

# Fuel Consumption Optimization: Early Experiments<sup>\*</sup>

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**Abstract:** The paper deals with a problem of fuel consumption optimization. Solutions existing in this field are mainly based on the various conceptual approaches such as hybrid and electric vehicles. However, it leads to high initial cost of a vehicle. The approach presented in this paper aims at conventional vehicles and is based on recursive algorithms of system identification and adaptive quadratic optimal control under Bayesian methodology. Experiments with real data measured on a driven vehicle are provided.

Keywords: Control applications; closed-loop control; adaptive systems; system identification; stochastic systems; autoregressive models; fuel control; vehicles; constraints; decision making.

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## 1. INTRODUCTION

Today's automotive industry suffers from lack of solutions to help a driver to optimize ride with regard to reducing fuel consumption and minimizing environmental pollution ( $CO_2$ , noise, vibration). Existing solutions are mainly based on the various conceptual approaches in the form of hybrid vehicles, electromobiles, etc. It is quite understandable: with increasing government regulations for emissions and fuel quality the hybrid vehicles become attractive. The increasing price of oil and the promise of fuel savings through the use of an electric motor also contribute to establishment of hybrid electric powertrains in the marketplace. Due to their potential, the hybrid electric vehicles and control strategy for them are recently widely studied. Many efforts are focused on developing novel concepts and low-cost systems for these eco-cars. A detailed overview is nicely presented by Chan [2007]. Comparison of technologies for hybrid and telematics-equipped vehicles (so called "intelligent") from the point of view of fuel economy improvement is described by Manzie et al. [2007]. Problem of optimal control strategy for hybrid vehicles is discussed in the paper of Chan-Chiao Lin et al. [2004].

However, fuel savings associated with hybrid vehicles are connected with a rather high initial cost of such a vehicle due to the increased complexity of the powertrain. Moreover, it is typical for any novel technology that some time is needed to refine it and make it reliable, get rid of the errors and learn to repair it.

Recently electric vehicles (electromobiles) came to the market. Their advantage from the environmental point of view is surely significant. However, the initial cost of

the electromobile is high again. Moreover, there still exist problems with long out-of-town traveling and charging stations.

Therefore, optimization of fuel consumption for conventional vehicles is still strongly desired and it could bring significant environmental and economic savings. Many papers are devoted to this problem. For instance, Raubitschek et al. [2011] deal with driving strategies based on prediction of urban traffic situations. The main idea is to reduce the dynamics in the velocity profiles of driving situations and, respectively, the fuel consumption in urban traffic. Reducing the velocity dynamics is proposed via a situation-adaptive reaction to every predictively known forthcoming traffic event. The proposed algorithm calculates a fuel consumption optimized driving trajectory at each route section of the vehicle provided that predictive information about the traffic events is available. Used input parameters are temporal and spatial depending constraints of the driving situation as well as other restrictions like a speed-limit.

The publication of Saboohi et al. [2009] is devoted to modeling an eco-driving strategy of a vehicle based on minimization of fuel consumption in a given route. Vehicle speed and gear ratio are identified as control variables. The effect of working load is considered according three engine running processes of idle, part-load and wide open throttle.

An approach proposed in the presented paper is oriented at conventional vehicles. The paper deals with optimization of fuel consumption based on data continuously measured on a driven vehicle and on external observations. Dynamic driving values are proposed to be treated via multivariate autoregression model under Bayesian methodology, see Kárný et al. [2005]. Algorithms of recursive (on-line) sys-

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<sup>\*</sup> The research was supported by projects TAČR TA01030123 and MŠMT 1M0572.

tem identification and adaptive quadratic optimal control are used for solution to the fuel consumption optimization task. The algorithms used run in an on-line mode and are based on explicit solutions avoiding numerical computations as far as possible.

The main features of the proposed approach are as follows.

- Driving data are divided among modeled output variables, input controlled ones and external measurements.
- The control algorithm handles the input variables (namely, pressing the gas and the brake pedal and selected gear of transmission).
- Estimation of the modeled variables (fuel consumption, average rear wheels speed, engine speed, engine torque, etc.) runs in time with the currently given input controlled variables and external measurements (road altitude, etc.).
- The key estimated variables are the fuel consumption and the average rear wheels speed. Minimization of the fuel consumption under condition of the prescribed speed is reached via setpoints and penalizations in criteria of the control algorithm.

