is the input to the controller  $K$  to generate the control action.

Such an approach requires to respect the basic properties of a network channel when transmitting the signal. Data packet dropout is a well-known frequent phenomenon. The quantity of dropped packets is cumulated from the last update at the time  $t_k$ . Denoting some sampling interval-dependent integer  $d_{ik} \geq 1$  at time  $t_k$ , then the output from the buffer yields  $\underline{x}_i(t_k) = x_i(t_k - d_{ik}T)$ .

Suppose the resulting input time-varying delay consists of the constant communication delay denoted as  $d_c$  and the delay caused by packet dropout  $d_{ik}T$ . The input of the controller is  $\underline{x}_i(t_k) = x_i(t_k - d_{ik}T - d_c)$ , where  $1 \leq d_k \leq (t_{k-1} - d_c)/T$ .  $K$  is the gain matrix to be determined. Denoting  $\tau_i(t) = t - t_k - d_{ik}T - d_c$ , then (4) can be rewritten as

$$u_i(t) = Kx_i(t - \tau_i(t)) \quad t \in [t_k, t_{k+1}) \quad k = 1, 2, \dots \quad (5)$$

### 2.2 Quantizers

Let us follow a common definition of a quantizer so as introduced in the reference Liberzon [2003]. A quantizer is a device that converts a real-valued signal into a piecewise constant one taking into account only a finite-set of values. There are available different classes of quantizers. A common definition of a quantizer can be introduced as follows. Denote the variable to be quantized as  $y \in \mathbb{R}^n$  and  $|\cdot|$  the standard Euclidean norm in  $\mathbb{R}^n$ . A *quantizer* is defined as a piecewise constant function  $q : \mathbb{R}^n \rightarrow \mathcal{Q}$ , where  $\mathcal{Q}$  is a finite subset of  $\mathbb{R}^n$ . This leads to a partition of  $\mathbb{R}^n$  into a finite number of quantization regions of the form  $\{\forall y \in \mathbb{R}^n : q(y) = i\}$ ,  $i \in \mathcal{Q}$ . The quantization regions are not assumed to have any particular shapes.

*Assumption 2.* Suppose that there exist positive real numbers  $M$  and  $\Delta$ ,  $\Delta < M$ , such that the following conditions hold

a) If

$$|y| \leq M \quad (6)$$

then

$$|q(y) - y| \leq \Delta \quad (7)$$

b) If

$$|y| > M \quad (8)$$

then

$$|q(y)| > M - \Delta \quad (9)$$

Condition a) gives a bound on the quantization error when the quantizer does not saturate. Condition b) provides a way to detect the possibility of saturation. The quantities  $M$  and  $\Delta$  are called *the range of  $q$*  and the *quantization error*, respectively. Suppose that  $q(x) = 0$  holds for  $x$  in some neighborhood of the origin. For instance, the quantizer with rectangular quantization regions satisfies the above requirements with its values in each quantization region belonging to that region.

Consider the quantized measurements in the form

$$q_\mu(y) \triangleq \mu q\left(\frac{y}{\mu}\right) \quad (10)$$

where  $\mu > 0$  is a scalar parameter. Supposing  $\mu = 0$  means that the output of the quantizer equals zero. The range of the quantizer is  $M\mu$  and the quantization error is  $\Delta\mu$ . The parameter  $\mu$  can be understood as a “zoom” variable. It means that when  $\mu$  increases, then the zooming goes out and a new quantizer with larger range and larger quantization error is obtained. When  $\mu$  decreases, then the zooming goes in and a quantizer with smaller range but also with smaller quantization error is constructed.  $\mu$  can be updated in the dependence on the system local state (or the local measurement output). In this sense, another state of the closed-loop system can be reached.

Now, let us follow an application of this notion to quantizer design for decentralized feedback systems. The approach presented by Zhai et al. [2009] and Chen et al. [2010] is extended for the feedback design of the system (1).

