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On-line Parameter Tuning of Model Predictive Control

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Abstract: Model Predictive Control is a powerful control approach suitable for industrial applications due to its simplicity and flexibility. The aim of the paper is to introduce principal points for on-line tuning and automatic set-up of predictive control parameters. A proposed way of the tuning arises from the fact that in all control tasks, there is a certain degree of uncertainty with respect to the controlled system including its neighborhood. The strategy takes advantage of an analogy of the discrete Predictive Control and Linear Quadratic Control (LQ Control) connected with the probability calculus. The parameter tuning is demonstrated on multidimensional robotic system.

1. INTRODUCTION

In common practice, the majority of the controllers generates control actions with fixed parameter scaling, independent of controlled system character, considering only differences between system outputs and their reference, required signals. It is sufficient in almost cases, where controlled systems are linear or slightly non-linear in their behavior and they have single or several few pairs of inputs and outputs.

If the controlled systems have a considerable internal relations or a different number of their inputs and outputs, then, in such cases, it is necessary to involve this knowledge in used controller. This is feasible in model-based controllers. Especially suitable strategies for these cases are multistep strategies based on Linear Quadratic Control (LQ Control) and Model Predictive Control (Ordys et al., 1993). These strategies work well having correct physical or on-line identified model. The problem arises if this fact is not respected and control algorithm does not take into account the changes in the system response.

Usually, from mathematical point of view, the model-based predictive algorithms are able to manage response changes by resetting (tuning) their parameters in each optimization. However, it is not usual due to a number of parameters (elements of parameters) and unknown systematic procedure of their tuning. This paper deals with one possible way of on-line parameter tuning of Predictive Control if the system model is temporary imperfect or the measurement of system outputs is suddenly disturbed by rapid increase of noise.

Model Predictive Control is a powerful control approach suitable for industrial applications for its simplicity and flexibility (Rossiter, 2003). The aim of this paper is to introduce principal points for on-line tuning and automatic set-up of Model Predictive Control. The proposed tuning strategy arises from the fact that in all control tasks, there is a certain degree of uncertainty with respect to the system to be controlled and its neighborhood (Wittenmark, 1995). The strategy takes advantage of an analogy of the discrete Predictive Control and LQ Control connected with the probability calculus (Kárný, 1996). The tuning strategy will be demonstrated on multidimensional robotic system representing a system with internal nonlinear relations of parameters, states, inputs and outputs.

2. MODEL-BASED APPROACH TO CONTROL

Model-based approach to control represents a specific way, which consists in design (preliminary) steps and main real-control steps. The steps can be outlined as follows.

2.1 Design steps of model-based control

– *Data pre-processing*: input and output signal specification, removing outliers and possible signal scaling;
– *Prior information specification*: definition of supposed model order and model parameters of controlled system, possibly mathematical-physical analysis;
– *Model initialization*: model structure or order estimation, inclusion of prior information in that model structure;
– *Estimation/Calculation of model parameters*;
– *User ideal specification*: pre-processing of user requirements and their transformation into suitable (unified) form for control computation;
– *Control design*: set of controller parameters, initialization.

2.2 Main real-control steps

– *Model innovation*: on-line model parameter innovation/identification or recalculation of state dependent parameters;
– *Control computation*: adjustment of controller parameters, computation of control actions relative to the system outputs and their reference, and realization of computed actions.

Sequence of outlined steps and their procedures correspond to the character of given control process and selected control strategy. This paper is focused on control design and control computation steps in view of the controller parameter tuning.

3. MODEL PREDICTIVE CONTROL

Predictive Control is based on a minimization of a quadratic criterion (1), in which the future system outputs are substituted by their predictions (2) expressed by means of the system model (3) (Rossiter, 2003; Ordys et al. 1993):

$$J = \mathbf{J}^T\mathbf{J} = E\,[(\hat{\mathbf{y}} - \mathbf{w})^T \mathbf{Q}_y^T \mathbf{Q}_y (\hat{\mathbf{y}} - \mathbf{w}) + \mathbf{u}^T \mathbf{Q}_u^T \mathbf{Q}_u \mathbf{u}] \tag{1}$$

$$\hat{\mathbf{y}} = \mathbf{f} + \mathbf{G}\,\mathbf{u} \tag{2}$$

where $\hat{\mathbf{y}}$, $\mathbf{w}$ and $\mathbf{u}$ are vectors of predictions (future predicted system outputs), references and control actions (system inputs) for a given prediction horizon N:

