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VEHICLE POSITION ESTIMATION USING GPS/CAN DATA BASED ON NONLINEAR PROGRAMMING

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ABSTRACT

The paper solves a problem of the estimation of the moving vehicle position. The position is measured by global position system (GPS) but outages sometimes occur in the measurements. During these outages, the actual position is estimated using data from vehicle sensors. A moving vehicle is described by a discrete-time state-space model with bounded noise. This model is constructed using kinematics laws and it can be used for arbitrary type of ground vehicle. Bayesian approach is applied to obtain position estimates. The maximum a posteriori (MAP) estimation converts to the nonlinear programming. The paper also discusses a setting of initial conditions for successful running of estimation process.

KEY WORDS

nonlinear systems, state-space model, filtering, bounded noise, incomplete data, vehicle position estimation

1 Introduction

Today, an importance of the precise tracking of moving vehicle is growing. The knowledge of the vehicle position is required in many practical application, e.g. in a navigation or in a vehicle trajectory reconstruction. There are many methods how to solve this task using various data sources and their combination, see eg. [3], [5],[7].

Frequently, the position of a moving vehicle is determined by means of global positioning system (GPS). The GPS provides the position directly in the Cartesian coordinates but signal outages can occur in the data that are caused e.g. by big trees or buildings that inhibit the signal receiving.

Inertial navigation system (INS) is often an alternative data source. INS is a navigation aid that uses a computer, motion sensors (accelerometers) and rotation sensors (gyroscopes) to continuously calculate via dead reckoning the position, orientation, and velocity (direction and speed of movement) of a moving object without the need for external references. The starting location and the initial orientation of a vehicle (azimuth) have to be known. But the data is relative and therefore, the estimation error has a cumulative character.

Many estimation algorithms combining the GPS/INS data were developed. GPS corrects the errors in INS, INS supplies data during GPS signal outages. For estimation purposes, a model is usually constructed that uses kinematics laws, i.e., it is not concerned with the causing forces. The estimation algorithms mostly use a various modification of Kalman filter (KF), see e.g. [10], [11], [12]. Nevertheless, KF is very sensitive to tuning covariances.

In [4], a scheme to integrate GPS/INS is proposed using a constructive neural network to overcome the limitations of schemes based on KF.

In [9], the vehicle position is estimated using discrete-time state-space model with non-linear state equation and with uniformly distributed noise. The problem of MAP estimation converts there into the problem of the non-linear programming.

Here, we generalize this model by introducing likewise nonlinear output equation and we construct alternative vehicle model that better utilizes available measurements. We focus on the precise off-line estimation based on a simple algorithm that uses readily available data sources and avoids demanding initial setting.

As additional data source during GPS data outages, we use data from vehicle sensors instead of commonly used INS data. These data are provided by controller area network (CAN) that is a bus network that connects devices, sensors and actuators in a vehicle for control applications. The CAN data include a complete information about vehicle velocity, yaw rate and lateral acceleration. The CAN measurements are at disposal during the whole driving time. These data are an adequate alternative to above mentioned INS data.

2 Preliminaries

2.1 Notation

Throughout, the following notation is used:

$\equiv$	equality by definition
$\propto$	equality up to a constant factor
$\mathbf{z}^*$	a set of $\mathbf{z}$-values, $\mathbf{z} \in \mathbf{z}^*$, $\mathbf{z}$ is a column vector
$\mathbf{z}_t$	value of $\mathbf{z}$ in discrete time instant t; $t \in t^* \subset \{0, 1, 2, \dots T\}, T < \infty$

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$\hat{\mathbf{z}}_t$	point estimate of $\mathbf{z}_t$;
$\mathbf{z}^{k:l}$	the ordered sequence; $\mathbf{z}^{k:l} \equiv [\mathbf{z}'_k, \mathbf{z}'_{k+1}, \ldots, \mathbf{z}'_l]'$, $0 \le k \le l$
$\mathbf{z}^{k:a;b:l}$	subsequence of $\mathbf{z}^{k:l}$; for $a < b$ $\mathbf{z}^{k:a;b:l} \equiv [\mathbf{z}'_k, \mathbf{z}'_{k+1}, \ldots, \mathbf{z}'_a, \mathbf{z}'_b, \ldots, \mathbf{z}'_l]'$
$'$	transposition
$\underline{\mathbf{z}}$, $\overline{\mathbf{z}}$	lower and upper bound on $\mathbf{z}$, respectively; inequalities like $\mathbf{z} \ge \underline{\mathbf{z}}$ are meant entry-wise
$\mathbf{0}_{(\alpha)}$, $\mathbf{1}_{(\alpha)}$	column vector of zeros and ones of the indicated size, respectively
$\mathcal{R}^n$	n-dimensional real space
$f(\cdot\|\cdot)$	probability density functions (pdf); respective pdfs are distinguished by the argument names; no formal distinction is made between a random variable, its realization and pdf argument

Integrals used are always definite and multivariate ones. The integration domain coincides with the support of the pdf in its argument.