Layout of the paper is as follows. Section 2 provides basic facts about a model used and Bayesian estimation and control algorithm. Section 3 describes application of the mentioned approach to a problem of fuel consumption optimization. Section 4 provides experiments with real data samples and discussion. Conclusion in Section 5 closes the paper.

## 2. PRELIMINARIES

### 2.1 Model

Let us consider a system, which produces observable output variables  $y_t$  and  $z_t$  influenced by input controlled variable  $u_t$  and external variable  $v_t$  at discrete time moments  $t \in \{1, \dots, T\} \equiv t^*$ . In general, all the variables are multivariate. A relation among the variables is modeled via the conditional probability density function (pdf)

$$f(y_t, z_t | u_t, v_t, \phi_{t-1}, \Theta), \quad (1)$$

where  $\phi_{t-1} = [u_{t-1}, v_{t-1}, \dots, u_{t-n}, v_{t-n}, 1]$  is a regression vector,  $\Theta$  is a vector of model parameters. Throughout the paper this pdf is treated as a static linear multivariate regression model with normal noise, i.e.,

$$[y_t, z_t]' = [u_t, v_t, \phi_{t-1}]' \Theta + e_t, \quad (2)$$

where  $e_t$  is normal noise with zero mean and covariance matrix  $R$ .

The main tasks addressed in this paper are: (i) to estimate model parameters and predict output variables; (ii) to propose the control inputs so that to keep values of chosen outputs near the desired values.

### 2.2 Estimation and output prediction

To estimate parameters  $\Theta$  of model (1), a well-known Bayesian approach presented, for example, in Peterka [1981] is used in the following way.

$$f(\Theta | D(t)) \propto f(y_t, z_t | u_t, v_t, \phi_{t-1}, \Theta) f(\Theta | D(t-1)), \quad (3)$$

where  $D(t) = (d_1, \dots, d_t)$  with  $d_t \equiv (y_t, z_t, u_t, v_t)$ ,  $\propto$  means proportionality and  $f(\Theta | D(t-1))$  denotes a prior pdf. In case of normal noise in model (2) the parameter estimation algorithm is a straightforward recursive computation of statistics with a chosen prior Gauss-inverse-Wishart pdf that can be found in many sources, e.g. Peterka [1981], Kárný et al. [2005].

Prediction of output variables  $y_t$  and  $z_t$  is performed in the following way:

$$\begin{aligned} & f(y_t, z_t | u_t, v_t, D(t-1)) \\ &= \int_{\Theta^*} f(y_t, z_t | u_t, v_t, \phi_{t-1}, \Theta) f(\Theta | D(t-1)) d\Theta. \end{aligned} \quad (4)$$

### 2.3 Control

To control the considered system, an approach called *the fully probabilistic design* (FPD) described in Kárný, Guy [2006] is used. The mentioned algorithm works with the model of the closed-loop of the controlled system and a feedback controller. The closed-loop model of the system and its controller are given by the joint pdf

$$\begin{aligned} & f(y_t, z_t, u_t, v_t | \phi_{t-1}) \\ &= f(y_t, z_t | u_t, v_t, \phi_{t-1}) f(u_t, v_t | \phi_{t-1}), \end{aligned} \quad (5)$$

which is factorized according to the chain rule, see e.g. Kárný et al. [2005]. Here the output variable  $y_t$  stands for outputs to be optimized, while  $z_t$  denotes the non-optimized part of the output vector.

The exploited FPD algorithm requires to define the so called *ideal* model of the introduced closed-loop (5) with the same structure, i.e.,

$$\begin{aligned} & \tilde{f}(y_t, z_t, u_t, v_t | \phi_{t-1}) \\ &= \tilde{f}(y_t, z_t | u_t, v_t, \phi_{t-1}) \tilde{f}(u_t, v_t | \phi_{t-1}) \end{aligned} \quad (6)$$

which represents wishes of a decision-maker about behavior of the original closed-loop.