$\hat{\mathbf{y}} = [\hat{\mathbf{y}}_{k+1}, \cdots, \hat{\mathbf{y}}_{k+N}]^T$, $\mathbf{w} = [\mathbf{w}_{k+1}, \cdots, \mathbf{w}_{k+N}]^T$, $\mathbf{u} = [\mathbf{u}_k, \cdots, \mathbf{u}_{k+N-1}]^T$;

and matrices $\mathbf{Q}_y$ and $\mathbf{Q}_u$ are weighting control parameters: output and input penalizations; E is an operator of mean. The predictions $\hat{\mathbf{y}}_{k+1}, \cdots, \hat{\mathbf{y}}_{k+N}$ in appropriate time instants of the prediction horizon can be expressed recurrently by (2) using the following model function in general form:

$$\hat{\mathbf{y}}_{k+1} = f(\mathbf{u}_k, \mathbf{x}_k) \tag{3}$$

The function $f(\mathbf{u}_k, \mathbf{x}_k)$ can represent either input-output model (e.g. autoregressive model) or state-space model. In the former case, $\mathbf{x}_k$ denotes the vector of delayed system inputs and outputs, the number of which corresponds to the system order, and in the latter case, $\mathbf{x}_k$ means usual state vector of the state-space model.

The minimization of the criterion (1) can be provided in one shot as a least squares problem solution of algebraic system of equations (Golub et al., 1989):

$$\mathbf{J} = \begin{bmatrix} \mathbf{Q}_y & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_u \end{bmatrix} \begin{bmatrix} \hat{\mathbf{y}} - \mathbf{w} \\ \mathbf{u} \end{bmatrix} = \mathbf{0}, \text{ with } \hat{\mathbf{y}} = \mathbf{f} + \mathbf{G}\,\mathbf{u}$$

$$\text{i.e. } \begin{bmatrix} \mathbf{Q}_y & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_u \end{bmatrix} \begin{bmatrix} \mathbf{G} & \mathbf{w} - \mathbf{f} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ -\mathbf{I} \end{bmatrix} = \mathbf{0} \tag{4}$$

For further explanation, let us consider the notation defined in the equations above. The following subsections outline common features of Predictive Control and LQ Control including its probabilistic interpretation. The interpretation will be use for on-line parameter tuning.

3.1 Analogy of Predictive Control and LQ Control

LQ Control, as well as Predictive Control, is a multistep control strategy. Its basis is a quadratic criterion usually expressed as follows

$$J = E\left[\sum_{j=k+1}^{N} \{(\hat{\mathbf{y}}_j - \mathbf{w}_j)^T \mathbf{Q}_y^T \mathbf{Q}_y (\hat{\mathbf{y}}_j - \mathbf{w}_j) + \mathbf{u}_{j-1}^T \mathbf{Q}_u^T \mathbf{Q}_u \mathbf{u}_{j-1}\}\right] \tag{5}$$

The minimization is not provided in one-shot as in predictive control design, but it is performed by recursive dynamic programming procedure as follows (Bobál et al., 2005)

$$\min_{\mathbf{u}_k} J = \min_{\mathbf{u}_k} \{E\,[\|(\hat{\mathbf{y}}_{k+1} - \mathbf{w}_{k+1})\,\mathbf{Q}_y\|^2 + \|\mathbf{u}_k\,\mathbf{Q}_u\|^2] + S_{k+1}^0\} \tag{6}$$

In it, the terms S_{k+i}^0, $i = 1, 2, \cdots, N$ represent running losses

$$S_{k+1}^0 = \min_{\mathbf{u}_{k+1}} \{E\,[\|(\hat{\mathbf{y}}_{k+2} - \mathbf{w}_{k+2})\,\mathbf{Q}_y\|^2 + \|\mathbf{u}_{k+1}\,\mathbf{Q}_u\|^2] + S_{k+2}^0\}$$

$$\vdots$$

$$S_{k+N-1}^0 = \min_{\mathbf{u}_{k+N-1}} \{E\,[\|(\hat{\mathbf{y}}_{k+N} - \mathbf{w}_{k+N})\,\mathbf{Q}_y\|^2 + \|\mathbf{u}_{k+N-1}\,\mathbf{Q}_u\|^2] + S_{k+N}^0\}$$

being gradually accumulated in the cost value J of the quadratic criterion (1).

If the minimization procedure starts every time with zero initial running loss, i.e. $S_{k+N}^0 = 0$, evaluation of obtained control law of LQ control leads to identical values of control actions as in case of Predictive Control.

In the both forms of the criterion i.e. (1) and (5), there operate parameters $\mathbf{Q}_y$ and $\mathbf{Q}_u$. As mentioned, they balance various control aims assigned by user, determine the quality of control process and character of controller response. Thus more precisely, penalization matrix $\mathbf{Q}_y$ serves weighting of differences among expected system outputs and user set points. The higher values of $\mathbf{Q}_y$ push the controller to generate control actions, usually energy-demanding, for more precise meeting the user references (leading to higher precision) and vice versa. On the other hand, penalization matrix $\mathbf{Q}_u$ weighs quantity, range or distribution of input energy. The lower values lead to higher, more independent control actions against the higher values of $\mathbf{Q}_u$, which lead to zero control. The both penalizations are mutually related.