The only properties of pdfs exploited within the paper are as follows, e.g., [6]. Let us consider the joint pdf $f(a,b,c)$, then the *chain rule* $f(a,b,c) = f(a|b,c)f(b|c)f(c)$ relates it to the conditional pdfs $f(a|b,c)$, $f(b|c)$ and marginal pdf $f(c)$. All pdfs are non-negative and *normalised* $\int f(a,b|c)\mathrm{d}a\mathrm{d}b = \int f(a|b,c)\mathrm{d}a = \int f(b|a,c)\mathrm{d}b = 1$. These properties imply *Bayes rule*

$$f(b|\,a,c) = \frac{f(a|b,c)f(b|c)}{\int f(a|b,c)f(b|c)\mathrm{d}b} \propto f(a|b,c)f(b|c). \quad (1)$$

Note that a factor in the denominator is determined uniquely by normalisation.

2.2 Basics of Bayesian learning

In Bayesian view [1], [6], the system is described by probability density functions (pdfs). The quantities describing the system consist generally of observable outputs $\mathbf{y}^{1:T}$, optional inputs $\mathbf{u}^{1:T}$ and internal quantities $\mathbf{X}$ that are never observed directly. The internal quantities $\mathbf{X}$ consist of system states $\mathbf{x}^{0:T}$ and a time invariant unknown parameters $\mathbf{\Theta}$. The collection of the outputs and inputs is called data and denoted $\mathbf{d}^{1:T}$ with $\mathbf{d}_t = (\mathbf{y}_t, \mathbf{u}_t)$, $t \in t^* = \{1, \ldots, T\}$.

Using the commonly accepted definition of the state as the collection of quantities comprising whole past history and taking into account that admissible input generator cannot exploit unknown quantities, the chain rule factorizes the joint pdf of all quantities as follows

$$f\left(\mathbf{d}^{1:T}, \mathbf{X}\right) = f\left(\mathbf{d}^{1:T}, \mathbf{x}^{0:T}, \mathbf{\Theta}\right) = f\left(\mathbf{x}_0, \mathbf{\Theta}\right) \prod_{t=1}^{T} \quad (2)$$
$$f\left(\mathbf{y}_t|\,\mathbf{x}_t, \mathbf{u}_t, \mathbf{\Theta}\right) f\left(\mathbf{x}_t|\,\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{\Theta}\right) f\left(\mathbf{u}_t\middle|\,\mathbf{d}^{t-1}\right).$$
$$\propto f(\mathbf{x}_0, \mathbf{\Theta}) \prod_{t=1}^{T} f(\mathbf{y}_t|\mathbf{x}_t, \mathbf{u}_t, \mathbf{\Theta}) f(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{\Theta}).$$

The individual factors are, $t \in t^*$,

prior pdf: $f(\mathbf{x}_0, \mathbf{\Theta})$ (3)

observation model: $f(\mathbf{y}_t|\mathbf{x}_t, \mathbf{u}_t, \mathbf{\Theta})$ (4)

state evolution model: $f(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{\Theta})$ (5)

controller: $f(\mathbf{u}_t|\mathbf{d}^{1:t-1})$. (6)

As the controller does not depend on the internal quantities $\mathbf{x}_t$, $\mathbf{\Theta}$ it plays no role in the estimation. Therefore, the knowledge of the controller is not required. Only, the generated input values have to be known.

The Bayesian learning works with characteristics of the joint pdf (2). This pdf combines prior information in $f(\mathbf{x}_0, \mathbf{\Theta})$, theoretical knowledge described by both observation (4) and state evolution (5) models and observed data $\mathbf{d}^{1:T}$ by using deductive rules of the calculus with pdfs (1). Bayesian learning consists of evaluation of characteristics of the posterior pdf on (a part of) internal quantities conditioned on (a part of) observed data. For $\mathbf{X}$ consisting of $\mathbf{x}^{0:T}$ and $\mathbf{\Theta}$, the Bayes rule provides immediate formal solution

$$f\left(\mathbf{X}\middle|\,\mathbf{d}^{1:T}\right) = \frac{f\left(\mathbf{d}^{1:T}, \mathbf{X}\right)}{\int f\left(\mathbf{d}^{1:T}, \mathbf{X}\right)\mathrm{d}\mathbf{X}}. \quad (7)$$

3 Discrete-time state-space model with bounded noise and partially missing data

We consider a discrete-time state space model that describes a system without an input, with state $\mathbf{x}_t$ of length m and output $\mathbf{y}_k$ of length n by the following non-linear state (8) and output (9) equations in the discrete time instants $t \in t^* = 1, 2, \ldots, T$, $k \in k^* \subset t^*$

$$\mathbf{x}_t = g(\mathbf{x}_{t-1}) + \mathbf{w}_t \quad (8)$$
$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{e}_k, \quad (9)$$

where g is a real vector function, $g: \mathcal{R}^m \to \mathcal{R}^m$;
h is a real vector function, $h: \mathcal{R}^m \to \mathcal{R}^n$;
$\mathbf{w}_t$, $\mathbf{e}_k$ are vectors of the state and output noises of lengths m and n, respectively, i.e., they are zero mean, mutually conditionally independent and identically distributed.