The key idea of FPD is to design the controller so that the closed-loop model (5) would be as close as possible to its ideal description (6) in the sense of minimizing the Kullback-Leibler divergence (KLD)

$$K(f || \tilde{f}) = \int_{[y, z, u]^*} f \ln \frac{f}{\tilde{f}} d[y_t, z_t] du_t, \quad (7)$$

with  $f = f(y_t, z_t, u_t, v_t | \phi_{t-1})$  and  $\tilde{f} = \tilde{f}(y_t, z_t, u_t, v_t | \phi_{t-1})$ . KLD is presented in Kullback and Leibler [1951] and adopted as a theoretically justified proximity measure. This divergence is known to be an optimal tool within the adopted Bayesian approach.

In the case of linear normal autoregression models, the FPD coincides with a widely spread quadratically optimal control, where penalizations in the squares of variables in the optimality criteria are the main control options, see Kárný et al. [1985]. In the FPD, these penalizations are taken as the inversions of the noise variances of the corresponding factors of the joint pdf describing the closed-loop.

According to Feldbaum [1961], the dual control is not practically applicable. Therefore, a separate model estimation should be used with a subsequent substitution of the point parameter estimates during the control synthesis. It means, the control synthesis is considered with known model parameters. For minimization of (7) the dynamic programming is used. The detailed derivation of the control algorithm can be found in Kárný et al. [2005].

### 3. FUEL CONSUMPTION OPTIMIZATION

Let us apply the above approach to a problem of fuel consumption optimization. The main obvious aim here is to design values of the control variables so that to minimize the fuel consumption. A simple intuitive solution of this task causes a reducing the speed until full stop. Therefore, a speed of a vehicle should follow the recommended values and, at the same time, not exceed the existing limits. These demands are generally in contradiction and their balance makes the optimum of the solved task.

Experts in this field summarize general rules for driving with a lower fuel consumption as follows.

- When approaching a turn of the road, a vehicle should slow down gradually. At the beginning of the turn maneuver the vehicle should have just the proper speed for driving through it.
- After finishing the turn maneuver, the vehicle should increase speed slowly.
- When approaching an uphill road, it is necessary to increase the engine rotation speed in advance so that the vehicle's speed does not fall on the slope.
- On downhill road the vehicle must be driven either without pressing the gas pedal or with a minimum of it.

Other constraints as traffic rules and a smooth reducing of the speed in case of unexpected pedestrian in a road must be also satisfied.

It is obvious that there is a lot of aspects that can influence the actual fuel consumption. However, they can be divided among three groups of variables. The first group includes modeled variables related to a vehicle itself. The second one involves variables which can control the modeled ones. The third group of variables comprises the so called external ones which express influence of the outer world like approaching road turn, road downhill or uphill, etc.

For model (1), or precisely, closed-loop (5) the mentioned groups of variables can be treated in the following way. The first group of variables is entering (5) via notations  $y_t, z_t$ . Here vector of modeled output variables  $y_t$  includes the fuel consumption and the vehicle speed which are the key values to be optimized. Vector  $z_t$  contains the modeled quantities that are connected to  $y_t$  (i.e., engine speed, engine torque, lateral acceleration, etc.). However, they must be only properly constrained but not optimized.

The second group of variables in (5) is expressed by control input variable  $u_t$ . Here the entries that can directly influence the driving are taken, namely, pressing the gas pedal, pressing the brake pedal, selected gear of transmission, rotating steering wheel.

Vector of external variables  $v_t$  creates the third mentioned group used in (5). The considered entries of  $v_t$  include road altitude, distance travelled, time travelled, UTM information for vehicle position, etc. It should be noted that selection of measured variables that influence the fuel consumption is a separate complicated task (details about selection of variables are described in Section 4).

With selected  $y_t, z_t, u_t$  and  $v_t$ , the ideal pdf (6) can be defined. However, the optimization task (7) is now decomposed into two subtasks: (i) modeling of the recommended speed under current conditions, (ii) given the recommended speed, design the control variables keeping this speed and minimizing the fuel consumption.

For a known route the recommended speed can be defined "manually". It means, a sufficient way how to determine the recommended speed is to go through this route or to investigate a detailed map of the region and to assign the speed according to an expert from the transportation area. For an unknown road, one has to rely on the information from a navigation. It means that recommended speed must be estimated at each time instant. At the current stage of the project, the task is solved for the case with known route and the prescribed speed is given.

