Decentralization of Fully Probabilistic Control Design using Variational Bayes Approximation

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We are concerned with design of decentralization control strategy for stochastic systems with global performance measure. Such a strategy is required e.g. for creation of a distributed control systems for traffic light signalization [1]. The technique of Fully Probabilistic Control Design (FPD) [2, 3] is capable of solving the task for centralized control strategy via Kullback-Leibler divergence (KLD) minimization. In this contribution, we propose a way for decentralization of FPD using the Variational Bayes (VB) approximation. This approximation is a well known technique of Bayesian estimation [4, 5] and it is based on approximation of the true posterior density by a product of conditionally independent densities, VB-marginals. The fundamental principle is again minimization of the KLD between a product of conditionally independent densities and the true posterior. Typically, the solution is found in the form of a set of implicit equations. Solution of the set can be found by an iterative algorithm in which evaluation of shaping parameters of each VB-marginal requires evaluation of moments from the remaining densities.

We design imperfect decision makers for control of the distributed system by imposing a conditional independence restriction on the control strategy. The resulting optimum is found to be equivalent to application of the VB approximation on the result of the FPD. The resulting strategy is thus computed iteratively. Each decision-maker computes its own strategy using moments from its neighbors that are obtained by means of communication. The original algorithm operates on the decision horizon that is common to all decision makers. At each time step of the horizon, the decision makers iterate the resulting strategy using VB approximation. When a consensus is reached they proceed backwards as is typical for FPD. The communication load under this scheme is however rather high. Therefore, we propose to use an alternative way of solution of the implied set of implicit equations. Specifically, the decision makers design their own strategy for the whole decision horizon with respect to moments obtained from the other decision makers. The VB iterations are run on moments of the joint densities on the whole horizon.

We apply this methodology to distributed control of a linear Gaussian system with two inputs and three outputs with quadratic loss function. Each input is controlled by one decision maker. We study convergence of the terminal loss of the decentralized algorithms to the terminal loss obtained by the centralized FPD solution. The first algorithm is found to be faster in convergence, however, the number of sent messages is much higher then that of the second algorithm.

References

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