Here, $\mathbf{w}_t$, $\mathbf{e}_k$ are assumed to have uniform distribution on a multivariate box with the center $\mathbf{0}$ and unknown half-widths of the support intervals $\mathbf{q}$ and $\mathbf{r}$, respectively, i.e,.

$$f(\mathbf{w}_t|\mathbf{q}) = \mathcal{U}(\mathbf{0}, \mathbf{q}), \quad f(\mathbf{e}_k|\mathbf{r}) = \mathcal{U}(\mathbf{0}, \mathbf{r}). \quad (10)$$

In the following, we consider only single uninterrupted data outage, i.e. available data are $\mathbf{d}^{1:a;b:T}$, $a < b$, and $k^* = \{1, 2, \ldots, a, b, b+1, \ldots, T\}$; the number of all available data items is K, $K \le T$. Note that an extension to the more data outages is straightforward.

Further, we suppose that $\mathbf{x}_0$, $\mathbf{q}$, and $\mathbf{r}$ are a priori mutually independent and that it holds

$$\underline{\mathbf{x}}_0 \le \mathbf{x}_0 \le \overline{\mathbf{x}}_0,\ \mathbf{0} \le \mathbf{q} \le \overline{\mathbf{q}},\ \mathbf{0} \le \mathbf{r} \le \overline{\mathbf{r}},\ \{\underline{\mathbf{x}} \le \mathbf{x}_t \le \overline{\mathbf{x}}\}_{t \in t^*}. \quad (11)$$

Note that restrictions in (11) are defined by the user so that they reflect the reality. These known optional values specify user's prior information.

Equations (8) and (9) together with the assumptions (10) and (11), define the state uniform model (SU model).

We introduce the column vector $\mathbf{X}$ as follows

$$\mathbf{X} = \left[\begin{array}{ccc} \left(\mathbf{x}^{0:T}\right)' & \mathbf{q}' & \mathbf{r}' \end{array} \right]'. \tag{12}$$

The joint pdf (2) of data $\mathbf{y}^{1:a;b:T} \equiv \mathbf{d}^{1:T}$, the state trajectory $\mathbf{x}^{0:T}$, and unknown parameters $\mathbf{\Theta} = [\mathbf{q}', \mathbf{r}']'$ of the SU model takes the form

$$f\left(\mathbf{y}^{1:a;b:T}, \mathbf{x}^{0:T}, \Theta\right) \tag{13}$$

$$\propto f(\mathbf{x}_0, \mathbf{\Theta}) \prod_{t \in t^*} f(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{\Theta}) \prod_{k \in k^*} f(\mathbf{y}_k|\mathbf{x}_k, \mathbf{\Theta})$$

$$= \left[\prod_{i=1}^{m} q_i\right]^{-T} \left[\prod_{j=1}^{n} r_j\right]^{-K} \chi(\mathcal{S}).$$

where m, n are the lengths of the state and output vector, respectively,
T, K are numbers of available states and outputs, respectively
$\chi(\mathcal{S})$ is the indicator of the support $\mathcal{S}$ of this pdf.

The set $\mathcal{S}$ is a set of $\mathbf{X}$ (12) such that (for given realization $\mathbf{y}^{1:T}$) the noise terms in (8) and (9) are inside multivariate box defined by (10) and (11), i.e.,

$$\mathcal{S} = \{\mathbf{X} \in \mathcal{S}0; \forall t \in t^* : |\mathbf{x}_t - g(\mathbf{x}_{t-1})| \leq \mathbf{q}, \tag{14}$$

$$|\mathbf{y}_k - h(\mathbf{x}_k)| \leq \mathbf{r}\}.$$

where $\mathcal{S}0$ is the set of $\mathbf{X}$ that meet (11).

The linear version of SU model with complete data set was introduced in [8] and algorithms for both state filtering and parameters estimations were designed. In [9], this model was generalized by assuming of a non-linear state equation and algorithm for state and parameter estimation with missing data was proposed. Here, we continue generalization of the original model by allowing of nonlinearities in both state (8) and output (9) equations and by considering of missing measurements. The output outages are easily introduced by using different time indexes, t and k, in (8) and (9).

3.1 One-shot state and parameter estimation

We suppose that the considered system is described by the SU model (13). Using knowledge of data $\mathbf{d}^{1:T}$ and prior information, we aim to estimate states $\mathbf{x}^{0:T}$ and the noise bounds $\mathbf{q}, \mathbf{r}$, i.e. vector $\mathbf{X}$ (12). We focus on a maximum a posteriori (MAP) estimation, see e.g. [1], that provides a point estimate of the internal quantity $\mathbf{X}$

$$\hat{\mathbf{X}}_{MAP} = \text{arg max}_{X^*} f(\mathbf{d}^{1:T}|X) f(X)$$

The MAP estimate $\hat{\mathbf{X}}_{MAP}$ of $\mathbf{X}$ with negative logarithm of a posteriori pdf has the following form

$$\hat{\mathbf{X}}_{MAP} = \text{arg min}_{\mathbf{X} \in \mathcal{S}} \left(\sum_{i=1}^{m} \ln(q_i) + \frac{K}{T} \sum_{j=1}^{n} \ln(r_j) \right). \tag{15}$$

where m, n are the lengths of the state and output vector, respectively,
$\mathcal{S}$ is given by (14).