The elements of the penalizations are usually specifically selected and being constant for whole run of control process; predominantly only elements on diagonals are non-trivial non-zero values. It is sufficient for large number of cases with stationary properties and stable surrounding conditions. However, even in these cases, the parameters need to be initially set and during control process tuned so that the control actions correspond to topical system state. The following subsection explains possible physical meaning of the control parameters by which the automatic set-up and on-line tuning may be realized. It arises from the probability calculus.

3.2 Probabilistic interpretation of control parameters

To select the values of control parameters more properly, differently from their trial selection, it is useful to investigate their fundament. In (Kárný, 1996; Belda 2009), probabilistic approach to LQ Control design is shown. Using mentioned approach, the physical fundament or meaning of the parameters can be formulated and used for on-line tuning even for set-up. The approach is based on Kullback-Leibler divergence (*KL-divergence*)

$$\mathcal{D}(f_N \,\|\, {}^I\!f_N) \equiv E\left\{\ln\left(\frac{f_N}{{}^I\!f_N}\right)\right\} \tag{7}$$

where f_N and ${}^I\!f_N$ are joint probability density functions (*pdf*s) operating on their domains and the horizon N.

Let succeeding system state $\mathbf{x}_{k+1}$ is assumed that it follows from a previous system state $\mathbf{x}_k$ and system input $\mathbf{u}_k$ only, then *pdf*s f_N and ${}^I f_N$ are considered to be defined for values and their parameters within the horizon N :

$$\begin{aligned} f_N &\equiv f(\mathbf{x}_{k+N}, \mathbf{u}_{k+N-1}, \mathbf{x}_{k+N-1} \cdots, \mathbf{u}_k, \mathbf{x}_k) \\ &= \left\{ \prod_{j=k+1}^{k+N} f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1})\, f(u_{j-1} \mid \mathbf{x}_{j-1}) \right\} f(\mathbf{x}_k) \end{aligned} \tag{8}$$

$$\begin{aligned} {}^I f_N &\equiv {}^I f(\mathbf{x}_{k+N}, \mathbf{u}_{k+N-1}, \mathbf{x}_{k+N-1} \cdots, \mathbf{u}_k, \mathbf{x}_k) \\ &= \left\{ \prod_{j=k+1}^{k+N} {}^I f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1})\, {}^I f(u_{j-1} \mid \mathbf{x}_{j-1}) \right\} f(\mathbf{x}_k) \end{aligned} \tag{9}$$

By minimization of *KL-divergence*, as it is indicated in (10),

$$\{ {}^O f(\mathbf{u}_{j-1} \mid \mathbf{x}_{j-1}) \}_{j=k+1}^{k+N} \in \underset{\{f(\mathbf{u}_{j-1} | \mathbf{x}_{j-1})\}_{j=k+1}^{k+N}}{\arg\min} \mathcal{D}(f_N \,\|\, {}^I f_N) \tag{10}$$

optimal *pdf* ${}^O f(\mathbf{u}_k \mid \mathbf{x}_k)$ is obtained. It describes probabilistic distribution of required control actions $\mathbf{u}_k$. Let the controlled system has normally distributed character $\mathcal{N}(\boldsymbol{\mu}_y, \mathbf{C}_y)$:

$$f(\mathbf{y}_{k+1} | \mathbf{u}_k, \mathbf{x}_k) = \frac{1}{\sqrt{(2\pi)^s |\mathbf{C}_y|}} e^{-\frac{1}{2}(\mathbf{y}_{k+1} - \boldsymbol{\mu}_y)^T \mathbf{C}_y^{-1} (\mathbf{y}_{k+1} - \boldsymbol{\mu}_y)} \tag{11}$$

where $\boldsymbol{\mu}_y$ are expected values $\boldsymbol{\mu}_y = \hat{\mathbf{y}}_{k+1}$; $\mathbf{C}_y$ is a covariance matrix; in a similar way for *pdf* $\mathcal{N}({}^I\boldsymbol{\mu}_y, {}^I\mathbf{C}_y)$:

$${}^I f(\mathbf{y}_{k+1} | \mathbf{u}_k, \mathbf{x}_k) = \frac{1}{\sqrt{(2\pi)^s |\mathbf{C}_y|}} e^{-\frac{1}{2}(\mathbf{y}_{k+1} - {}^I\boldsymbol{\mu}_y)^T \mathbf{C}_y^{-1} (\mathbf{y}_{k+1} - {}^I\boldsymbol{\mu}_y)} \tag{12}$$

${}^I\boldsymbol{\mu}_y$ is a vector of references ${}^I\boldsymbol{\mu}_y = \mathbf{w}_{k+1}$; and $\mathcal{N}({}^I\boldsymbol{\mu}_u, {}^I\mathbf{C}_u)$:

$${}^I f(\mathbf{u}_k | \mathbf{x}_k) = \frac{1}{\sqrt{(2\pi)^v |\mathbf{C}_u|}} e^{-\frac{1}{2}(\mathbf{u}_k - {}^I\boldsymbol{\mu}_u)^T \mathbf{C}_u^{-1} (\mathbf{u}_k - {}^I\boldsymbol{\mu}_u)} \tag{13}$$

${}^I\boldsymbol{\mu}_u$ is a vector of control references usually ${}^I\boldsymbol{\mu}_u = \{\mathbf{u}_{k-1} \vee \mathbf{0}\}$.