Suppose the availability of local static output feedback controllers  $K$  for the system (1). The overall controller asymptotically stabilizes the closed-loop system (1)–(3) with the  $H_\infty$  disturbance attenuation level  $\gamma$ . More precisely, the closed-loop system is written as

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B_1 K x_i(t - \tau_i(t)) + \sum_{j=1, i \neq j}^N L_{ij} x_j(t) \\ &\quad + B_2 w_i(t) \\ z_i(t) &= C x_i(t) \quad i = 1, \dots, N \quad N > 2 \end{aligned} \quad (11)$$

Suppose that each local quantizer is located between sensors and a local controller in the local feedback loop, where each quantizer is described in the same way as a centralized quantizer by (6)–(9) with the quantized measurements (10). Suppose the availability of only quantized local output information. The state feedback (5) can be modified using the quantized information on  $x_i$  as

$$u_i(t) = K \mu_i q\left(\frac{x_i(t - \tau_i(t))}{\mu_i}\right) \quad (12)$$

Supposing given the gain matrix  $K$ , then the closed-loop system (1), (12) with any fixed positive scalars  $\mu_i$  has the form

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B_1 K x_i(t - \tau_i(t)) + \sum_{j=1, i \neq j}^N L_{ij} x_j(t) \\ &\quad + B_2 w_i(t) + F_i(\mu_i, x_i(t - \tau_i(t))) \end{aligned} \quad (13)$$

$$z_i(t) = C x_i(t) \quad i = 1, \dots, N \quad N > 2$$

where

$$\begin{aligned} F_i(\mu_i, x_i(t - \tau_i(t))) \\ = \mu_i B_1 K \left( q\left(\frac{x_i(t - \tau_i(t))}{\mu_i}\right) - \frac{x_i(t - \tau_i(t))}{\mu_i} \right) \end{aligned} \quad (14)$$

The goal is to find the gain matrix  $K$  and the parameters  $\mu_i$  depending on the local states  $x_i(t)$  guaranteeing the closed-loop system stability with the  $H_\infty$  disturbance attenuation level  $\gamma$ .

### 2.3 Overall System Description

The compacted system description of (1)–(3) has the form

$$\begin{aligned} \dot{x}(t) &= \bar{A}x(t) + \bar{B}_1 u(t) + \bar{B}_2 w(t) \quad x(t_o) = x_o \\ z(t) &= \bar{C}x(t) \end{aligned} \quad (15)$$

where  $x = (x_1^T, \dots, x_N^T)^T$ ,  $u = (u_1^T, \dots, u_N^T)^T$ ,  $w = (w_1^T, \dots, w_N^T)^T$ ,  $z = (z_1^T, \dots, z_N^T)^T$  are the system states, the control inputs, the disturbance inputs, and the controlled outputs, respectively. The matrices are defined as

$$\begin{aligned} \bar{A} &= (\bar{A}_{ij}) \quad \bar{A}_{ii} = A + L_p \quad \bar{A}_{ij} = L_q \\ \bar{B}_1 &= \text{diag}(B_1, \dots, B_1) \quad \bar{B}_2 = \text{diag}(B_2, \dots, B_2) \\ \bar{C} &= \text{diag}(C, \dots, C) \end{aligned} \quad (16)$$

Consider the stabilizing controller for the system (8)–(11) as

$$u(t) = \bar{K} \underline{x}(t_k) = \text{diag}(K, \dots, K) \underline{x}(t_k) \quad t \in [t_k, t_{k+1}) \quad (17)$$

where  $\underline{x}(t_k) = (x_1(t - \tau_1(t))^T, \dots, x_N(t - \tau_N(t))^T)^T$ .

### 2.3 Decentralized Networked Control Systems

The DNCS correspond generally with multiple packet simultaneous transmissions through parallel network channels, where each channel generally corresponds with a local feedback loop with individual time-varying delay. Multiple packet transmissions are used as an effective approach for multi-loop control systems with delays and packet dropouts. The availability of Acknowledgement (ACK) about data losses to the sender at each feedback loop is supposed by Kurose and Rose [2005].