3.2 Notes to the estimation with missing data

The previous text implies that the problem of missing measurement data can be easily incorporated into the estimation algorithm. If all measurements are available, then $k^* = t^*$, $K = T$, see (8), (9). The missing measurement causes that the corresponding entry of the output inequality (9) is missing in the respective time instant. The more is missing data, the smaller is the number of corresponding constraint conditions in the set $\mathcal{S}$ (14) and the smaller is the weight of the second term in (15).

To prevent the state-estimates divergence in the case of measurement outage, the data have to be present both at the beginning and end of estimated time interval.

3.3 MAP estimation as a problem of mathematical programming

The MAP estimate (15) can be solved by the following nonlinear programming (NLP) form [2]

$$\begin{aligned} &\text{Find } \hat{\mathbf{X}} \text{ so as to} \\ &\mathcal{J}(\hat{\mathbf{X}}) = \sum_{i=1}^{m} \ln(q_i) + \frac{K}{T} \sum_{j=1}^{n} \ln(r_j) \ \rightarrow \min \\ &\text{while } \mathcal{C}(\hat{\mathbf{X}}) \leq \mathbf{0},\ \underline{\mathbf{X}} \leq \hat{\mathbf{X}} \leq \overline{\mathbf{X}}, \end{aligned} \tag{16}$$

where
(i) $\mathcal{J} : \mathcal{R}^{(m+2)T+n} \rightarrow \mathcal{R}$ is a real function on $\hat{\mathbf{X}}$
(ii) $\mathcal{C}$ is a real vector function that corresponds to the inequalities describing $\mathcal{S}$ (14), i.e.,

$$\begin{array}{rrl} \mathbf{x}_1 - g(\mathbf{x}_0) & -\mathbf{q} & \leq \mathbf{0} \\ -\mathbf{x}_1 + g(\mathbf{x}_0) & -\mathbf{q} & \leq \mathbf{0} \\ \vdots & & \\ \mathbf{x}_T - g(\mathbf{x}_{T-1}) & -\mathbf{q} & \leq \mathbf{0} \\ -\mathbf{x}_T + g(\mathbf{x}_{T-1}) & -\mathbf{q} & \leq \mathbf{0} \\ & & \\ \mathbf{y}_1 - h(\mathbf{x}_1) & -\mathbf{r} & \leq \mathbf{0} \\ -\mathbf{y}_1 + h(\mathbf{x}_1) & -\mathbf{r} & \leq \mathbf{0} \\ \vdots & & \\ \mathbf{y}_T - h(\mathbf{x}_T) & -\mathbf{r} & \leq \mathbf{0} \\ -\mathbf{y}_T + h(\mathbf{x}_T) & -\mathbf{r} & \leq \mathbf{0} \end{array}$$

(iii) $\underline{\mathbf{X}}$, $\overline{\mathbf{X}}$ are known vectors; they stem from the set $\mathcal{S}0$ that is given by (11) and have the following form

$$\underline{\mathbf{X}} = \begin{bmatrix} \underline{\mathbf{x}}_0 \\ \mathbf{1}_{(T)} \otimes \underline{\mathbf{x}} \\ \mathbf{0}_{(m)} \\ \mathbf{0}_{(n)} \end{bmatrix}, \quad \overline{\mathbf{X}} = \begin{bmatrix} \overline{\mathbf{x}}_0 \\ \mathbf{1}_{(T)} \otimes \overline{\mathbf{x}} \\ \overline{\mathbf{q}} \\ \overline{\mathbf{r}} \end{bmatrix}. \qquad (17)$$

To solve (16), we use the function “fmincon” from optimization toolbox of MATLAB [1]. This function starts with searching of $\hat{\mathbf{X}}$ at user supplied point $\hat{\mathbf{X}}_0$. A starting point $\hat{\mathbf{X}}_0$ of the optimization has to be set appropriately. Improper setting of $\hat{\mathbf{X}}_0$ causes numerical instability. The setting of $\hat{\mathbf{X}}_0$ is discussed in Section 4.2.

4 Estimation of moving vehicle position

4.1 Non-linear SU model of a moving vehicle

Here, we construct the state (8) and output (9) equations that relate the vehicle movements to GPS/CAN data. The list of available data and considered internal quantities is summarized in Table 1. In the model, we do not consider altitude changes and use the Cartesian coordinate system.

Table 1. GPS/CAN data – usage in the model
(t. = true; m. = measured)

quantity [unit]	notation	source	in model as
t. position No.1 [m]	$(p_{x1},\, p_{y1})$	-	state
t. position No.2 [m]	$(p_{x2},\, p_{y2})$	-	state
t. azimuth [rad]	φ	-	state
t. velocity [m/s]	v	-	state
t. yaw rate [rad/s]	ω	-	state
m. position No.1 [m]	$(\tilde{p}_{x1},\, \tilde{p}_{y1})$	GPS	output
m. position No.2 [m]	$(\tilde{p}_{x2},\, \tilde{p}_{y2})$	GPS	output
m. azimuth [rad]	$\tilde{\varphi}$	GPS	output
m. velocity [m/s]	$\tilde{v}$	CAN	output
m. yaw rate [rad/s]	$\tilde{\omega}$	CAN	output

The position of a vehicle is measured in two spots, (p_{x1}, p_{y1}) and (p_{x2}, p_{y2}). These spots are placed on the lengthwise axis of the vehicle. This arrangement easily enables to estimate azimuth φ, which is defined as the horizontal angular distance between a direction of the vehicle moving and a northern direction, measured clockwise.