Then, the optimal *pdf* ${}^O f(\mathbf{u}_k \mid \mathbf{x}_k)$ leads to the following form (Kárný, 1996; Belda, 2009):

$${}^O f(\mathbf{u}_k | \mathbf{x}_k) = \frac{1}{\sqrt{(2\pi)^v |{}^O\mathbf{C}_u|}} e^{-\frac{1}{2} {}^O\mathbf{C}_u \{\mathbf{u}_k + f(\mathbf{C}_u^{-1}, \mathbf{C}_y^{-1}, \mathbf{x}_k, \mathbf{w})\}^2} \tag{14}$$

Obtained control law, which is involved in the exponent of the optimal *pdf* ${}^O f(\mathbf{u}_k \mid \mathbf{x}_k)$, is identical to the control law of LQ control (Bobál et al., 2005); i.e. in general:

probabilistic approach: $\quad {}^O\mathbf{u}_k = -f(\mathbf{C}_u^{-1}, \mathbf{C}_y^{-1}, \mathbf{x}_k, \mathbf{w}) \quad (15)$

LQ control: $\quad {}^O\mathbf{u}_k = -f(\mathbf{Q}_u, \mathbf{Q}_y, \mathbf{x}_k, \mathbf{w}) \quad (16)$

According to this indicated correspondence, it is possible to interpret physical meaning of the control parameters, penalizations as an inversion of covariance matrices (dispersion for single input single output systems):

$$\mathbf{Q}_u \propto \mathbf{C}_u^{-1}, \quad \mathbf{Q}_y \propto \mathbf{C}_y^{-1} \tag{17}$$

4. REALIZATION OF ON-LINE TUNING

As mentioned, the penalizations $\mathbf{Q}_y$ and $\mathbf{Q}_u$ balance individual terms in the criterion (1), thereby determine partly the quality of the control process and partly the character of controller response. In the previous section was shown, that their inversions closely relate to the dispersions or covariance matrices. They are usually selected from an experience or by an experimental tuning. However, considering the relation to the quality of the model, the selection is possible to be done more straightforwardly and to be provided on-line during control process.

4.1 Approximation of covariance matrices

The covariance matrix $\mathbf{C}_y$, in the fact its inversion, represents matrix of precisions of the system model. It includes cross relations among individual output signals. The matrix is proportional to output penalization $\mathbf{Q}_y$ as indicated in (17).

Let us return to the model function of controlled system (3). It represents only ideal deterministic relations of the system given by mathematical substance. However, the real systems contain number of stochastic components, which are usually involved into one noise term $\mathbf{e}_y$ as follows

$$\mathbf{y}_k = f(\mathbf{u}_{k-1}, \mathbf{x}_{k-1}) + \mathbf{e}_y \tag{18}$$

The term $\mathbf{e}_y$ represents a stochastic uncertainty, which is not modeled. However, it can be used in transferred interpretation for a quality evaluation of the model, i.e.

$$\mathbf{e}_{y_k} = \mathbf{y}_k - f(\mathbf{u}_{k-1}, \mathbf{x}_{k-1}) = \mathbf{y}_k - \boldsymbol{\mu}_{y_k} \tag{19}$$

where $\boldsymbol{\mu}_{y_k} = E\{\mathbf{y}_k\} = \hat{\mathbf{y}}_k$ represents deterministic relation, in stochastic point of view, expected values of the system outputs. Using this term, other descriptive statistics (e.g. error $\mathbf{e}_y = \mathbf{y} - \hat{\mathbf{y}}$, dispersion $E\{(y - \mu_y)^2\}$ or covariance matrix $\mathbf{C}_y = E\{(\mathbf{y} - \boldsymbol{\mu}_y)(\mathbf{y} - \boldsymbol{\mu}_y)^T\}$ can be evaluated in the relation to precision or precision matrices (Bernardo et al., 2000).

The matrix $\mathbf{C}_u$ may have similar interpretation. Nevertheless, for proper interpretation, note that in the ideal case, it, being diagonal, determines required (expected) square standard deviations of individual inputs (control actions) or in real case, the covariance matrix is confronted with additional measurement of really realized control actions, if possible.

This proposed idea is suitable for systems, where the system model together with the noise can change substantially. Possibly, due to additional interferences, these changes may occur randomly during the control. Inadequate choice of input and output penalizations can cause serious problems.

Unexpected system noise increase may force the controller to generate inputs that are suddenly out of any reasonable physical range of the controlled system or at least represent unreal magnitude change. It may cause serious device failures, e.g. system actuators (servo motors etc.) might not be able to achieve designed control interventions or may be damaged.