However, all the modeled variables are recursively estimated via the approach described in Section 2 in order to compare their course with the real one. It means that ideal values are also taken from the real data course. The upper constraints are also chosen for the inputs. The deviations from desired values of the key outputs  $y_t$  are penalized in the control step. The more important the value, the higher penalty. The rest of the modeled outputs are only estimated to ensure their adequate course.

The approach applied to these specific data has been implemented as a software EcoJob created in Matlab, see The Matlab Inc. [2000]. EcoJob is mainly based on functions of toolbox MixTools developed in authors' department by Nedoma et al. [2003] under project ProDaCTool [2003]. Results of experiments are provided below.

### 4. EXPERIMENTS

The experimental part of the work is carried out in collaboration with Škoda Auto a.s. (see [www.skoda-auto.com](http://www.skoda-auto.com)) which provided real observations. To ensure necessary dynamic, data were measured for driving both with a lower and a usual fuel consumption.

Originally, the available measurements contained significant number of variables measured for a selected out-of-town route. A basic data sample including 16 most important variables influencing the driving was selected to be used for experiments. These variables are as follows:

- (1) fuel consumption [ $\mu l$ ],
- (2) average rear wheels speed [ $km/h$ ],
- (3) angle of rotating steering wheel [ $degrees$ ],
- (4) pressing the gas pedal [%],
- (5) pressing the brake pedal [%],
- (6) selected gear of transmission,
- (7) engine torque [ $Nm$ ],
- (8) engine speed [ $rpm$ ],

- (9) lateral acceleration in multiples of gravimetric acceleration,
- (10) yaw rate [degrees],
- (11) distance travelled [m],
- (12) time travelled [s],
- (13) UTM-X coordinate for vehicle position [m],
- (14) UTM-Y coordinate for vehicle position [m],
- (15) road altitude [m],
- (16) vehicle course [degrees].

After the data preprocessing the whole available number of data items was about 20 thousands with a sampling period equal to 1 second. Various sequence and count of the above variables entering the modeled vectors were tested in order to find those which have more influence on fuel consumption under existing constraints. Configuration of the input vector remained the same for all the experiments as follows:  $u_t = [u_{1;t}, u_{2;t}, u_{3;t}]'$ , where  $u_{1;t}$  – pressing the gas pedal,  $u_{2;t}$  – pressing the brake pedal,  $u_{3;t}$  – selected gear of transmission. Upper constraints 100, 25, 6 were chosen for every input respectively. The optimized output  $y_t$  was always the same, i.e.,  $y_t = [y_{1;t}, y_{2;t}]'$ , where  $y_{1;t}$  is the fuel consumption,  $y_{2;t}$  – average rear wheels speed (identified with the vehicle's speed). Vectors  $z_t$  and  $v_t$  were changing for different experiments. The obtained results were compared with real data courses.

Two types of experiments were performed. For the first one, EcoJob constructed the closed-loop model with the help of functions of toolbox Mixtools only. This constructed and estimated model was used during the control steps.

For the second type of experiments, an external specific software for vehicle simulation was exploited in addition to EcoJob and Mixtools functions. This vehicle simulator including cruise control function was implemented in Matlab by experts from Škoda Auto a.s. It was used to test reactions of a vehicle for the given FPD-based inputs from the fuel saving point of view and to check a correct work of the closed-loop.

Both the types of experiments are described below.

#### 4.1 Experiments with the Mixtools functions

Currently the best results were reached with the following configuration:  $z_t = [z_{1;t}, z_{2;t}, z_{3;t}, z_{4;t}]'$ , where  $z_{1;t}$  – engine torque,  $z_{2;t}$  – engine speed,  $z_{3;t}$  – lateral acceleration,  $z_{4;t}$  – yaw rate. External variables exploited were contained in vector  $v_t = [v_{1;t}, v_{2;t}, v_{3;t}, v_{4;t}, v_{5;t}]'$ , where  $v_{1;t}$  – time travelled,  $v_{2;t}$  – UTM-X coordinate for vehicle position,  $v_{3;t}$  – UTM-Y coordinate for vehicle position,  $v_{4;t}$  – road altitude,  $v_{5;t}$  – vehicle course. Bigger penalization values 5 and 30 were used for optimized outputs  $y_{1;t}$  and  $y_{2;t}$ . For the rest of the outputs and for inputs the penalization values were rather small (about 0.1). The number of data used for the control time cycle during this experiment was 3450.