For measured yaw rate ω, an anti-clockwise direction is considered as a positive direction of rotation.

Using available data, a vehicle movement is characterized by a time evolution of the position vector $\mathbf{p} = (p_x, p_y)$, whereas the following relationships between $\mathbf{p}$ and velocity $\mathbf{v} = (v_x, v_y)$ hold

$$\dot{p}_x = v_x = v \sin\varphi, \;\; \dot{p}_y = v_y = v\cos\varphi, \;\; \dot{\varphi} = \omega, \qquad (18)$$

where v is absolute velocity value, $v = \sqrt{v_x^2 + v_y^2}$;
$\dot{z}$ means a time derivative of z.

The above relations cannot be used directly in the discrete-time equations (8) and (9). Therefore, we approximate the differential equations by the difference ones. Then, using the assignment from Table 1 and the relationships (18), the state equation (8) takes the following form, $t \in t^*$

$$\begin{array}{lllll} p_{x1;t} & = p_{x1;t-1} & +hv_{t-1}\sin\varphi_{t-1} & +w_{x1;t} \\ p_{y1;t} & = p_{y1;t-1} & +hv_{t-1}\cos\varphi_{t-1} & +w_{y1;t} \\ p_{x2;t} & = p_{x2;t-1} & +hv_{t-1}\sin\varphi_{t-1} & +w_{x2;t} \\ p_{y2;t} & = p_{y2;t-1} & +hv_{t-1}\cos\varphi_{t-1} & +w_{y2;t} \\ \varphi_t & = \varphi_{t-1} & -h\,\omega_{t-1} & +w_{\varphi;t} \\ v_t & = v_{t-1} & & +w_{v;t} \\ \omega_t & = \omega_{t-1} & & +w_{\omega;t}, \end{array} \qquad (19)$$

where h is the time difference between two subsequent time instants of measurement, labeled by t and $t-1$.

The last two equalities in (19) take into consideration that there are no fast changes of values of v and ω between two successive time instant.

The output equation (9) is as follows, $t \in t^*$

$$\begin{array}{ll} \mathcal{I}_t\, \tilde{p}_{x1;t} & = \mathcal{I}_t(p_{x1;t} + e_{x1;t}) \\ \mathcal{I}_t\, \tilde{p}_{y1;t} & = \mathcal{I}_t(p_{y1;t} + e_{y1;t}) \\ \mathcal{I}_t\, \tilde{p}_{x2;t} & = \mathcal{I}_t(p_{x2;t} + e_{x2;t}) \\ \mathcal{I}_t\, \tilde{p}_{y2;t} & = \mathcal{I}_t(p_{y2;t} + e_{y2;t}) \\ \mathcal{I}_t\, \tilde{\varphi}_t & = \mathcal{I}_t(\xi(p_{x1;t}, p_{y1;t}, p_{x2;t}, p_{y2;t}) + e_{\varphi;t}) \\ \tilde{v}_t & = v_t + e_{v;t} \\ \tilde{\omega}_t & = \omega + e_{\omega;t} \end{array} \qquad (20)$$

where $\mathcal{I}_t$ is a measurement indicator: $\mathcal{I}_t = 1$ if the GPS data are available at time instant t and $\mathcal{I}_t = 0$ otherwise; i.e., the entries with nonzero $\mathcal{I}_t$ correspond to available outputs $\mathbf{y}_k$, $k \in k^*$ in (9);
$\xi(.)$ is a nonlinear function that relates $\tilde{\varphi}_t$ with $(p_{x1;t}, p_{y1;t})$ and $(p_{x2;t}, p_{y2;t})$; it uses $\mathrm{arctg}\frac{p_{x1;t}-p_{x2;t}}{p_{y1;t}-p_{y2;t}}$, $\mathrm{sign}(p_{x1;t} - p_{x2;t})$, and $\mathrm{sign}(p_{y1;t} - p_{y2;t})$.

Note that the used indicator $\mathcal{I}_t$ simplifies the notation and unifies the utilization of time indexes.

This model can be estimated using the technique described in Section 3.3.

4.2 Setting of starting point for NLP

To successfully run NLP, a starting point $\hat{\mathbf{X}}_0$ of the optimization has to be set appropriately. In experiments, we set the $\hat{\mathbf{X}}_0$ in the following way. First, we estimate the approximate vehicle position with the undermentioned simplified linear SU model (21) and (22) and obtain $\hat{\mathbf{X}}_{\mathrm{LIN}}$ using the technique that is based on linear programming, for details

[1]MATLAB is a technical computing language and interactive environment for algorithm development, numeric computation, data visualization and analysis, see www.mathworks.com.

see [8]. Then we set $\hat{\mathbf{X}}_0 = \hat{\mathbf{X}}_{\mathrm{LIN}}$ and run NLP according to Sec. 3.3.