The information structure constraints on only sensor-actuator pairs in the gain matrices is sufficiently justified for symmetric composite systems. Much higher reliability of subsystems than that of the interconnections, an essential simplification of the DNCS design using LMIs, and the design requirement to keep the symmetry in the closed-loop system lead to the preference of decentralized control as pointed out by Stanković et al. [2007] and Bakule and de la Sen [2010]. In the case of symmetric composite systems, the identical interactions are given by construction of

real world systems. The structural properties of these systems are employed to reduce the complexity of the control design. Highly reliable processor operating only on local feedback loops, i.e. in parallel, are needed for fast control action in response to local inputs and perturbations. It is required to keep the identity of interconnection terms.

*Assumption 3.* The number of packet dropouts is bounded so that it satisfies the constraint

$$0 < \tau_i(t) \leq \bar{\tau} \quad (18)$$

where  $\bar{\tau}$  is a given positive constant.

Consider now the closed-loop overall system (15)–(17) with the quantizer  $\bar{F}(\mu, \underline{x}(t_k))$  in a compacted form as follows

$$\begin{aligned} \dot{x}(t) &= \bar{A}x(t) + \bar{B}_1 \sum_{i=1}^N \bar{K} E_i \underline{x}(t_k) + \bar{B}_2 w(t) + \bar{F}(\mu, \underline{x}(t_k)) \\ z(t) &= \bar{C}x(t) \quad x(t_o) = \bar{\Phi}_k(t_o) \quad t_o \in [-\bar{\tau}, 0] \end{aligned} \quad (19)$$

where

$$\begin{aligned} E_i &= \text{diag}(0, \dots, I_n, \dots, 0) \\ \bar{F}(\mu, \underline{x}(t_k)) &= \begin{pmatrix} F_1(\mu_1, x_1(t - \tau_1(t))) \\ \dots \\ F_N(\mu_N, x_N(t - \tau_N(t))) \end{pmatrix} \end{aligned} \quad (20)$$

The identity matrices appear at the  $i$ th block of the matrix  $E_i$  in (20).  $\bar{\Phi}_k(t_o)$  denotes the function of initial condition of the corresponding time instant.

*Assumption 4.* Acknowledgment ACK to the sender about local data losses is always available for each channel of the system (19).

## 2.4 The Problem

Consider the system (1), (2) (or equivalently (15)) and the controller (12) satisfying Assumptions 1-4. The goal is to design the gain matrix  $K$  of the controller and quantizer parameters  $\mu_i$  in (12) depending only on the local states  $x_i(t - \tau_i(t))$  guaranteeing the asymptotic stability with  $H_\infty$  disturbance attenuation level  $\gamma$  of the closed-loop system (13) (or equivalently (19)). Focus the DNCS design on the reduction of design complexity by using the structural properties of the system (15). Solve the problem by using the LMI approach.

## 3. MAIN RESULTS

The reduced-order control design model is constructed by using the transformation of states of the system (15) as follows

$$\tilde{x}(t) = T x(t) \quad (21)$$

where

$$T = \frac{1}{N} \begin{pmatrix} (N-1)I & -I & \dots & -I & -I \\ -I & (N-1)I & \dots & -I & -I \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -I & -I & \dots & (N-1)I & -I \\ I & I & \dots & I & I \end{pmatrix} \quad (22)$$

where  $I$  denotes the  $n \times n$  identity matrix.

The transformation of states defined by (21) yields

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A}\tilde{x}(t) + \bar{B}_1 u(t) + \bar{B}_2 w(t) \\ \tilde{z}(t) &= \tilde{C}\tilde{x}(t) \quad \tilde{x}(t_o) = \tilde{x}_o \end{aligned} \quad (23)$$

where

$$\tilde{A} = \text{diag}(A_s, \dots, A_s, A_c) \quad (24)$$

with

$$\begin{aligned} A_s &= A + L_p - L_q \\ A_c &= A_s + N L_q \end{aligned} \quad (25)$$

and  $\bar{B}_1, \bar{B}_2$  given by (16).

