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Comovement of Central European stock markets using wavelet coherence: Evidence from high-frequency data

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Abstract:
In this paper, we contribute to the literature on international stock market comovement. The novelty of our approach lies in usage of wavelet tools to high-frequency financial market data, which allows us to understand the relationship between stock market returns in a different way. Major part of economic time series analysis is done in time or frequency domain separately. Wavelet analysis can combine these two fundamental approaches, so we can work in time-frequency domain. Using wavelet power spectra and wavelet coherence, we have uncovered interesting dynamics of cross-correlations between Central European and Western European stock markets using high-frequency data. Our findings provide possibility of a new approach to financial risk modeling.

Keywords: comovement, stock market, wavelet analysis, wavelet coherence

JEL: C22, C40, E32, F30, G15

1 Introduction

During last decades and mainly during last several years, the interconnection between all stock markets has grown significantly. As the stock markets of Latin America, Central and Eastern Europe and South and Southeast Asia are becoming more open to foreign investors, the traded volumes have increased considerably. Such increase in liquidity and availability of stocks of transition and emerging economies enlarges the possibilities of an international portfolio diversification. The recent events on financial markets between years 2007 and 2009 raised some serious questions about a potential of the diversification during critical events.

In globalized financial markets with growing trading volumes and liquidity, the integration and comovements are becoming stronger in time so that the use of diversification has been becoming more limited. Therefore, examination and research on different types of comovements and correlations in time is of a great importance. In addition to the time dimension of the market dynamics, there are different types of investors who influence such dynamics. Starting with noise traders with an investment horizon of several minutes or hours, the spectrum of investors ranges through technicians with the horizon of several days to fundamentalists with the horizon of several weeks or months to pension funds with the investment horizon of several years. Thus, apart from the time domain, there is a frequency domain, which represents various investment horizons.

As both domains are equally important and valid for a deeper research of the dynamics of the financial markets, it is needed to contain them in the analysis. However, majority of used models focus on one of the domains solely. In the time domain, Dalkir (2009) finds that comovements increase during volatile periods but remain strong afterwards, which is explained by Bayesian learning theory. Forbes and Rigobon (1999) apply a time-dependent adjusted correlation analysis and argue that there are strong linkages between stock markets which become stronger during volatile periods. Khan and Park (2009) use cross-country time-varying correlation coefficients and apply it on Asian crisis of 1997 and show that even after controlling for economic fundamentals, strong herding contagion remains. Similarly, Leong and Felmingham (2003) use time-varying correlations on different East-Asian stock indices and finds increasing integration in time with the strongest correlations during the crisis of 1997. Morana and Beltratti (2008) examine four developed markets (the USA, the UK, Germany and Japan) and uncover that the comovements in the markets are increasing in time for prices, returns, volatility and correlations. Savva et al. (2005) research on dynamic conditional correlations between the US, English, German and French stock markets and show that correlations increased after Euro adoption whereas the increase was the most profound for Germany and France comovements.

The literature is less numerous for the frequency domain. One of a few examples is Bonfiglioli and Favero (2005) who examine relationship between the US and German stock markets and argue that there is only a short-run and no long-run interdependence. Moreover, the authors show that contagion exists only between abnormal fluctuations and specifically in a non-linear and a regime-switching manner.

In our research, we combine both time and frequency domain and we apply cross-wavelet analysis on high-frequency (5 minute) data of Czech (PX), Hungarian (BUX) and Polish (WIG) stock indices with a benchmark of German stock index (DAX). By analyzing wavelet power spectra and cross-wavelet transform, which were recently used in financial analysis by Aguiar-Conraria et al. (2008); Rua and Nunes (2009); Dalkir (2004), we show how correlations are changing in time and across frequencies, continuously. The novelty of our approach lies in the usage of these wavelet tools to high-frequency financial market data, which allows us to understand the relationship between Central European stock market returns in a different way.

The paper is structured as follows. We present a literature review of wavelets application on financial data and of literature dealing with comovements and interdependence. Further on, we present a brief introduction to the methodology of the wavelet analysis. We explain the main tools of our analysis - wavelet power spectrum and cross-wavelet transform. After the methodology is set, we employ high-frequency, 5 minute data of Czech (PX), Hungarian (BUX), Polish (WIG) and German (DAX) stock indices and study their interdependence in both time and frequency domain. To complete the analysis, we also include British (FTSE 100) index and the U.S. (S&P 500) index and study dependence of all pairs on daily data.

2 Literature review

In this section, we review the recent literature concerning the topic. The section is divided into two parts. In the first part, we discuss the current financial literature which uses a tool of the wavelet analysis. In the second part, a review of broad recent literature on comovements and interdependence on financial markets is presented.

2.1 Wavelets literature

Continuous wavelet analysis and a use of cross-wavelet transforms is not a common tool for the analysis of economic and financial data. Only recently, few applications emerged.

Aguiar-Conraria et al. (2008) research on the dependencies between monthly interest rates and industrial production, inflation and monetary aggregates M1 and M2 of the US economy between 1920s and 2000s. The authors show that the relationships between economic variables change in time and are not homogeneous across frequencies.

Rua and Nunes (2009) examine the comovements between the stock markets of the USA, the UK, Germany and Japan between 1973 and 2007. Again, the authors show that the interdependence changes in time and varies across frequencies. The comovements between the US and the European markets are the strongest whereas Japan is independent at almost all times and frequencies. The interdependence of Germany with the USA and the UK increases in time and from 2000 onwards, the wavelet coherence is significant for all frequencies for US-Germany and UK-Germany pairs. The same results are found for separate sectors of the markets.

