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How do skilled traders change the structure of the market

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ABSTRACT

We extend the original heterogeneous agent model of Brock and Hommes (1998) by introducing the concept of skilled traders. The idea of skilled traders is based on the endeavor of market agents to estimate future price movements. We distinguish between the three groups of skilled traders according to their trading strategies. The first group consists of skilled traders who estimate the trend parameter and have randomly generated bias. The second group has fixed bias to zero, and the third group, most advanced one, is able to estimate the bias parameter. The most interesting result from simulations is that for all model settings the stock market changes its structure at some point with growing number of skilled traders.

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1. Introduction

Models with heterogenous beliefs are generally perceived as a counterparts to the traditional financial models. While traditional models are based on fully rational agents operating on efficient markets (Lucas, 1971; Fama, 1970), heterogeneous agent models represent behavioral approach in financial markets modeling. Behavioral finance use a concept of bounded rationality in agent's decisions (Simon, 1957), assumption of full rationality of agents is relaxed and markets are viewed as a dynamic and complex system with many interacting agents having heterogeneous trading strategies (Brock & Hommes, 1997). Furthermore, psychology plays important role in agents' investment decisions (Shleifer, 2000; Barberis & Thaler, 2003).

Even simple heterogenous agents models are able to explain stylized facts observed on real financial markets, more specifically fat tails in returns distribution, volatility clustering and long memory (Lux & Marchesi, 2000; Farmer & Joshi, 2002; Giardina & Bouchaud, 2003). In basic heterogeneous agent model framework, there are typically two trading strategies; fundamentalists and chartists. While fundamentalists base their expectations about future prices on market fundamentals, chartists or technical analysts try to use observed patterns in past prices for future investment decisions. A heterogeneous agent model consisting of the two investor types can generate a very complex nonlinear

dynamics. A large amount of literature on heterogeneous agent models has developed, e.g. (Brock & Hommes, 1997), (Brock & Hommes, 1998), (Chiarella & He, 2000), (Gauersdorfer, Hommes, & Wagener, 2008), for an extensive survey on this topic see (Tesfatsion & Judd, 2006).

This paper contributes to the literature with a model that uses a new trading strategy—skilled trader. The skilled traders use simple predictions to predict future price changes. A concept of heterogeneous agent models with traders using a simple prediction was introduced in Barunik, Vacha, & Vosvrda (2009) and Vacha, Barunik, & Vosvrda (2009).

Proposed model is a system of interacting agents with different trading strategies. Each trading strategy, also called predictor, is defined by a unique set of parameters, altogether they form a heterogeneous system, which replicates behavioral patterns on a real financial market. Agents make decision about future trading strategy every period. Decision process of all market participants depends on the performance reached by predictors in previous periods. Experience from real financial markets shows that investors differ in their trading horizons. We can observe active intraday investors as well as automatic trading systems operating in fractions of seconds on real financial markets. On the other hand, we can see long term investors who base their decisions on the fundamental market values. For this reason, we add a new heterogeneity parameter that defines the length of evaluation horizon for determining the performance measure of a specific predictor.

Brock & Hommes (1998) use basically two parameters defining a trading strategy; the trend and the bias. Even a very simple setting with two predictors can generate complicated pricing dynamics.

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Brock & Hommes (1998) show that the endogenous dynamics in the model can lead to large price changes in the market or even market crashes. In their simple approach all solutions can be computed analytically. Since our model with many predictors is much more complicated and contains noise, we study the model properties using simulations. For every specific simulation, we generate parameters of predictors as draws from the normal or the uniform distribution.

Our model considers price predictions and studies their impact on market dynamics. We define a new predictor type, called skilled trader who can predict either the trend parameter or both the trend and the bias parameter simultaneously. A skilled trader predicts the model parameters on information set consisting of last price observations. Further, the skilled traders simply assume that the price deviations follow an AR(1) process and they use the maximum likelihood method to estimate the parameters. The length of the information set used for estimation is generated randomly for every trading strategy.

Important parameter is the proportion of skilled traders on the market, so we run simulations to study behavior of the model for all possible ratios of skilled traders on the market. We start to simulate the market with zero skilled traders, i.e., all predictors are random and end with 100% of skilled traders on the market. We expect that with more skilled traders in the market, the market will be less efficient in a sense of persistent patterns in generated price time series. In the paper, we study three different settings. First setting assumes that the skilled trader predicts only the trend parameter, while the bias parameter is random. In the second case, the skilled trader again predicts the trend parameter while the bias parameter is set to zero. The third setting uses a complete skilled trader, who predicts both the trend and bias parameters. Simulations show that there is substantial change in the simulated market structure when skilled traders are present. All the different settings also lead to structural change in the market at some point with increasing number of skilled traders.