In such cases, where the model quality is low, it is usually acceptable to decrease control quality in order to achieve at least some reasonable values. The reduction of the control quality, i.e. the smaller reflection of difference between real measured output and its appropriate set point (reference), may be achieved just by adequate immediate on-line retuning of the penalizations.

The actual tuning can be realized by on-line evaluation of the model quality (19), on the basis of which output penalization $\mathbf{Q}_y$ is changed using proportional relation to the output covariance matrix $\mathbf{C}_y$ or practically to its estimate

$$\hat{\mathbf{C}}_{\mathrm{y}\,(i)} = \mathbf{e}_{\mathrm{y}(i)}\,\mathbf{e}_{\mathrm{y}(i)}^T \tag{20}$$

which is calculated from current outputs $\mathbf{y}_i$. The effect is that during periods of increased output noise, output set point is set to be less strict. It causes the output to be tracked less closely. It allows the input to stay in its reasonable ranges.

4.2 Filtration of approximation

Topical values of output covariance matrix can change very quickly. It can cause big changes of $\mathbf{Q}_y$. In order to avoid this variability, $\hat{\mathbf{C}}_{\mathrm{y}(i)}$ should be filtrated. As a suitable filter, exponential forgetting is proved as follows

$$\tilde{\mathbf{C}}_{\mathrm{y}(1)} = (1-\lambda)\,\varphi\,\hat{\mathbf{C}}_{\mathrm{y}\,(1)} \tag{21}$$

$$\tilde{\mathbf{C}}_{\mathrm{y}(i)} = \lambda\,\tilde{\mathbf{C}}_{\mathrm{y}(i-1)} + (1-\lambda)\,\varphi\,\hat{\mathbf{C}}_{\mathrm{y}\,(i)}, \quad i = 2,\cdots,k \tag{22}$$

where λ is a forgetting factor, which influences decrease speed of individual weight contributions to $\hat{\mathbf{C}}_{\mathrm{y}\,(i)}$; φ is a proportional coefficient shifting the values to values in no tuned mode in usual system behavior. Furthermore, φ unifies the ratio of both penalization matrices. In order to find a reasonable value for the parameter λ, the suitable number of time instants ℓ has to be defined in the correspondence to the character of control process.

The contribution of $\hat{\mathbf{C}}_{\mathrm{y}(i)}$ to $\tilde{\mathbf{C}}_{\mathrm{y}\,(i)}$ drops to the given level during these ℓ time instants. The usual choice is to select the number of instants (denoted by $\ell_{½}$) that cause dropping the contribution of $\hat{\mathbf{C}}_{\mathrm{y}\,(i)}$ to one half of its original. It implies that $\ell_{½}$ satisfies

$$\lambda^{\ell_{1/2}}(1-\lambda)\,\varphi\,\hat{\mathbf{C}}_{\mathrm{y}\,(k)} = \tfrac{1}{2}(1-\lambda)\,\varphi\,\hat{\mathbf{C}}_{\mathrm{y}\,(k)} \tag{23}$$

So called 'half-time' $\ell_{½}$ is user friendly way for selection of the factor λ, because it can easily imagined what is the time needed for a contribution of $\hat{\mathbf{C}}_{\mathrm{y}\,(i)}$ to drop to one half.

As indicated at description of proportional coefficient φ, the control parameters are mutually related. For their selection, their ratios are important and significant in spite of utilization of base units SI in the criterion.

5. SIMULATION EXAMPLES AND RESULTS

This section demonstrates the application of the investigated physical interpretation of the control parameters to the design of Model Predictive Control. It was indicated at the beginning of the section 3. The Predictive Control with on-line tuning of the parameters (output penalization $\mathbf{Q}_y$) is applied to a multidimensional robotic system 'Moving Slide'. It is shown in Fig. 1 (Belda, 2010).

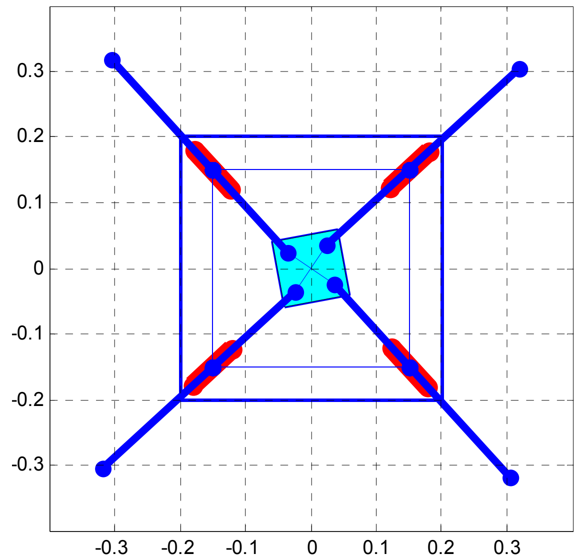

Fig. 1. Robotic system 'Moving Slide'