The state-trajectory solution of the  $i$ th subsystem can be described by the system determined by the states  $x_i = \tilde{x}_i - \tilde{x}_N$  for  $i = 1, \dots, N - 1$  and  $x_N = \tilde{x}_N - \sum_{i=1}^{N-1} \tilde{x}_i$  as

$$\begin{aligned} \dot{\hat{x}}_i(t) &= \hat{A}_i \hat{x}_i(t) + \hat{B}_1 \hat{u}(t) + \hat{B}_2 \hat{w}(t) \\ \hat{z}(t) &= \hat{C} \hat{x}(t) \quad \hat{x}_i(t_o) = \hat{x}_{i_o} \quad i = 1, \dots, N - 1 \end{aligned} \quad (26)$$

where  $\hat{x}_i = (\hat{x}_i^T, \hat{x}_N^T)^T$ ,  $\hat{u}_i = (\hat{u}_i^T, \hat{u}_N^T)^T$ ,  $\hat{A}_i = \text{diag}(A_s, A_c)$ ,  $\hat{B}_1 = \text{diag}(B_1, B_1)$ ,  $\hat{B}_2 = \text{diag}(B_2, B_2)$ , and  $\hat{C} = \text{diag}(C, C)$ . Therefore, the dynamics of the original overall system can be described by the subsystem model (21) consisting of two parts operating in parallel. All the phenomena encountered in the whole system can be studied by means of the model (26). It leads finally to two systems of order  $n$ . To simplify the notation, denote  $x_s(t)$  a generic state for any  $\hat{x}_i(t)$  in (26) and  $x_c(t) = x_N(t)$ . We get

$$\begin{aligned} \dot{x}_s(t) &= A_s x_s(t) + B_1 u_s(t) + B_2 w_s(t) \quad x_s(t_o) = x_{s_o} \\ \dot{z}_s(t) &= C x_s(t) \end{aligned} \quad (27)$$

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_1 u_c(t) + B_2 w_c(t) \quad x_c(t_o) = x_{c_o} \\ \dot{z}_c(t) &= C x_c(t) \end{aligned}$$

The application of  $T$  on the system (15) results in the system (27) with block diagonal structure where the first  $N - 1$  blocks are identical. This fact reflects (26). The generic system (27) is directly based on (26). It means that the dynamic properties of (15) and (27) are equivalent.

Let us consider the state feedback  $H_\infty$  controller design without quantizers now. Note that each subsystem state of the closed-loop system can be thought as the closed-loop system consisting of the systems (27) and the controller  $K$  simultaneously stabilizing both these plants, i.e.  $u_s(t) = K x_s(t - \tau_s(t))$  and  $u_c(t) = K x_c(t - \tau_c(t))$  satisfying the bounds  $0 < \tau_s(t) \leq \bar{\tau}$  and  $0 < \tau_c(t) \leq \bar{\tau}$  where  $\bar{\tau}$  as given in Assumption 3.

The problem of simultaneous stabilization by (27) is solved using the robust stabilization approach with a central plant. The central plant serves as a nominal system, while the uncertain terms enable to include both plants in (27) when reaching the uncertainty bounds. This approach leads to the construction of the  $n$ -dimensional system

$$\begin{aligned} \dot{x}_m(t) &= (A_m + \Delta A_m) x_m(t) + B_1 u_m(t) + B_2 w_m(t) \\ z_m(t) &= C x_m(t) \end{aligned} \quad (28)$$

where  $x_m \in \mathbb{R}^n$ ,  $u_m \in \mathbb{R}^m$ , and  $z_m \in \mathbb{R}^p$  are the system states, the control inputs, and the controlled outputs, respectively.  $w_m \in \mathcal{L}_2[0, \infty)$  is  $h$ -dimensional vector of exogenous disturbance signal. The nominal plant is the system (28) with  $\Delta A_o = 0$ , while the  $\Delta$ -term denotes the

uncertainty. The terms in (28) are constructed to include the systems (27) as follows

$$A_m = \frac{A_c + A_s}{2} = A + L_p + \left(\frac{N}{2} - 1\right)L_q \quad (29)$$

$$\Delta A_m = G\delta(t)H$$

where  $GH = \frac{A_c - A_s}{2} = \frac{N}{2}L_q$ . Note that the factorization into matrices  $G, H$  is not unique.  $\delta(t) \in \mathbb{R}^{q \times q}$  denotes a time-varying matrix satisfying the relation  $\delta^T(t)\delta(t) \leq I$ , where  $I$  denotes the  $q$ -dimensional identity matrix.