Dalkir (2004) shows that there is a strong relationship between the US money supply and income where the former leads the dependency. Such result is in favor of monetary policy as it shows its effectiveness. However, such effectiveness is argued to be the strongest in 1980s and early 1990s. Again, the dependence changes in time and across frequencies. Ramsey and Lampart (1998) examine a set of macroeconomic variables between 1960 and 1994. The authors show that there is a proportional relationship between consumption and income whereas monetary aggregates M1 and M2 have different wavelet power (variance) but move together throughout the whole examined period. Interestingly, M1 and income are highly coherent (correlated) while the latter leads the relationship at low frequencies but the former leads it at business cycle scales.

2.2 Comovements, interdependence and correlation

The largest part of the literature examines the interdependence between the USA and countries of the Western Europe. Baele (2005) and Baele and Inghelbrecht (2010) apply switching models to show that the intensity of comovements and spillovers increased during 1980s and 1990s with no evidence of significant contagion other than small effect during the 1987 crisis. Bonfiglioli and Favero (2005) research on the US-Germany interrelations with a use of various models (cointegration tests, error-correction model and instrumental variables regression) and argue that long-run international diversification is efficient whereas short-run diversification needs to take non-linearities and regime switching into consideration. Connolly and Wang (2003) examine intraday dynamics of the US, the UK and Japanese stock markets and the effect of news announcements with an interesting conclusion of a possible use of private information across the markets. Connolly et al. (2007) research on comovements between the US, the UK and German stock and bond markets and show that during high (low) implied volatility periods, the comovements are stronger (weaker) whereas stock-bond comovements tend to be positive (negative) following low (high) implied volatility days. The benefits of international diversification are emphasized. Morana and Beltratti (2008) examine stock markets of the USA, the UK, Germany and Japan between 1973 and 2004 where the comovements are increasing in time for all markets while the US-Europe relations are stronger. Savva et al. (2005) use multivariate EGARCH model with dynamic conditional correlations on the stock markets of the USA, the UK, Germany and France and show increasing correlations whereas the increase is most significant after the Euro adoption.

The second most widely discussed region is the Central and Eastern Europe. Egert and Kocenda (2007) examined high-frequency stock market comovements of Czech, Hungarian, Polish, German, French and English stock markets between 2003 and 2005 and find no significant cointegration with signs of short-term spillovers in both returns and volatility. Gilmore et al. (2008) study the Central European (CE) stock markets and find strong cointegration but argue that signs of convergence to the Western Europe are lacking after the EU accession. Hanousek and Kocenda (2009) analyze high-frequency data of the CE stock markets and show that these are strongly influenced by developed economies. Jokipii and Lucey (2007) study unadjusted conditional correlations in banking sector of CE stocks and uncover strong interdependence; after adjusting for high volatility, only influence from the Czech to the Hungarian stock market remains significant.

The comovements of Asian stock markets are analyzed in Leong and Felmingham (2003). The authors study the markets of Singapore, South Korea, Japan, Taiwan and Hong Kong between 1990 and 2000 and argue that the integration increases in time and correlations strengthened significantly during the 1997 Asian crisis. Latin America is studied in Edwards and Susmel (2001) where the authors examine weekly returns and volatility with univariate and bivariate switching volatility models and find strong evidence of volatility comovements, which are, however, short-lived.

When we try to summarize findings in the literature, there is a strong agreement that countries are becoming more interdependent.

In this paper, we contribute to the discussion on the comovement and use a model-free approach which is more straightforward to interpret. Introduction of the wavelet approach enables to study the interdependencies of financial time series in time as well as in frequency domain and thus allows to understand the structure of the possible dependencies more deeply. We introduce the methodology in the following section.

3 Wavelet analysis

The application of spectral techniques to economic and financial data has focused on the finding of complex but stable frequency components. The spectral analysis has uncovered little evidence of stable frequencies in financial and economic data. The main problem is with stationarity of the examined time series, more specifically, the stability of cycles. Popular method for investigating spectra of time series is the Fourier transform. The key problem with the Fourier transform applied to financial data is that the estimated spectrum of time series is rather global than localized. As a consequence, the time information in examined time series is lost. Therefore, an analysis of structural changes, shocks or transient behavior is impossible. Conversely, the wavelet transform offers localized time–frequency decomposition. As a result, wavelets have significant advantages over basic Fourier analysis when the object under study is locally non-stationary and inhomogeneous, see Gençay et al. (2002); Percival and Walden (2000).

Wavelet analysis can be divided to two main parts: the discrete wavelet analysis and the continuous wavelet analysis. For a long time, the discrete wavelet analysis was dominating in economic applications (Gençay et al., 2002; Ramsay, 2002; Gallegati and Gallegati, 2007). However, in last couple of years, the continuous wavelet analysis is also becoming popular in economic applications. Important part of the continuous wavelet analysis is the ability to study interactions or comovement between two time series in time-frequency domain using the cross-wavelet tools (Aguiar-Conraria et al., 2008; Rua and Nunes, 2009). In our paper, we use the continuous wavelet analysis tools: wavelet power spectrum for measuring local variance of time series at various scales (frequencies) and wavelet coherence measuring the local correlation of two time series in time-frequency domain. In wavelet terminology it is common to use scale instead of frequency. It should be noted, that the scale has an inverse relation to frequency. In the rest of the paper, high scales correspond to low frequencies, and low scales correspond to high frequencies.

3.1 Wavelet

A wavelet is a real-valued or a complex-valued function $\psi(.)$ defined over the real axis. Furthermore, we assume the wavelet to be square integrable function, $\psi \in L^2(\mathbb{R})$[1]. There are several conditions that the wavelet must fulfill. Admissibility condition that allows for

[1]A function $x(t)$ is called squared integrable if $\int_{-\infty}^{\infty} x(t)^2 dt < \infty$.

reconstruction of a time series from its continuous wavelet transform:

$$C_\psi = \int_0^\infty \frac{\mid \Psi(f) \mid^2}{f} df < \infty, \tag{1}$$

where $\Psi(f)$ is the Fourier transform of the wavelet $\psi(.)$ defined as $\Psi(f) = \int_{-\infty}^{\infty} \psi(t) e^{-i2\pi f t} dt$. Since the admissibility condition implies that the wavelet has no zero frequency component, the wavelet must have zero mean:

$$\int_{-\infty}^{\infty} \psi(t) dt = 0 \tag{2}$$

This condition assures that any movement above zero must be cancelled out by movement to negative values so that the wavelet looks like a wave. The wavelet is usually normalized to have unit energy, i.e. the square of $\psi(.)$ integrates to unity

$$\int_{-\infty}^{\infty} \psi^2(t) dt = 1 \tag{3}$$

Equation 3 implies that the wavelet makes some excursion away from zero.