5.1 Description of robotic system

The robotic system used for the demonstration represents planar parallel robot serving for top milling operations. It has four inputs (for drives) and three outputs (Cartesian coordinates *x*, *y* and angular displacement ψ around perpendicular *z*-axis to *xy*-plane). The control is focused on movable platform, i.e. just on Cartesian coordinates of its centre and its angular displacement. The system itself is a system with sophisticated nonlinear internal relations and, as mentioned, with different number of inputs and outputs. The relations can be described by a system of nonlinear differential equations as pure equations of motion (Stejskal et al., 1996)

$$\ddot{\mathbf{y}} = \mathbf{f}(\mathbf{y},\dot{\mathbf{y}}) + \mathbf{g}(\mathbf{y})\,\mathbf{u} \tag{24}$$

where vector of outputs is $\mathbf{y} = [x, y, \psi]^T$.

The system (24) was linearized or decomposed and transformed to the ordinary state-space model (Valášek et al., 1999) and discretized to the following discrete form:

$$\begin{aligned}\mathbf{x}_{k+1} &= \mathbf{A}_k\,\mathbf{x}_k + \mathbf{B}_k\,\mathbf{u}_k \\ \mathbf{y}_k &= \mathbf{C}\,\mathbf{x}_k\end{aligned}, \quad \mathbf{x}_k = \begin{bmatrix}\mathbf{y}_k \\ \dot{\mathbf{y}}_k\end{bmatrix} \tag{25}$$

where individual elements of state matrix $\mathbf{A}_k$ and input matrix $\mathbf{B}_k$ are dependent on topical system state $\mathbf{x}_k$. They are recalculated in every time step prior to every evaluation of the quadratic criterion (1), from which topical control actions arise (Belda, 2003 and 2005).

5.2 Time histories of Model Predictive Control process

Tests were performed on trajectory in Fig. 2 with one repetitive run and initial and final point in the workspace centre.

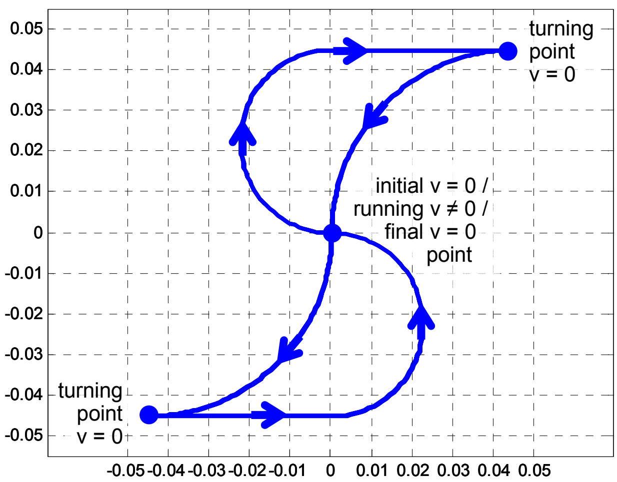

Fig. 2. Testing ‘S-shape’ trajectory

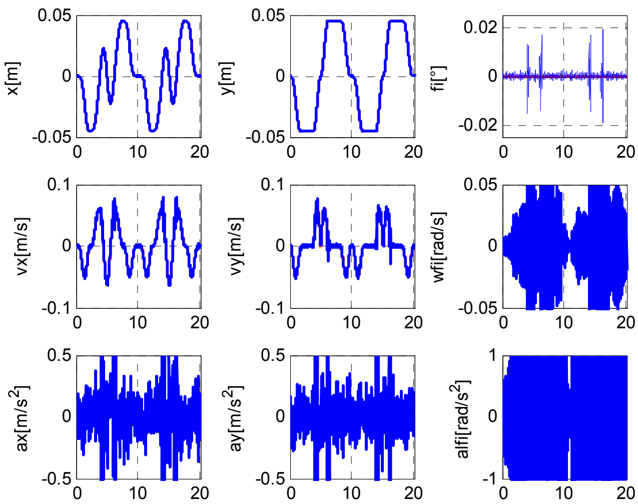

Fig. 3. Time histories of kinematical quantities

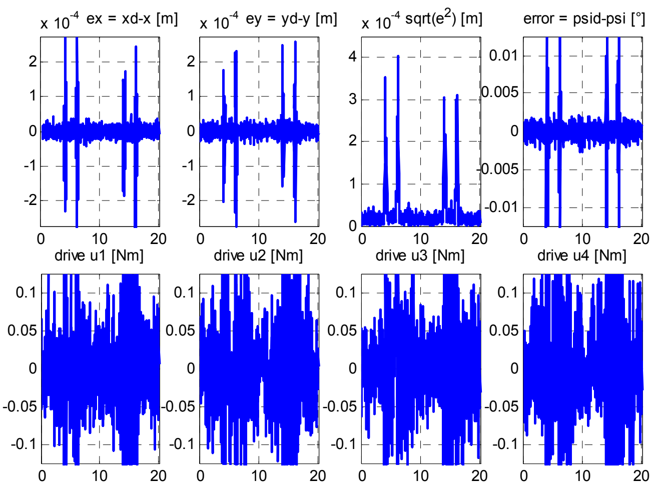

Fig. 4. Time histories of errors and control actions

Fig. 3 - Fig. 4 show no tuned case (i.e. $\mathbf{Q}_y = const.$) and other Fig. 5 - Fig. 7 illustrate on-line tuned Model Predictive Control. In Fig. 5, there are time histories of the penalization elements responding to the noise increase for 0.4s in 4, 6, 14, 16s.