The simplified linearized SU model has the following state equation

$$\begin{array}{llll} p_{x1;t} = & p_{x1;t-1} & +hv_{x1;t-1} & +w_{x1;t} \\ p_{y1;t} = & p_{y1;t-1} & +hv_{y1;t-1} & +w_{y1;t} \\ p_{x2;t} = & p_{x2;t-1} & +hv_{x2;t-1} & +w_{x2;t} \\ p_{y2;t} = & p_{y2;t-1} & +hv_{y2;t-1} & +w_{y2;t} \end{array} \tag{21}$$

where

$v_{x1;t} = v_{x2;t} = v_t \sin\hat{\varphi}_t$

$v_{y1;t} = v_{y2;t} = v_t \cos\hat{\varphi}_t,\ t \in t^*$

with

$$\hat{\varphi}_t = \begin{cases} \xi(p_{x1;t}, p_{y1;t}, p_{x2;t}, p_{y2;t}), & t = k \in k^* \\ \hat{\varphi}_{t-1} - h\omega_{t-1}, & t \neq k \in k^* \end{cases}$$

Note that $v_{i;t}$, $i \in \{x1, y1, x2, y2\}$ and ω_t are considered here as model inputs. The function $\xi(.)$ is defined in (20).

The output equation of the simplified linearized SU model is as follows

$$\begin{array}{ll} \mathcal{I}_t\ \tilde{p}_{x1;t} & = \mathcal{I}_t(p_{x1;t} + e_{x1;t}) \\ \mathcal{I}_t\ \tilde{p}_{y1;t} & = \mathcal{I}_t(p_{y1;t} + e_{y1;t}) \\ \mathcal{I}_t\ \tilde{p}_{x2;t} & = \mathcal{I}_t(p_{x2;t} + e_{x2;t}) \\ \mathcal{I}_t\ \tilde{p}_{y2;t} & = \mathcal{I}_t(p_{y2;t} + e_{y2;t}) \end{array} \tag{22}$$

4.3 Additional restrictions for state estimates

The knowledge of distance d between measuring spots $(p_{x1;t}, p_{y1;t})$ and $(p_{x2;t}, p_{y2;t})$ can be included into the estimation algorithm by imposing of additional restriction in the following form

$$\begin{array}{lll} & (p_{x1;t} - p_{x2;t})^2 + (p_{y1;t} - p_{y2;t})^2 - d^2 - \alpha & \leq 0 \\ - & (p_{x1;t} - p_{x2;t})^2 - (p_{y1;t} - p_{y2;t})^2 + d^2 - \alpha & \leq 0 \end{array} \tag{23}$$

where α is user supplied tolerance. We choose α in accordance with GPS measurement errors that are in centimeters.

5 Experiments

5.1 Data description

For experiments, we use real GPS and CAN data according to Table 1.

The GPS data are sampled with frequency 10 Hz. Their precision is given by “circular error probable” (CEP) that corresponds to the radius of a circle inside which the true position is lying with 50% probability. Here, CEP is in centimeters.

The CAN data are provided with the frequency 50 Hz but we use only part of these data in accordance with GPS data frequency.

During the experiments, we have a complete GPS data set at disposal. We simulate data outages by an artificial omission of some GPS data items. In each experiment, only one continuous outage is considered.

5.2 Evaluation of experiments

The estimation of the vehicle position is performed using the proposed algorithm described in Section 3.3. The resulting estimates are compared with the actual values – we check whether the estimated values are within desired tolerance area that is 0.5 m. To evaluate the quality of the estimates, the absolute error of estimates Δ_{z_t}, $t \in t^*$, is used. It is defined as the difference between the measured quantity $\tilde{z}_t$ and its estimated value $\hat{z}_t$, $z \in \{p_{x1}, p_{y1}, p_{x2}, p_{y2}, \varphi\}$, i.e.,

$$\Delta_{z_t} \equiv |\tilde{z}_t - \hat{z}_t|,\ t \in t^*. \tag{24}$$

The maximum entry of the sequence $\{\Delta_{z_t}\}_{t\in t^*}$ is denoted by $\max(\Delta_z)$. The mean of this sequence is denoted by $\mathrm{mean}(\Delta_z)$.

5.3 Results

The estimation of moving vehicle position during simulated GPS signal outages was performed. Figure 1 shows the whole vehicle trajectory where the triangle denotes a start of the vehicle movement. Note that the origin of coordinates lies past the considered area. We run the set of experiments with the same lengths of simulated data outages that were placed on different parts of the total vehicle trajectory. The length of the outages is $\delta = 5\,\mathrm{s}$, i.e. 50 measurements are missing. During the set of experiments, a particular estimation run on the part of total trajectory that contains the data outage and some measurements before and after outage.

Figures 2 – 4 are illustrative. They show the courses of state estimation errors for one selected data outage. On Figure 2 is the course of $\Delta_{p_{x1;t}}$, $\Delta_{p_{y1;t}}$, $t \in t^*$, on Figure 3 is the course of $\Delta_{p_{x2;t}}$, $\Delta_{p_{y2;t}}$, $t \in t^*$. Figure 4 shows the course of Δ_{φ_t}, $t \in t^*$.