The fuel savings with the control inputs obtained during this experiment was about 19% in comparison with the real measurements.

The results presented in Figure 1 demonstrate comparison of the FPD-computed input variables, i.e., pressing the gas

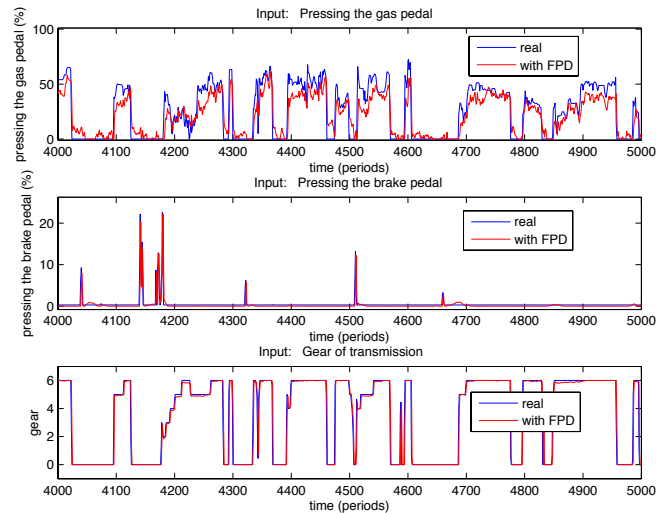


Fig. 1. Control inputs obtained with Mixtools: pressing the gas pedal (top), pressing the brake pedal (middle) and gear (bottom).

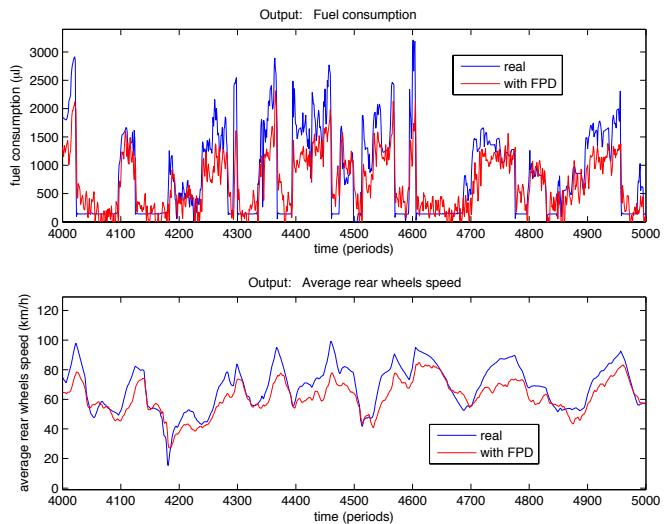


Fig. 2. Optimized outputs obtained with Mixtools: fuel consumption (top) and the average rear wheel speed (bottom).

pedal (top), pressing the brake pedal (middle) and selected gear (bottom), with the real ones. For better illustration a fragment of results for 1000 data items already after estimation is shown. At the beginning of the estimation the results of the on-line algorithm were worse. However, after learning the computed inputs correspond to the real values with a bit lower pressing both the gas and the brake pedals that is in accordance with general rules of fuel economy from Section 3. A sensitive point for the used configuration was computation of gear. This is planned to be improved in the next version of EcoJob.

The fuel consumption reduced about 19% and the recommended speed are shown in Figure 2. The estimated non-optimized outputs, i.e., engine torque, engine speed, lateral acceleration, yaw rate are provided in Figure 3. Their courses are only followed to be properly constrained.