Let us define a wavelet as:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi \left(\frac{t-u}{s} \right) \tag{4}$$

where $1/\sqrt{s}$ denotes a normalization factor ensuring the unit variance of the wavelet, $\|\psi_{u,s}\|^2 = 1$. The location parameter u gives an exact position of the wavelet. For example the translation of the wavelet $\psi(u-3, s)$ means shifting of the wavelet three units to the right. Scale or dilatation parameter s defines how the wavelet is stretched or dilated. For example, higher scale means more stretched wavelet that is suitable for detection of lower frequencies.

There are many different types of wavelets available. Every wavelet type has specific characteristics and is useful for different purposes. For an interesting discussion about wavelet types, see Percival and Walden (2000); Adisson (2002). In our analysis of financial markets comovement, we use the Morlet wavelet[2] defined as:

$$\psi^M(t) = \frac{1}{\pi^{1/4}} e^{i\omega_0 t} e^{-t^2/2} \tag{5}$$

where ω_0 denotes a central frequency of the wavelet[3]. In our analysis, we use $\omega_0 = 6$, which is the setting often used in economic and financial applications, for example Aguiar-Conraria et al. (2008); Rua and Nunes (2009). The Morlet wavelet is a complex or analytic wavelet[4] within a Gaussian envelope with a good time-frequency localization. The wavelet has both real and imaginary part which allows for studying both an amplitude and a phase.

The morlet wavelet is centered at the point $(0, \omega_0/2\pi)$ in time-frequency domain, i.e. for $\omega_0 = 6$, we obtain the frequency center as $\mu_f = 6/2\pi \approx 1$ (Aguiar-Conraria et al., 2008). For the frequency – scale relationship, we can write:

$$f = \frac{\mu_f}{s} \approx \frac{1}{s} \tag{6}$$

[2]We present the simple form that satisfies admissibility condition for $\omega_0 > 5$.
[3]In our analysis we consider only mother wavelets.
[4]It has zero Fourier transforms for negative frequencies.

Hence, for the central frequency $\omega_0 = 6$, the wavelet scale s and the corresponding Fourier period are almost identical (Maraun and Kurths, 2004).

3.2 The continuous wavelet transform

The continuous wavelet transform $W_x(u,s)$ is obtained by projecting the specific wavelet $\psi(.)$ onto the examined time series $x(t) \in L^2(\mathbb{R})$. The continuous wavelet transform is defined as:

$$W_x(u,s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \overline{\psi\left(\frac{t-u}{s}\right)} dt. \tag{7}$$

Important aspect of the wavelet transforms is the ability to decompose and then subsequently perfectly reconstruct the function $x(t) \in L^2(\mathbb{R})$:

$$x(t) = \frac{1}{C_\psi} \int_0^{\infty} \left[\int_{-\infty}^{\infty} W_x(u,s) \psi_{u,s}(t) du \right] \frac{ds}{s^2}, \qquad s > 0 \tag{8}$$

A key feature of the wavelet transform is the energy preservation of the examined time series. We use this property for the wavelet power spectrum analysis, which defines the variance:

$$\|x\|^2 = \frac{1}{C_\psi} \int_0^{\infty} \left[\int_{-\infty}^{\infty} |W_x(u,s)|^2 du \right] \frac{ds}{s^2}. \tag{9}$$

3.3 Wavelet power spectrum

The wavelet power spectrum defined as $|W_x(u,s)|^2$ measures the local variance of the time series $x(t)$ at various scales s. Hence we get variance decomposition with a good time localization of the time series under investigation (Torrence and Compo, 1998).

We test the statistical significance of the peaks in the power spectrum against a null hypothesis that the examined time series is generated by a white noise process. Statistically significant areas at the 5% significance level are bordered by a black bold line. For a detailed discussion about significance testing and derivation of background spectra see, Grinsted et al. (2004); Torrence and Compo (1998).

Since we analyze finite time series with finite wavelets, the beginning and the end of the wavelet power spectrum and the wavelet coherence will be affected by errors caused mainly by the discontinuities at the edges. In our paper, we pad the time series with sufficient number of zeroes, which increases the length of the analyzed time series N up to the next higher power of two. The area where the errors caused by discontinuities of the continuous wavelet transform cannot be ignored, i.e. the edge effects become important, is called the cone of influence[5] (Grinsted et al., 2004). The cone of influence lies under the cone which is bordered by a thin black line.

3.4 Wavelet coherence

Now we move from the univariate wavelet analysis to cross-wavelet approach and we introduce the main tool for our analysis – wavelet coherence. First, we define the cross wavelet

[5] The cone of influence is highly dependent on the type of wavelet used Torrence and Compo (1998).

transform of two time series $x(t)$ and $y(t)$ with the continuous wavelet transforms $W_x(u,s)$ and $W_y(u,s)$, (Torrence and Compo, 1998; Grinsted et al., 2004):

$$W_{xy}(u,s) = W_x(u,s)W_y^*(u,s), \tag{10}$$

where u is a position index and s denotes the scale, symbol * denotes a complex conjugate. Furthermore, we define cross wavelet power as $|W_{xy}(u,s)|$ (Hudgins et al., 1993; Torrence and Compo, 1998). The cross wavelet power uncovers areas in time–scale space where the time series show high common power, i.e. it represents the local covariance between the time series at each scale. In the analysis of financial time series, we are also interested in areas, or regions, where the two time series in time–scale space comove, but does not necessarily have high power. Useful wavelet tool that can uncover these comovements is the wavelet coherence.