Table 2 summarizes the results of the set of experiments for the estimation of the position of the spot (p_{x1}, p_{y1}). It contains shapes of the courses of Δ_{z_t} with demarcated tolerance area 0.5 m, values of $\max(\Delta_z)$ and $\mathrm{mean}(\Delta_z)$, $z \in \{p_{x1},\ p_{y1}\}$. Each row corresponds to one experiment. A shape of the relevant part of total trajectory is added. Note that estimation results for (p_{x2}, p_{y2}) are very similar.

5.4 Discussion

In most of the experiments, a maximum of the absolute estimation error of vehicle position was less than $1\mathrm{m}$, mean values of the absolute estimation error within a particular

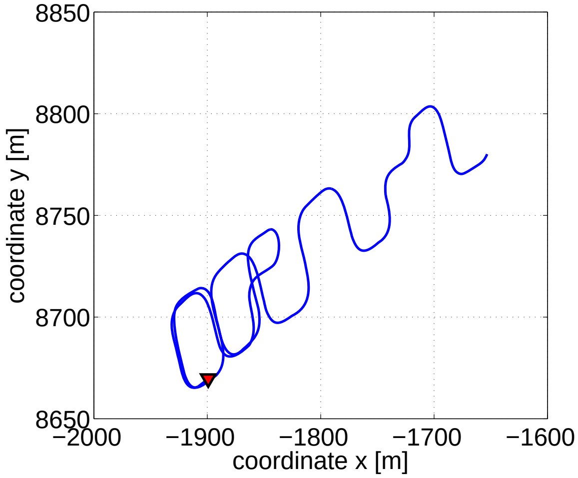

Figure 1. The whole trajectory of a moving vehicle in cartesian coordinates where the triangle denotes a start of the vehicle movement

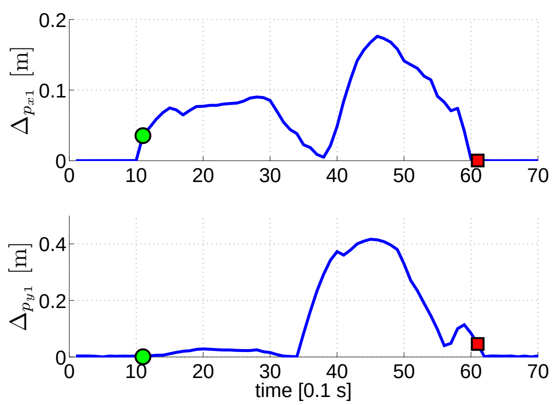

Figure 2. Courses of absolute estimation errors $\Delta_{p_{x1;t}}$ and $\Delta_{p_{y1;t}}$ for a single experiment with marked beginning (bullet) and end (square) of the data outage.

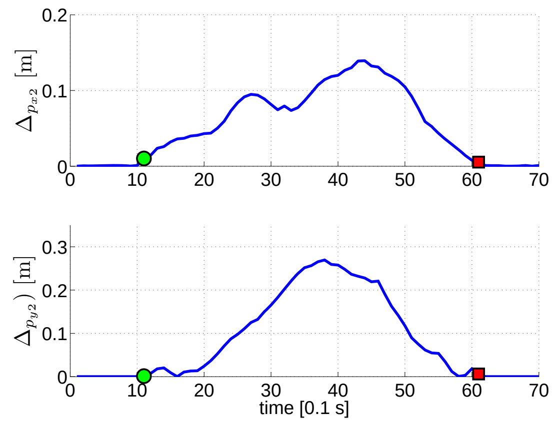

Figure 3. Courses of absolute estimation errors $\Delta_{p_{x2;t}}$ and $\Delta_{p_{y2;t}}$ for a single experiment with marked beginning (bullet) and end (square) of the data outage.

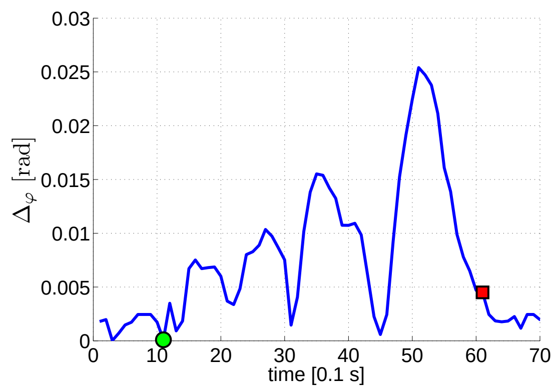

Figure 4. Course of absolute estimation errors Δ_{φ_t} for a single experiment with marked beginning (bullet) and end (square) of the data outage.

experiment are from ca $5\mathrm{cm}$ up to $30\mathrm{cm}$. A maximal deviation 0.5 m from true position coordinates is required. We observe that the quality of estimation results depends strongly on the trajectory shape. The less accurate results occur mostly when an outage is placed on the sharp turns of the trajectory. The used experimental data were obtained during a training drive that took place outside of road. A common drive is usually more quiet without such a sharp turns.

Further, the estimation quality depends on the amount of available measurements before (NB) and after (NA) data outage. In our experiments, $NA = NB = 10$ was appropriate for 50 missing measurements.