High ratio of skilled traders on the market simulates the situation where majority of strategies has a simple prediction rule and as a consequence all market participants have similar beliefs. We show that the market exhibit completely different behavior in case the market participants' prediction mechanism shrinks to a similar simple prediction tool. This can represent, for example, crisis period when all participants estimate similar direction of the price movement. Another example which can be close to this behavior is the period of increased uncertainty (risk) where the market participants do not know which (fundamental) price is correct.

The structure of the paper is as follows. After the introduction, a heterogeneous agent model is defined. Next part briefly introduces the implementation of skilled traders into the heterogeneous agent model framework. The last part of the paper investigates how the presence of skilled predictors qualitatively changes the market behavior.

2. Model

We use basic principles of the heterogeneous agent model introduced by (Brock & Hommes, 1998). They consider a system with one risky asset and one risk free asset. Let p_t denote the ex-dividend price of the risky asset at time t and let $\{y_t\}$ be the stochastic dividend process of the risky asset.¹ The supply of the risk free asset is perfectly elastic and pays a fixed return r . Let us define the dynamics of the wealth as:

$$W_{t+1} = (1+r)W_t + (p_{t+1} + y_{t+1} - (1+r)p_t)z_t, \tag{1}$$

where z_t denotes the number of shares of the asset purchased at time t . Let us further define $E_{h,t}$ and $V_{h,t}$ as the beliefs of investor type h

about the conditional expectation and conditional variance of the excess return $p_{t+1} + y_{t+1} - (1+r)p_t$ on the information set $F_t = \{p_t, p_{t-1}, \dots; y_t, y_{t-1}, \dots\}$. Investors are supposed to be myopic mean-variance maximizers, so the demand of a trader type h for the risky asset is given as:

$$z_{h,t} = \frac{E_{h,t}[p_{t+1} + y_{t+1} - (1+r)p_t]}{a\sigma^2}, \tag{2}$$

the risk aversion coefficient, $a > 0$, is defined to be the same for all traders. We further assume the conditional variance of excess returns to be constant for all investor types, i.e., $V_{h,t} = \sigma^2$.

Let us consider z_t^s as supply of outside risky shares. Further we define $n_{h,t}$ as the fraction of investors of type h on the market at time t . The equilibrium of demand and supply for all trader types H is given as

$$\sum_{h=1}^H n_{h,t} \left\{ \frac{E_{h,t}[p_{t+1} + y_{t+1} - (1+r)p_t]}{a\sigma^2} \right\} = z_t^s, \tag{3}$$

We assume the case of zero supply of outside shares, i.e., $z_t^s = 0$, so the market clearing equilibrium yields:

$$(1+r)p_t = \sum_{h=1}^H n_{h,t} \{E_{h,t}[p_{t+1} + y_{t+1}]\}. \tag{4}$$

In a market where all agents are rational, the price is given by the discounted sum of future dividends

$$p_t^* = \sum_{k=1}^{\infty} \frac{E_t[y_{t+k}]}{(1+r)^k}. \tag{5}$$

The fundamental price p_t^* depends on the dividend process. Assuming the special case of an IID dividend process $\{y_t\}$, with constant mean $E_t\{y_t\} = \bar{y}$, the fundamental price is given by

$$p^* = \sum_{k=1}^{\infty} \frac{\bar{y}}{(1+r)^k} = \frac{\bar{y}}{r}. \tag{6}$$

We assume the expectations about future dividends to be the same for all trading strategies, i.e., $E_{h,t}[y_{t+1}] = E_t[y_{t+1}]$. In the special case of an IID dividend process, we get $E_t\{y_{t+1}\} = \bar{y}$.

2.1. Heterogeneity

So far, the model was rational and all traders or trading strategies were equal. Important feature of the Brock & Hommes(1998) approach is an implementation of heterogeneity. Next step is thus abandoning the idea of rationality and moving to the real financial market, allowing prices to deviate from their fundamental value p_t . In general form, beliefs about the future price $E_{h,t}[p_{t+1}]$ have the following form

$$E_{h,t}[p_{t+1}] = E_t[p_t^*] + f_h(x_{t-1}, \dots, x_{t-k}), \text{ for all } h, t. \tag{7}$$

Beliefs about the future price consist of the fundamental price and a market model of a predictor $f_h(x_{t-1}, \dots, x_{t-k})$, defined on an information set that consists of past price deviations with length k . In another words, a predictor h believes or predicts that the market price will deviate from its fundamental value p_t .