The wavelet coherence can be perceived as a measure of local correlation of the two time series both in time and scale. This procedure is analogous to coherence in Fourier analysis. Following approach of Torrence and Webster (1999), we define the wavelet coherence as the squared absolute value of the smoothed cross wavelet spectra normalized by the product of the smoothed individual wavelet power spectra of each series i.e.

$$R^2(u,s) = \frac{|S(s^{-1}W_{xy}(u,s))|^2}{S(s^{-1}|W_x(u,s)|^2)S(s^{-1}|W_y(u,s)|^2)}, \tag{11}$$

where S is a smoothing operator. Without smoothing, the wavelet coherence is equal to one at all scales[6]. For more details about smoothing, see Torrence and Webster (1999); Hudgins et al. (1993); Torrence and Compo (1998).

The squared wavelet coherence coefficient is in the range $0 \leq R^2(u,s) \leq 1$, values close to zero indicate weak correlation while values close to one are an evidence of strong correlation. Thus it provides a useful tool for the analysis of comovement across the stock markets in the next section.

Since theoretical distributions for the wavelet coherence is not known, the 5% statistical significance level is determined using Monte Carlo methods. Similarly to the wavelet power spectrum, a thick contour delimits the areas where the wavelet coherence is significant.

3.5 Phase

We use wavelet coherence phase differences to characterize relationship between the two time series. The phase difference gives details about delays of oscillation (cycles) of the two examined time series. Following Torrence and Webster (1999), we define the wavelet coherence phase difference as

$$\phi_{xy}(u,s) = \tan^{-1}\left(\frac{\Im\{S(s^{-1}W_{xy}(u,s))\}}{\Re\{S(s^{-1}W_{xy}(u,s))\}}\right). \tag{12}$$

Phase is indicated by arrows on the wavelet coherence plots. A zero phase difference means that the examined time series move together at a particular scale s. Arrows pointing to

[6] Smoothing is achieved by convolution in both time and scale. The time convolution is performed with a Gaussian window, the scale convolution is done with a rectangular window Grinsted et al. (2004).

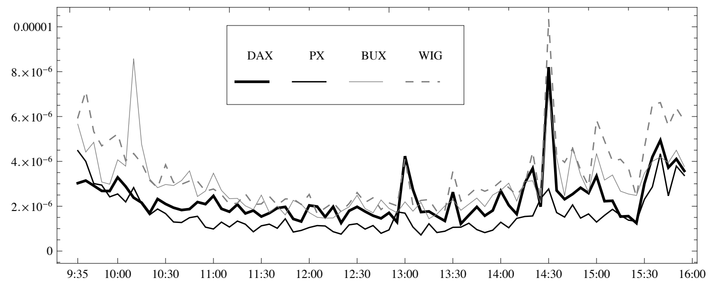

Figure 1: Intraday pattern of average squared returns (volatility).

the right (left) when the time series are in-phase (anti-phase), i.e. positively (negatively) correlated. Arrow pointing up means that the first time series leads the second one by 90°, arrow pointing down indicates that the second time series leads the first one by 90°. Usually, we have a mixture of positions, for example, arrow pointing up and right means that the time series are in phase with the first times series leading the second one.

4 High-frequency data analysis

4.1 Data description

In our analysis, we use 5-minute high-frequency data of Czech (PX), Hungarian (BUX) and Polish (WIG) stock indices with a benchmark of German stock index (DAX). Central European stock markets data were collected over a period of 2 years beginning with January 2, 2008 and ending by November 30, 2009. The data were obtained from TICK data.

When looking at the data, one quickly observes that number of observations for each trading day differs among the indices. This problem arises due to different stock market opening hours. Prague Stock Exchange, as well as Warsaw Stock Exchange, is open from 9:30 to 16:00 Central European Time (CET). Budapest Stock Exchange is open from 9:00 to 16:30 CET. Finally, Frankfurt Stock Exchange is open from 9:00 to 17:30 CET. Thus we need to adjust the dataset by including only the periods of day where the data is available for all analyzed stock indices. We compute logarithmic returns for the period from 9:30 to 16:00 CET for each day separately in order to avoid overnight returns. Finally, we are left with 77 return observations for each stock market for each day of the analyzed period. By discarding major public holidays, the final sample includes 450 trading days.

Another issue, which may bias the results of our analysis, is a known effect of news arrival during the first hours of the trading day as well as higher activity during the end of the trading day (McMillan and Speight, 2002). Variance at the open is found to be more than three times the midday variance and variance at the close is about 1.5 times the midday variance. Figure 1 shows average variances through the trading day for all analyzed stock market series. It confirms the U-shape of volatility during the trading day. It is also interesting to observe quite strong activity of DAX, BUX and WIG at 14:30 CET. This activity may be contributed by the usual U.S. macroeconomic announcements during 14:30 CET. Our analyzed period includes the mortgage crisis thus the stock market volatility

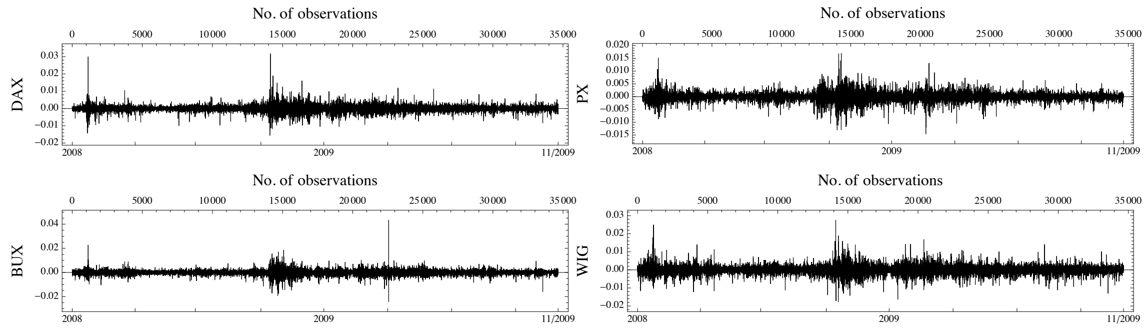

Figure 2: Plots of 5-min logarithmic returns for DAX, PX, BUX and WIG indices.