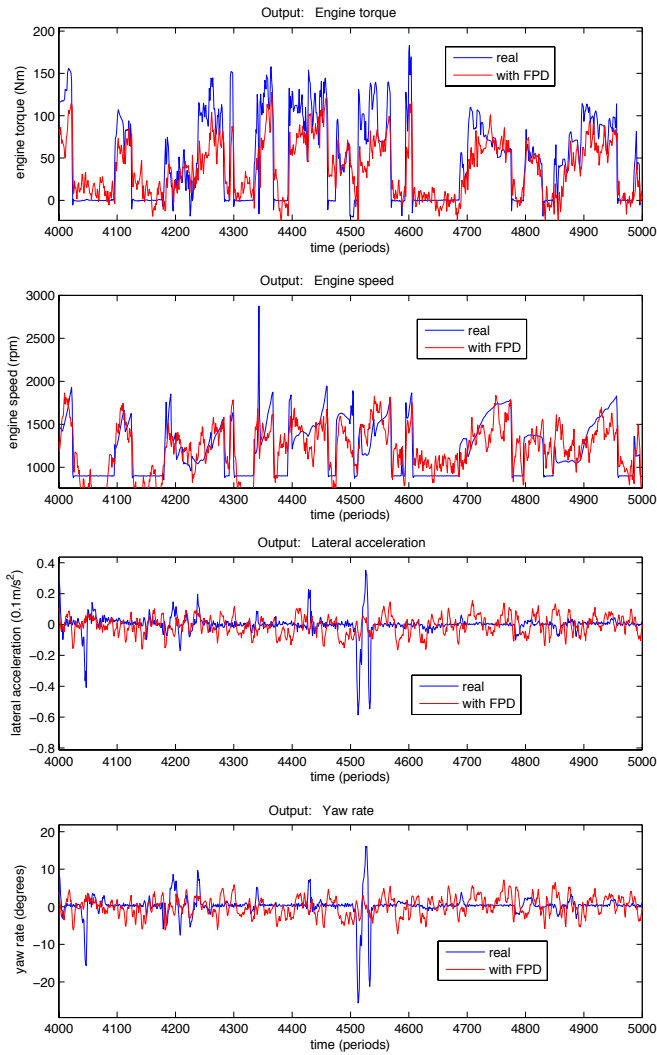


Fig. 3. Non-optimized outputs obtained with Mixtools: engine torque, engine speed, lateral acceleration and yaw rate.

#### 4.2 Experiments with the external vehicle simulator

For this experiment the best results were achieved with the following variables chosen:  $z_t = [z_{1;t}, z_{2;t}]'$ , where  $z_{1;t}$  was engine torque,  $z_{2;t}$  was engine speed. External variable  $v_t$  considered was road altitude. Penalization values were 5 and 20 for optimized outputs  $y_{1;t}$  and  $y_{2;t}$  respectively. The number of data used for the control time cycle was 3650.

Comparison of the FPD-based input variables with the real measurements is demonstrated in Figure 4. Similarly as in the previous experiment, it can be seen that the obtained values of pressing the gas pedal are, in general, lower than the real ones. The minimized fuel consumption and the average rear wheel speed are shown in Figure 5 (top) and (bottom) respectively. The fuel consumption is reduced about 30% against the real data. The recommended speed obtained via FPD is also lower than the original one. The estimated engine torque and engine speed are demonstrated in Figure 6 (top) and (bottom) respectively. Fragments with their lower values in Figure 6 correspond

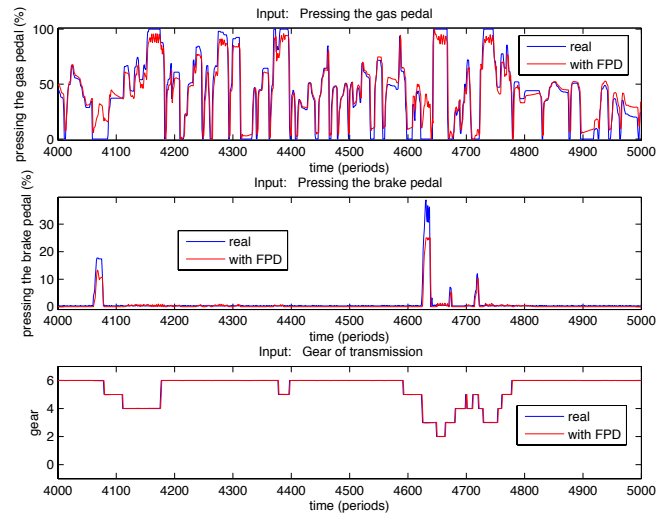


Fig. 4. Control inputs obtained with the external simulator: pressing the gas pedal (top), pressing the brake pedal (middle) and selected gear (bottom).