The necessity of properly chosen starting point $\mathbf{X}_0$ for NLP was discussed in Section 4.2. We set $\mathbf{X}_0$ using state estimate of simplified linearized SU model (21) and (22).

Proposed model of moving vehicle uses following CAN data during outages - velocity v and yaw rate ω. It does not use a knowledge of $3D$ acceleration that is also at disposal. The reason is that these measurements are very unprecise because of temperature-dependence and influence of gravity. Further, by using v and ω we avoid the transformation between local and global coordinate systems.

In proposed model, all involved quantities are considered as states/outputs. The model has no input. In such a way, we prevent the unprecise data to deteriorate the estimation results. We define inputs only in a linearized model that serves for setting of the starting point for NLP.

The proposed algorithm is an alternative to the KF based algorithms - it is simple to perform and it need no demanding initial setting. This feature is very important for practitioners. With proposed model, they avoid a complicated theoretical setting that is necessary for successful run of KF based algorithms.

Introduced model uses readily available data. CAN is commonly placed in present-day cars.

Note that idea of the estimation of the non-linear state space model with bounded uncertainty and missing data was introduced in [9]. Here, we construct alternative vehicle model that better utilizes available measurements by considering only outputs without inputs as discussed above. Here, we generalize this model by assuming non-linearity both in the state and output model equations. Further, the moving vehicle model is constructed in an alternative way that prevent an unprecise data to deteriorate estimation results. To properly set the initial point for NLP, a simplified linear model is constructed.

6 Conclusion

This paper presents an algorithm for the estimation of a moving vehicle position based on nonlinear programming. The proposed algorithm is advisable in the case of short GPS data outages when the precise reconstruction of moving vehicle position is required.

Future research aims a further improvement of the state estimates by imposing additional restrictions on state values. For this purpose, more data from vehicle sensors as a wheel angle or a distance moved will be included into the estimation algorithm. As well utilization of a fact that CAN data are provided with higher frequency than GPS data will be taken into account.

Acknowledgement

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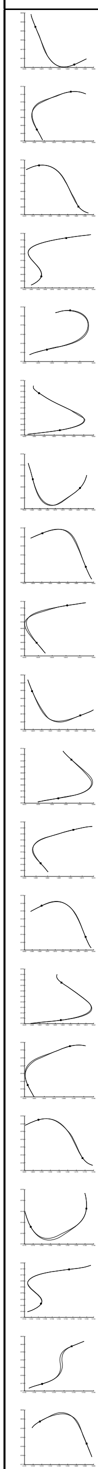
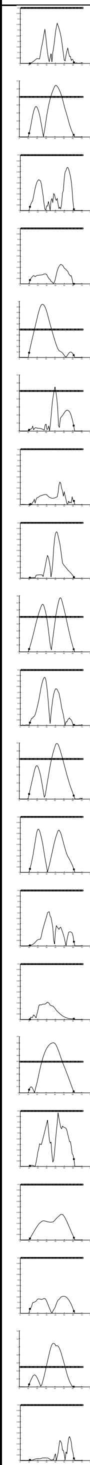
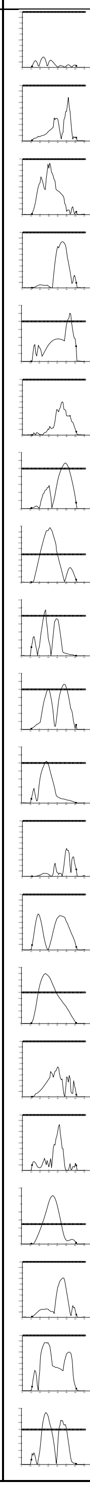
Table 2. Course of the absolute estimation errors Δ_{z_t} (24) with demarcated tolerance area 0.5 m and values of $\max(\Delta_z)$ and $\mathrm{mean}(\Delta_z)$, $z \in \{p_{x1}, p_{y1}\}$ with respective part of trajectory ("TR")

"TR"	$\Delta_{p_{x1;t}}$ course	$\Delta_{p_{x1;t}}$ max [m] (mean [m])	$\Delta_{p_{y1;t}}$ course	$\Delta_{p_{y1;t}}$ max [m] (mean [m])
		0.36 (0.08)		0.09 (0.02)
		0.64 (0.23)		0.39 (0.07)
		0.39 (0.11)		0.46 (0.13)
		0.18 (0.05)		0.41 (0.10)
		0.94 (0.28)		0.60 (0.18)
		0.55 (0.10)		0.30 (0.07)
		0.20 (0.04)		0.56 (0.17)
		0.42 (0.08)		0.97 (0.30)
		0.77 (0.29)		0.57 (0.13)
		0.43 (0.12)		0.56 (0.19)
		0.69 (0.24)		0.52 (0.12)
		0.39 (0.14)		0.25 (0.04)
		0.31 (0.08)		0.32 (0.13)
		0.16 (0.05)		0.80 (0.29)
		0.80 (0.30)		0.27 (0.08)
		0.48 (0.15)		0.41 (0.07)
		0.23 (0.10)		1.20 (0.34)
		0.15 (0.07)		0.35 (0.08)
		1.08 (0.34)		0.43 (0.17)
		0.21 (0.03)		0.73 (0.24)