is generally higher indicating higher risk and also nervousness of investors. This clearly corresponds to the sudden increase in the activity at 14:30 CET for all markets. It is interesting to note that Prague Stock Exchange does not react to the U.S. announcements at 14:30 CET so strongly. In order to prevent the bias from this effect, we tried also to estimate different periods which do not include the U.S. announcements time and do not include the beginning and end of the trading so the U-shape volatility effect is eliminated. The results have not been affected, thus we finally utilize all available data consisting of 34 650 observations for each analyzed index.

Table 1 provides descriptive statistics for our final sample of 5-minute high-frequency returns. Figure 2 shows plots of the data.

4.2 Results

Before discussing our results, let us briefly discuss how the wavelet analysis can be helpful in financial time series modeling. As the stock markets are complex systems of interacting agents with different term objectives, financial time series resulting from this process is a combination of different components operating on different frequencies. Standard time series econometric methods usually consider only frequency or time component separately. Wavelets allow us to study the frequency components of time series without loosing the time information. Thus we are able to uncover the interactions which can hardly be visible from any other modern econometric methods and which would stay hidden otherwise. Moreover, the wavelet analysis approach is model-free which gives very powerful tool in comparison

	DAX	PX	BUX	WIG
Mean	-2.98577×10^{-6}	-1.66586×10^{-5}	-2.63242×10^{-5}	-1.76341×10^{-5}
St.dev	0.00152547	0.0012775	0.00169863	0.0018456
Skewness	0.426297	-0.144049	0.131315	0.208045
Kurtosis	20.3099	16.3973	27.0218	12.824
Min	-0.0156048	-0.0146523	-0.0243584	-0.0177151
Max	0.0317693	0.0170459	0.0434668	0.0276198

Table 1: Descriptive Statistics for 5-min high-frequency data.

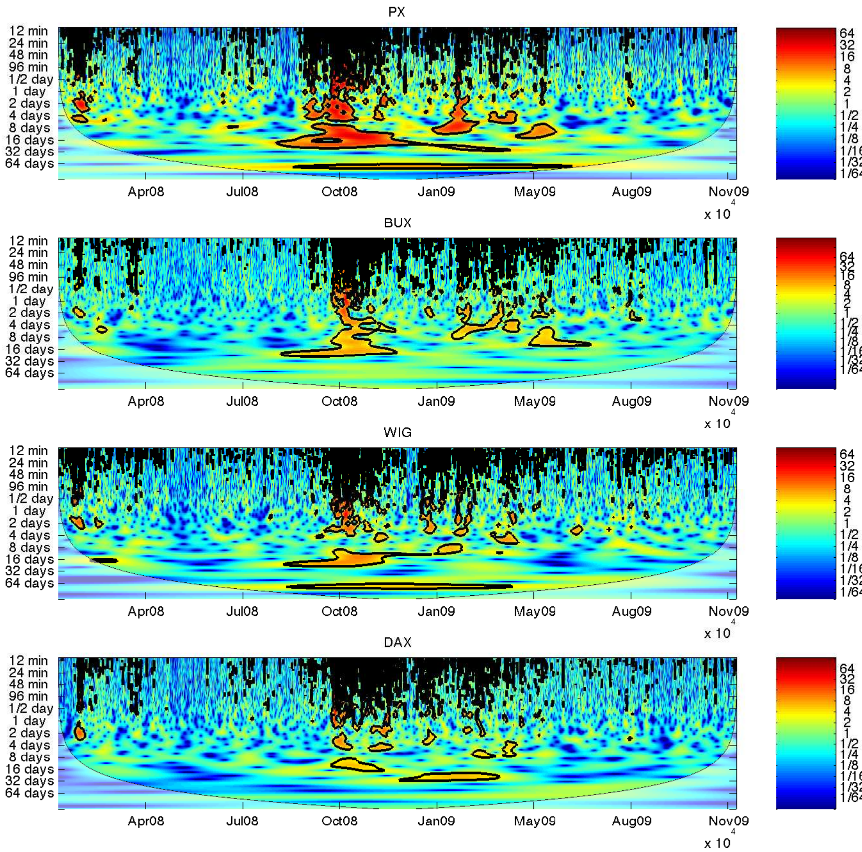

Figure 3: Wavelet power spectra of PX, BUX, WIG and DAX indices on the 5 minutes high-frequency returns. Horizontal axis shows time, while vertical axis shows scale in minutes/days.

with other methods relying on parameters as well as to comparison statistics with other possible model specifications.

In our analysis, we utilize the wavelet transform to analyze the time-scale properties of stock returns. Moreover, the main analysis of comovement of the studied stock markets is the wavelet coherence[7] as it allows to quantify the relation between two time series in the time-frequency domain.

Figure 3 shows the wavelet power spectra of the 5 minutes high-frequency indices. Looking at the time-scale decomposition of the indices, some interesting facts are revealed. We can see the most significant energy during October 2008 in all stock markets nearly through all frequencies. PX index shows highest volatility while DAX shows weaker activity. But it is interesting that using high-frequency data, we are able to observe quite a long period of strong dependences on all frequencies around October 2008. This period of consecutive large drops has been the most devastating to world stock markets and volatility naturally increased. During the largest world market turmoil, stock market reveals very strong patterns on the lower scale frequencies up to several months. Significant increase in variance is also

[7]For estimation, we use the MatLab wavelet coherence package developed by A.Grinsted, J.C.Moore and S.Jevrejeva.