Consider a stabilizing controller for the system (28) as

$$u_m(t) = Kx_m(t - \tau_m(t)) \quad (30)$$

where the delay  $\tau_m(t)$  satisfies the relation  $0 < \tau_m(t) \leq \bar{\tau}$ .

The closed-loop system (28)–(30) for  $t \in [t_k, t_{k+1})$  has the form

$$\dot{x}_m(t) = (A_m + \Delta A_m)x_m(t) + B_1Kx_m(t - \tau_m(t)) + B_2w_m(t) \quad (31)$$

$$z_m(t) = Cx_m(t) \quad x_m(t_o) = \Phi_{mk}(t_o) \quad t_o \in [-\bar{\tau}, 0]$$

where  $\Phi_{mk}(t_o)$  denotes the function of initial condition for the corresponding time instant. The gain matrix  $K$  can be determined by the procedure resulting in the robust delay-dependent stable overall closed-loop system with guaranteed performance level  $\gamma$  by using the Liapunov-Razumikhin approach.

Consider the Liapunov-Razumikhin function candidate for the system (31) in the form

$$V(t, x_m) = x_m(t)^T P x_m(t) \quad (32)$$

The standard Liapunov stability machinery, when applying (32) on the system (31), confirms that (32) is a Liapunov-Razumikhin function for the system (32). It leads to the following result.

*Theorem 1.* Given the system (28), an integer  $\bar{\tau} > 0$  and a constant  $\gamma > 0$ . Suppose that there exist symmetric positive definite matrices  $X, R1, R2 \in \mathbb{R}^{n \times n}$ , a symmetric positive definite matrix  $Q \in \mathbb{R}^{m \times m}$ , matrices  $Y \in \mathbb{R}^{m \times n}$ ,  $Z \in \mathbb{R}^{n \times q}$ , and a constant  $\varepsilon > 0$  satisfying the following relations

$$\begin{pmatrix} \Gamma & B_1Q & B_2 & XC^T & G & ZH^T \\ \bullet & -Q & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\gamma^2 I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & I & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\varepsilon I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\varepsilon I \end{pmatrix} < 0 \quad (33)$$

$$\begin{pmatrix} -X & Y^T B_1^T \\ \bullet & -R2 \end{pmatrix} \leq 0$$

$$\begin{pmatrix} -X & Y^T A_m^T \\ \bullet & -R1 \end{pmatrix} \leq 0$$

$$\begin{pmatrix} -Q & Y \\ \bullet & -X \end{pmatrix} \leq 0$$

$$R1 + R2 - X \leq 0$$

where  $\Gamma = \frac{1}{\bar{\tau}}(XA_m^T + A_mX + Y^TB_1^T + B_1Y)$ .

Then, the system (28) is asymptotically stabilized by the controller (30) with the disturbance attenuation level  $\gamma$ , where the gain matrix is given as

$$K = YX^{-1} \quad (34)$$

for the packet dropout  $d(k)$  satisfying the interval bounds  $0 \leq d(k) \leq \frac{\bar{\tau}}{T} - 1$  and  $X^{-1} = P$ .

Now we focus attention on the quantizer design for the system (28). Note that it supposes the availability of the gain matrix  $K$ . The closed loop system (28) with quantized signal has the form

$$\dot{x}_m(t) = A_m x_m(t) + B_1 K x_m(t - \tau_m(t)) + B_2 w_m(t) + \mu_m B_1 K \left( q \left( \frac{x_m(t - \tau_m(t))}{\mu_m} \right) - \frac{x_m(t - \tau_m(t))}{\mu_m} \right)$$

$$z_m(t) = C x_m(t) \quad x_m(t_o) = \Phi_{mk}(t_o) \quad t_o \in [-\bar{\tau}, 0] \quad (35)$$

*Assumption 5.* Suppose that  $\frac{x_m(t - \tau_m(t))}{\mu} \leq M$ , then in virtue of the assumption A2

$$\left| q \left( \frac{x_m(t - \tau_m(t))}{\mu_m} \right) - \frac{x_m(t - \tau_m(t))}{\mu_m} \right| \leq \Delta \quad (36)$$

holds.