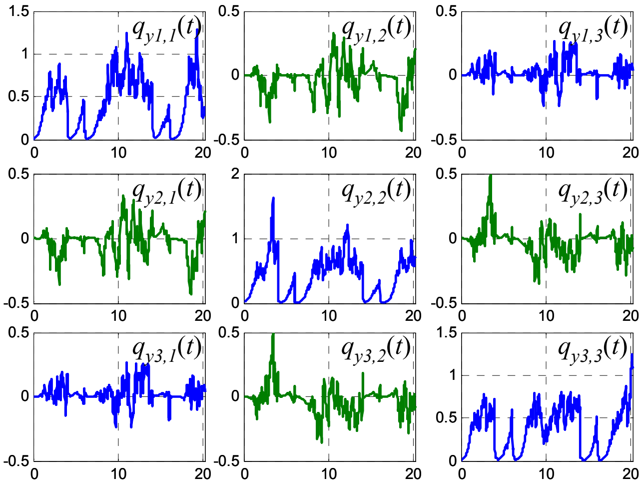

Fig. 5. Time histories of $\mathbf{Q}_y$ elements as are indicated.

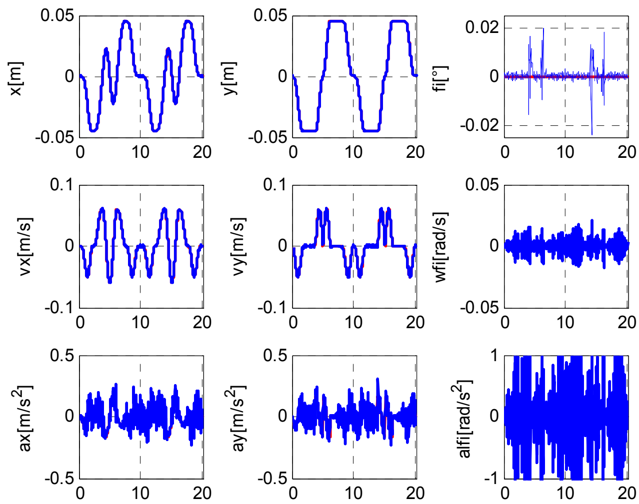

Fig. 6. Time histories of kinematical quantities

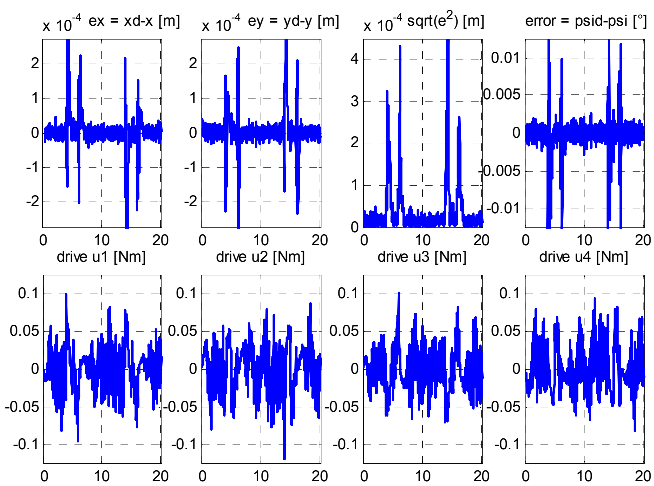

Fig. 7. Time histories of errors and control actions

5.3 Tests' conditions and result description

The conditions of the tests were the following. The movable platform was required not to rotate i.e. required angular displacement was $\psi = 0$ for whole time range 20s (= 2×10s per one cycle). The maximum tangential speed of the centre of movable platform was $v = 0.06\ \mathrm{m\,s^{-1}}$. The noise increase in mentioned time ranges was around ten times the normal amount. The sampling period was $Ts = 0.02s$ for the model (25) and for control experiments as well. The prediction horizon was $N = 10$.

The penalizations $\mathbf{Q}_\mathbf{y}$ and $\mathbf{Q}_\mathbf{u}$ for the test without tuning were selected as usual to be diagonal $\mathbf{Q}_\mathbf{y}$: $q_{\mathbf{y}(i,i)} = 1, i = 1, 2, \cdots, s$ and $\mathbf{Q}_\mathbf{u}$: $q_{\mathbf{u}(i,i)} = 10^{-4}, i = 1, \cdots, v$. In case of the test with on-line parameter tuning of Predictive Control, the output penalization is a function of estimated covariance matrix $\mathbf{Q}_{y_k} = f(\hat{\mathbf{C}}_{y_k}^{-1})$ with exponential forgetting $\lambda = 0.93$. $\mathbf{Q}_\mathbf{u}$ is a diagonal matrix as in case without tuning. It is that only the ratio of both weighting factors $\mathbf{Q}_\mathbf{y}$ and $\mathbf{Q}_\mathbf{u}$ is important.