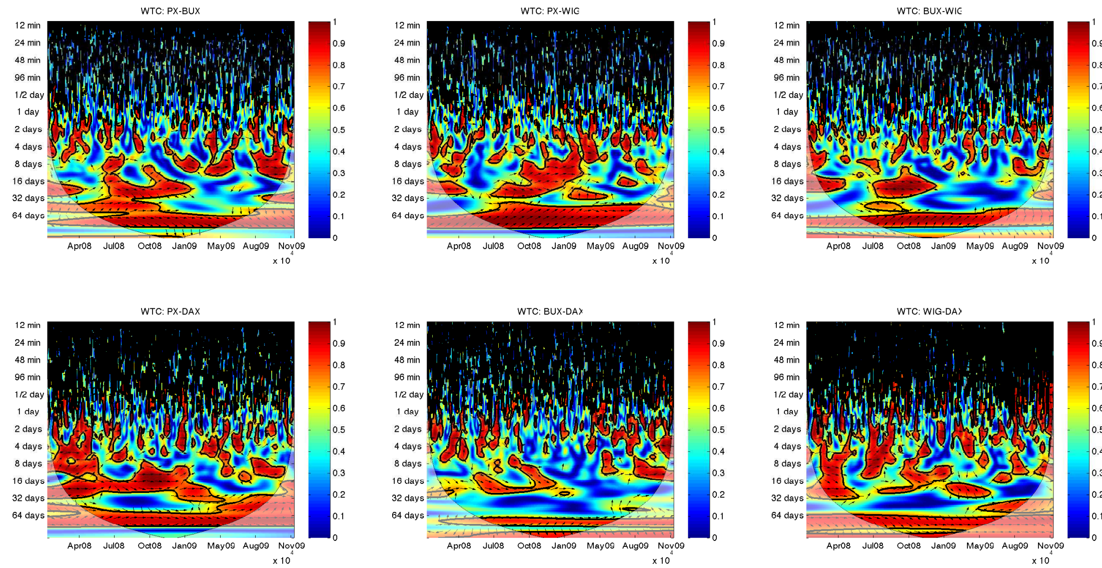

Figure 4: Wavelet coherence of PX, BUX, WIG and DAX indices pairs on the 5 minutes high-frequency returns. Horizontal axis shows time, while vertical axis shows scale in minutes/days. The warmer the color of region, the higher the degree of dependence between the pair.

visible on high-frequency scales up to 5 minutes.

We now concentrate on wavelet coherence which provides a picture on the comovement of the analyzed stock markets. To asses the statistical significance, we use the Monte Carlo simulations. Figure 4 shows the estimated wavelet coherence and the phase difference for all examined pairs of indices. Time is on the horizontal axis while vertical axis refers to scale. The wavelet coherence finds the regions in time–scale space where the two time series co-vary (but do not necessarily have high power). Regions inside the black lines plotted in warmer colors represent regions where significant dependence has been found. The colder the color is the less dependent the series are. Thus the plot clearly identifies both frequency bands and time intervals where the series move together.

From the analysis of the wavelet coherence, we can observe interesting results. First of all, there are quite large significant comovement periods among all tested stock markets through several scales. At the low scales, it is hard to see from the pictures as the black regions consist of many small periods of significant comovement at various scales (5min, 10min, etc.). Each of the pairs also shows strong comovement periods on several daily scales up to two to three weeks as well as periods where pairs comove on the several months scales.

When looking at the comovement of PX, BUX and WIG (Figure 4), we can observe that PX is positively correlated with WIG on lower frequencies up to several months. PX-WIG pair also shows very interesting development of coherence from the second half of year 2008 until the end of the first half of 2009. Local correlations are strongly significant through this time period but they change from the month scale to scale of one week. This dynamics of interdependence visible from the wavelet coherence of high-frequency data is unique and allows us to understand the relationship between the analyzed stock markets in a different way. Moreover, phases represented by arrows reveal that WIG is positively influenced by

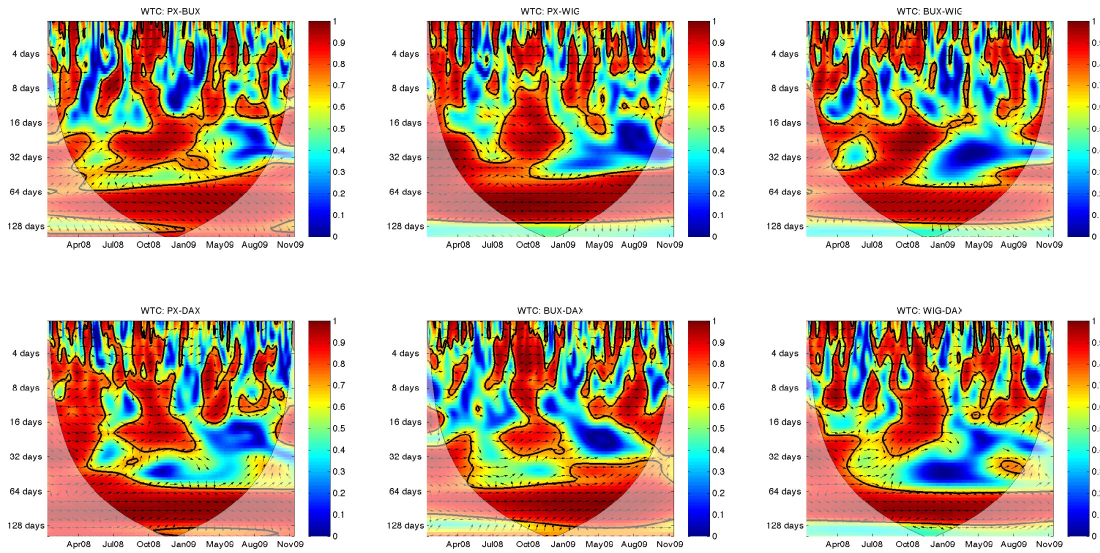

Figure 5: Wavelet coherence of PX, BUX, WIG and DAX indices pairs on the daily returns. Horizontal axis shows time, while vertical axis shows scale in minutes/days. The warmer the color of region, the higher the degree of dependence between the pair.