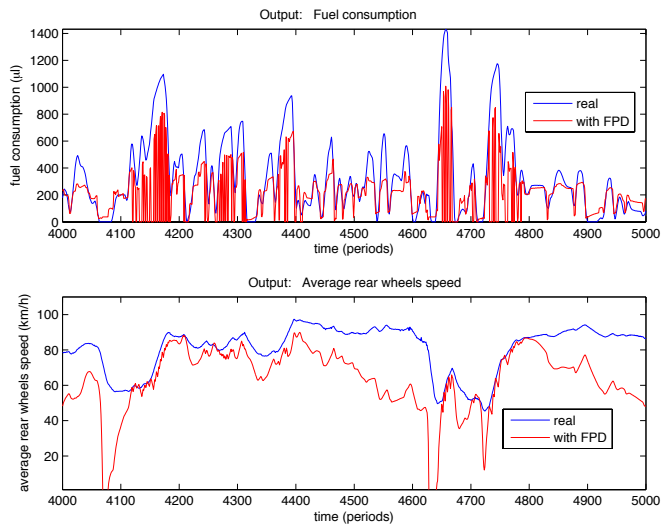


Fig. 5. Optimized outputs obtained with the external simulator: fuel consumption (top) and the average rear wheel speed (bottom).

to the lower speed and the reduced fuel consumption in Figure 5.

During this experiment, simulation of real-time data bus connection between a computer with running FPD algorithm and an electronic control unit (ECU) was also successfully tested.

#### 4.3 Discussion

The described experiments are the “early” ones at the initial phase of the project. To summarize this part of the work, one can say that the results concerning the fuel savings look promising. The considered closed-loop is influenced by the used control inputs as expected.

In these early experiments, the resulting fuel economy may seem to be obtained partially due to the lower speed of the vehicle. It indicates that a problem of more

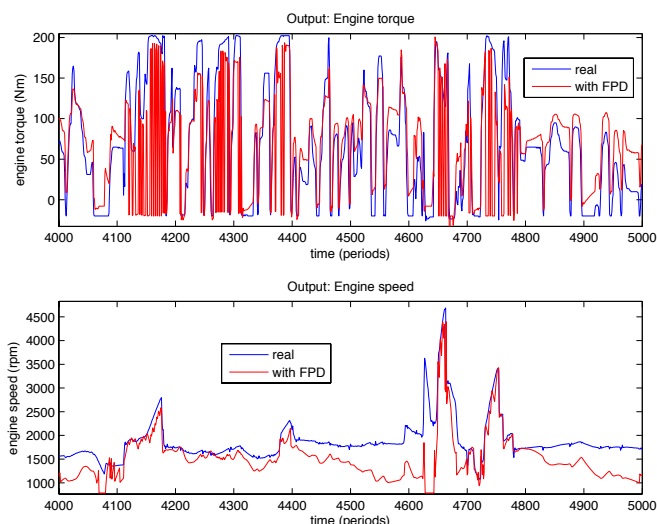


Fig. 6. Non-optimized outputs obtained with the external simulator: engine torque (top) and engine speed (bottom).

precise tracking the recommended speed and “dragging” the values up to the desired ones is still not solved completely.

An adequate choice of the recommended speed is a separate complicated task. This speed should be neither too fast nor too slow in order to ensure safe driving according to traffic rules and, simultaneously, reaching the destination on time. Usage of the recommended speed as a setpoint to be tracked using restrictions in the form of maximal allowed speed is expected to contribute to improving the control quality. It provides a chance that a driving without any sharp changes of the speed, but with slight modifications in pressing the gas pedal, can bring fuel savings. This improvement will be published soon.

## 5. CONCLUSION

The paper is devoted to a problem of fuel consumption optimization for conventional vehicles. Optimization of fuel consumption based on data continuously measured on a driven vehicle and on external observations is proposed. The proposed approach is based on recursive algorithms of estimation and control under Bayesian approach. Illustrative results of experiments with real measurements including tests of adaptive control loop are presented.

Generally this research project aims at optimization of fuel consumption both from the economical and ecological points of view. It means ecological criteria are planned to be introduced.

Future working plans of the project also includes exploitation of setpoints for the recommended speed based on a route identification. Approaching route elements (turns, hills, speed-restricting road signs, crossings, etc.) are to be identified in order to have time to change a speed gradually. In this case, angle of rotating steering wheel will be probably included in the control input vector.

## ACKNOWLEDGEMENTS

The research was supported by projects TAČR TA01030123 and MŠMT 1M0572.

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