The following theorem presents the quantizer up-dating strategy for  $\mu$  to keep the asymptotic stability of the system (35) with the disturbance attenuation level  $\gamma$ .

*Theorem 2.* Given the system (35). Suppose that the relation

$$\lambda_{\min}(R) > 0 \quad (37)$$

holds, where  $\lambda_{\min}(R)$  denotes the smallest value of the matrix  $R$  and  $-R = P(A_m + B_1K) + (A_m + B_1K)^T + \varepsilon PGG^T + \varepsilon^{-1}H^TH + \bar{\tau}(2P + PB_1QB_1^T P + \gamma^{-2}PB_2B_2^T P + C^T C)$ . Suppose also that  $M$  is selected enough large compared to  $\Delta$  so that the following relation

$$M > 2\Delta \frac{\|PB_1K\|}{\lambda_{\min}(R)} \quad (38)$$

holds. Then, there is a control strategy of updating  $\mu_m$ , which depends on the measurement state such that the closed-loop system (35) is asymptotically stable with the disturbance attenuation level  $\gamma$ .

Note that a possible updating strategy can be selected as

$$\frac{|x_m(t - \tau_m(t))|}{M} \leq \mu_m \leq \frac{\lambda_{\min}(R)}{2\Delta\|PB_1\|\|C\|} |x_m(t - \tau_m(t))| \quad (39)$$

Consider the closed-loop overall system (35) with the gain matrix  $K$  determined by (34) and a quantizer satisfying the assumption A5. A simple choice of  $\mu_m$  has the form

$$\mu_m = \frac{2|x_m(t - \tau_m(t))|}{M + 2\Delta(\|PB_1\|\|C\|/\lambda_{\min}(R))} \quad (40)$$

Such a choice corresponds with that one for delay-less systems in the reference Chen et al. [2010].

Local controllers (5) are modified as

$$u_i(t) = Kq(x_i(t - \tau_i(t))) \quad t \in [t_k, t_{k+1}) \quad (41)$$

when substituting  $\mu_m \rightarrow \mu_i$  and  $x_m \rightarrow x_i$  for all  $i$  in (40). The following theorem states the main result.

*Theorem 3.* Given the system (15)–(16) satisfying Assumption 1 and a constant  $\gamma$ . First, construct the reduced-order design system (28)–(29). Then

a) Select the controller matrix  $K$  given by (34) in the controller (30) for the reduced-order system (28)–(29) satisfying Assumption 3 within the inequalities (33) for a given time delay bound  $\bar{\tau} > 0$ .

b) Select  $M$  and  $\Delta$  for the quantizer satisfying Assumptions 2, 5 and the relations (37)–(38).

c) Synthesize the gain matrices  $K$  and quantizers obtained for the design system (28)–(29) as local feedback controllers (41) satisfying Assumptions 4, 5 into the original system (1)–(3).

Then, the overall closed-loop system is asymptotically stable with the disturbance attenuation level  $\gamma$ .

Proofs of theorems are omitted due to the space limitation.

#### 4. CONCLUSION

The paper contributes with a new quantizer design complexity reduced scheme when designing decentralized quantized NCS controller for a class of symmetric composite systems. Communication delays, bounded packet dropouts and quantization issues are included in the design of a controller. A delay-dependent LMI-based approach is used to select a stabilizing  $H_\infty$  state feedback controller. A sufficient condition guaranteeing the asymptotic stability with a disturbance attenuation level  $\gamma$  of the overall closed-loop system with implemented identical local quantized controllers and is proved. Those controllers are synthesized from the quantizer obtained for the design model.

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