PX; these markets also have the largest period of comovement through time and scales. PX is also positively correlated with BUX at several large time periods but the phases do not point to any directional influence. As to the dependence of these markets on DAX, pair PX-DAX shows the largest time periods of comovement. WIG is dependent on DAX while BUX again shows the weakest dependence through different time periods and scales.

We also perform the same analysis on the daily data including the same time period. The wavelet power spectra provide the same results. Much more interesting is the look at the cross wavelet coherence plotted in Figure 5. Daily data show large time-scale areas of comovement on lower frequencies around 3-6 months. During the second half of the year 2008, all the indices show strong comovement at the scale of 2-4 weeks, while this dependence disappears in the year 2009. Comovement periods on the daily scales up to one week are also visible for all pairs.

5 World stock markets comovement: Analysis of the daily data

We complete the analysis by adding London FTSE index and New York S&P 500 index. We also extend the time interval to the beginning of the year 2004 so we can study the comovement of all stock markets also during the period of a bubble before the current mortgage crisis. Table 2 provides descriptive statistics for the data and Figure 6 shows their plots. Figure 7 shows the wavelet coherence for all 6 stock markets.

We can observe quite large areas of significant comovement through time and scales for all indices pairs. The most notable is the comovement of DAX and FTSE pair, which shows the comovement almost at all scales through the whole time period. DAX and FTSE

	PX	BUX	WIG	DAX	S&P 500	FTSE
Mean	0.000419128	0.000617861	0.000293809	0.000270246	8.7991813×10^{-6}	0.000136383
St.dev	0.0190119	0.0195505	0.018148	0.0151573	0.014752	0.0137604
Skewness	-1.25761	0.523092	-0.124687	0.0950789	-0.370482	0.0403534
Kurtosis	21.6994	18.0163	7.30919	11.4186	13.251	12.6605
Min	-0.19902	-0.126489	-0.116855	-0.0839631	-0.0946951	-0.0926557
Max	0.123641	0.220164	0.108961	0.107975	0.109572	0.0964165

Table 2: Descriptive Statistics for daily returns.

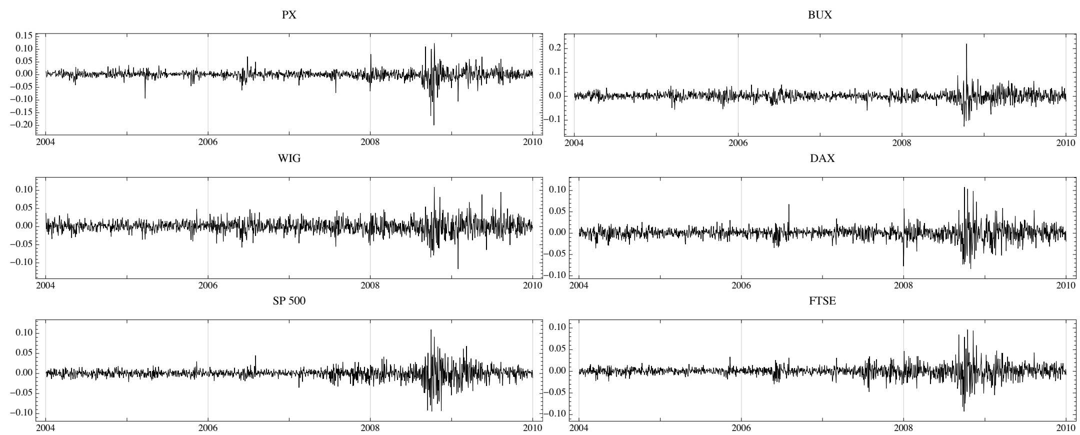

Figure 6: Plots of daily logarithmic returns for PX, BUX, WIG, DAX, S&P 500 and FTSE indices.

are also very positively correlated with S&P 500 on the long-term basis. It is interesting to note that in the scales up to one week, the comovement of the DAX and S&P 500 and FTSE and S&P 500 is visible only at some short time periods.

Interesting is also the comovement behavior of PX, WIG and DAX pairs. PX and WIG show strong comovement through the whole time period on the scales of one month to one year as well as some significant periods on lower, daily scales. But PX and DAX and WIG and DAX show weaker long-term dependence. PX and WIG also show comovement with S&P 500 stock market but interestingly only during the second half of the period – during the mortgage crisis. Dependence on the FTSE is little bit stronger. Last, BUX shows the weakest comovement with all the other stock markets.

6 Conclusion

In this paper, we contribute to the literature on the international stock market comovement by researching the interconnections between Central European stock markets in time-frequency space. The novelty of our approach lies in the usage of the wavelet tools to high-frequency financial market data, which allows to understand the relationship between stock market returns in a different way.