From set-up point of view, the suitable ratio of the penalizations or elements of $\mathbf{Q}_\mathbf{y}$ penalization are adjusted automatically. The penalization at the beginning of the control was set in correspondence to generally high output dispersion e.g. $E\{(y_i - \mu_y)^2\}_{i=1,\cdots,s} = 100$. That selection determines diagonal of $\mathbf{Q}_\mathbf{y}$: $q_{\mathbf{y}(i,i)} = 10^{-2}, i = 1, 2, \cdots, s$ and represents low output penalization neglecting initial identity of the mathematical model and system outputs due to initial position with zero speed. However, due to continuous tuning, the diagonal elements of $\mathbf{Q}_\mathbf{y}$ themselves and remaining elements of $\mathbf{Q}_\mathbf{y}$ (being initially zeros) are tuned to appropriate operational values.

At the comparison of the quality of both cases without tuning and with tuning (Fig. 4 and Fig. 7), the control actions of tuned case are more smoothed and smaller in magnitudes at the comparable profiles of control errors. The differences in robot behavior (see Fig. 3 and Fig. 6) look hardly noticeable. However, under detail investigation, the behavior of the controlled system in tuned case is calmer. It corresponds to appropriate control actions in Fig. 4 and Fig. 7.

6. CONCLUSION

The paper deals with the automatic set-up and mainly with automatic on-line tuning of control parameters of Model Predictive Control. On the basis of fundamental analogy of Predictive Control and LQ control, where LQ control was derived using probability calculus, the physical interpretation of control parameters (penalization matrices) was explained.

The presented interpretation was used as the basis for on-line tuning of mentioned control parameters. It complies with mathematical conditions of minimization procedures. In comparison with on-line adaptive approaches, the tuning can react more quickly against model identification, where retuning of whole model is slower. Furthermore, the proposed way may be feasible in real-time for fast-dynamic systems without increased computational demands e.g. unlike constrained predictive control (Maciejowski, 2002).

The practical notes to tuning realization were given by means of estimated parameter filtering in section 4. The section on examples and results demonstrated the on-line tuning with multidimensional robotic system. In it, there was a comparison with no-tuned case.

Nowadays, there exist a lot of interesting control approaches, but their automatic set-up and tuning based on some universal base is lacking. It is a limiting factor for their use in real industrial applications.

This paper proposes one possible systematic way for general use, which does not lead to increase control algorithm complexity or computational demands. It does not change conditions of usual control design. Only current information obtained from the topical measurement of system outputs and possibly of realized system inputs is considered.

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REFERENCES

Belda, K. (2010). Robotic Device (in Czech), Patent certification No. 301781 CZ, 5 pp.

Belda, K. (2009). Probabilistic design and tuning of LQ control. *Archives of Control Sciences*. **Vol. 19**, No. 4, pp. 371-384.

Belda, K., J. Böhm and M. Valášek (2005). Predictive Control Applied to Planar Parallel Robots. *Preprints of the 16th IFAC World Congress*, Prague, 6 pp.

Belda, K., J. Böhm, and M. Valášek (2003). State-space generalized predictive control for redundant parallel robots. *Mechanics Based Design of Structures and Machines*, **Vol. 31**, No. 3, pp. 413-432.

Bernardo, J. M. and A.F.M. Smith (2000). *Bayesian Theory*, Wiley.

Bobál, V., Böhm, J.,Fessl, J. and Macháček (2005). *Digital Self-tuning Controllers*, Springer.

Golub, H.G. and Ch. F. L. Van (1989). *Matrix Computations*, The Johns Hopkins Univ. Press.

Kárný, M. (1996). Towards fully probabilistic control design. *Automatica*, **Vol. 32**, No. 12, pp. 1719-1722.

Maciejowski, J., M. (2002). *Predictive Control with Constrains*, Prentice Hall.

Ordys, A. and D. Clarke (1993). A state - space description for GPC controllers. *Int. J. Systems SCI.*, **Vol. 24**, No. 9, 1727-1744.

Rossiter, J., A. (2003). *Model-Based Predictive Control A Practical Approach*, CRC Press.

Stejskal, V. and M. Valášek (1996). *Kinematics and Dynamics of Machinery*, Marcel Dekker, New York.

Valášek, M. and P. Steinbauer (1999). Nonlinear Control of Multibody Systems, *Euromech*, Lisabon, pp. 437-444.

Wittenmark, B (1995). Adaptive Dual Control, *Seminar handout*, Lund Institute of Technology, 11 pp.