In our research, we combine both time and frequency domain and we apply the cross-

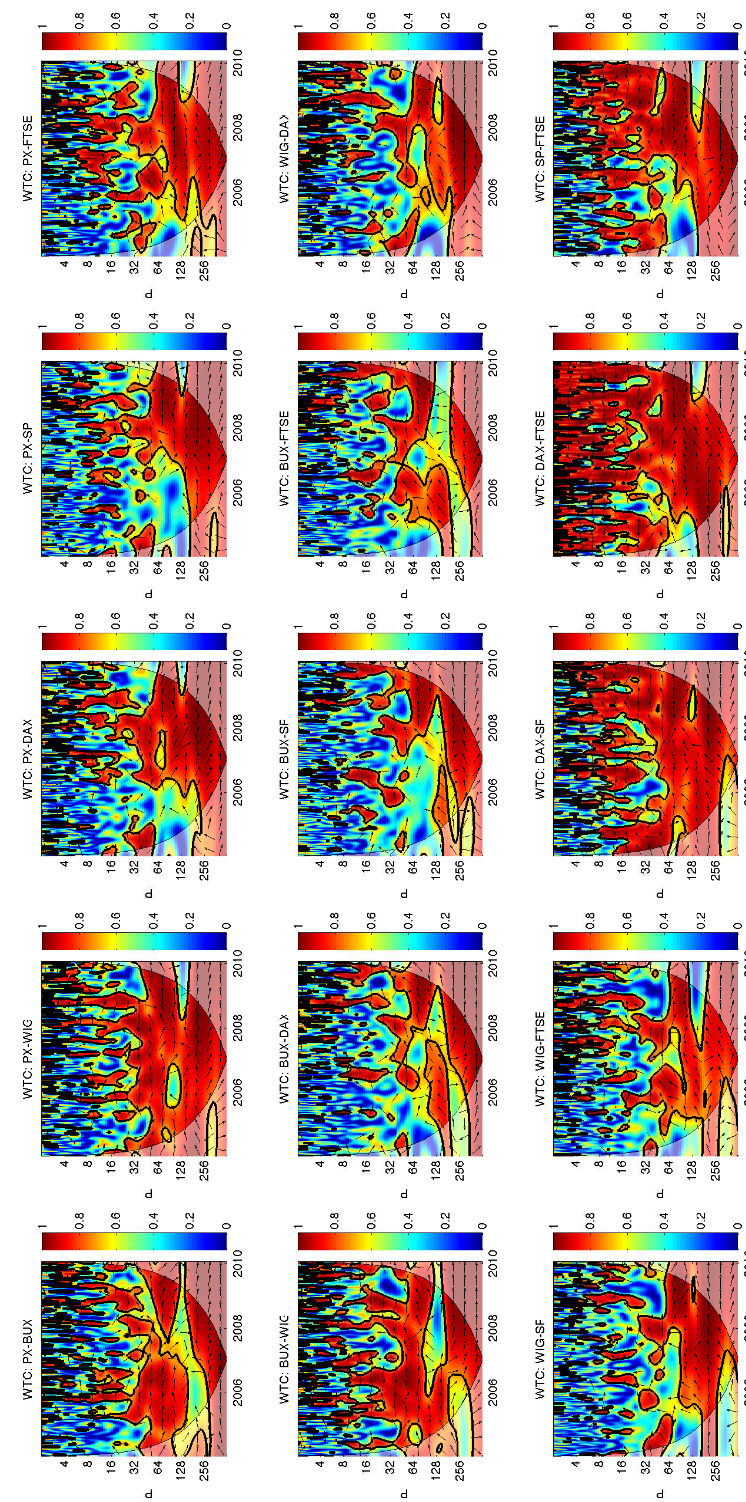

Figure 7: Wavelet coherence of PX, BUX, WIG and DAX, S&P 500 and FTSE indices pairs on the daily returns for 2004-2010 period. Horizontal axis shows time, while vertical axis shows scale in minutes/days. The warmer the color of region, the higher the degree of dependence between the pair.

wavelet analysis as a main tool of studying the comovements. Using the wavelet coherence, we show how local correlations are changing in time and across scales, continuously. In the first part of empirical analysis, we employ the high-frequency (5 minute) data of Czech (PX), Hungarian (BUX) and Polish (WIG) stock indices with a benchmark of German stock index (DAX) in the period of 2008-2009. In the second part, we also include British (FTSE 100) index and the U.S. (S&P 500) index and study interdependencies of all pairs on the daily data with widened period from 2004 until the end of 2009.

Our analysis finds that interconnection between all stock markets changes significantly in time and varies across scales. Using the 5 minutes high-frequency data, we find the strongest interdependencies among Czech (PX) and Polish (WIG) stock markets. Correlations, represented by wavelet coherence, were significant through various frequencies starting at intraday scale and ending at scales up to three months. PX-WIG pair also shows very interesting development of changing correlations from the second half of year 2008 until the end of the first half of 2009. Correlations are strongly significant through this time period but they change from the one month scale (lower frequency) to the shorter scale of one week (higher frequency). This dynamics of interdependence visible from the continuous wavelet transform of high-frequency data is unique and allows to understand the relationship between analyzed stock markets in a new way.

After studying the high-frequency data, we perform the same analysis on broader dataset to get more complex picture. The wavelet coherence reveals the difference of the interdependencies between Central European stock markets and benchmark the U.S. and Western European countries. Daily data of 2004-2009 period show very strong correlations of German (DAX) and London (FTSE) stock markets. These two markets show also large significant comovement on low scales up to one week. This kind of comovement can not be observed between DAX and S&P 500 for example, even though this pair uncovers also very strong comovement on higher scales. We suppose that this may be contributed by the large time difference between trading hours of these stock markets. On the contrary, Central European markets show different dynamics of comovement. The most interesting is the interdependence between Central European stock markets and benchmark German, British and the U.S. stock markets. Common feature is that during the first half of the studied period, these markets show relatively weak interdependence. While PX-WIG pair shows quite long period of interdependence through most of the frequencies, PX-DAX and WIG-DAX pairs show weaker dependencies. Thus, PX and WIG are strongly dependent while their relationship to DAX is not so strong. On the other hand, BUX shows much weaker interdependencies with all other stock markets.

To conclude, we have uncovered very interesting dynamics of correlations between Central European and Western European stock markets using a novel approach. Our findings are model-free and provide a possibility of the new approach to financial risk modeling. Thus, they have strong implications to portfolio management.

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