¹ random variables at time $t+1$ are denoted in bold

The market equilibrium, defined in Eq. (4), can be reformulated with heterogeneous predictors as:

$$(1 + r)x_t = \sum_{h=1}^H n_{h,t} \{E_t[p_t^*] + f_h(x_{t-1}, \dots, x_{t-k})\} \quad (8)$$

Furthermore, let us reformulate the market equilibrium using deviations from the fundamental price $x_t = p_t - p_t^*$,

$$(1 + r)x_t = \sum_{h=1}^H n_{h,t} E_{h,t} [x_{t+1}] \equiv \sum_{h=1}^H n_{h,t} f_{h,t}. \quad (9)$$

2.2. Selection of predictors

Every single iteration, the fraction $n_{h,t}$ of a predictor h at time t is updated. The model is closed system, therefore selection of predictors is controlled by endogenous market forces only. The fraction of a predictor h on the market depends on its performance measure $U_{h,t-1}$ and the intensity of choice β . Performance measures are calculated in every period, so the traders make their decisions based on updated performance measures. If the intensity of choice is high, then all agents use the trading strategy with the highest performance measure in the last period. Contrary to this case, when $\beta = 0$ agents are evenly distributed across the set of available trading strategies.

The fitness measure of strategy h is evaluated at the beginning of period t . As the fitness measure of trading strategies we use the moving averages of realized profits, where m_h denotes the length of the moving average filter. In a real market this parameter can be interpreted as the memory length or evaluation horizon for trading strategy h . The fitness measure $U_{h,t}$ is defined as

$$U_{h,t} = \frac{1}{m_h} \sum_{l=0}^{m_h-1} \left[(x_{t-l} - (1+r)x_{t-1-l}) \frac{(f_{h,t-1-l} - (1+r)x_{t-1-l})}{\sigma^2} \right]. \quad (10)$$

$n_{h,t}$ is given by the multinomial logit probabilities of discrete choice

$$n_{h,t} = \exp(\beta U_{h,t-1}) / Z_t, \quad (11)$$

$$Z_t = \sum_{h=1}^H \exp(\beta U_{h,t-1}), \quad (12)$$

where Z_t is a normalization factor ensuring that $n_{h,t}$ sum up to one.

3. Trading strategies

Basic framework for trading strategies in the heterogeneous agent models was investigated by Brock & Hommes (1998), who proposed a simple linear forecasting rule with fixed g_h and b_h ,

$$f_{h,t} = g_h x_{t-1} + b_h. \quad (13)$$

If $b_h = 0$ and $g_h > 0$, the investor is called a pure trend chaser. If $b_h = 0$ and $g_h < 0$, the investor is called a contrarian. Moreover, if $g_h = 0$, and $b_h > 0$ ($b_h < 0$), the investor is said to have an upward (downward) bias in his beliefs. In the special case of $g_h = b_h = 0$, the investor is fundamentalist, i.e., the investor believes that the price always returns to its fundamental value.

We extend the basic heterogeneous agent model by introducing a skilled trader, who is able to forecast the future trend parameter $g_{h,t}$ and $b_{h,t}$, which are in our setting variant in time. Three groups of skilled trading strategies are distinguished altogether. The first group

consists of skilled trading strategies which use simple linear predictions of $g_{h,t}$ and have randomly generated bias $b_h \sim N(0, 0.09)$, i.e., we use the same distributional parameters as for the stochastic beliefs. The second group consists of skilled trading strategies which use simple linear predictions of $g_{h,t}$ and have fixed bias $b_h = 0$. Thus, if skilled trader from this group estimates nonzero $g_{h,t}$, the trader is pure trend chaser. Finally, the most advanced trading strategies are in the third group, where strategies use simple linear predictions for both trend $g_{h,t}$ and bias $b_{h,t}$. The concept of skilled trader is a huge simplification of reality. However, we can expect there are traders that use simple predictions in real markets. In our work we simulate the situation when we are close to the extreme cases (financial crisis, terrorist attacks, etc.) where all market participants have the same beliefs about future price movements.

3.1. Stochastic beliefs

An important part of the model is trading strategy with stochastic beliefs. Parameters defining these trading strategies are generated stochastically at the beginning of the simulation and remain fixed for all the iterations. The trend parameter g_h and the bias parameter b_h of trader type h are realizations from the normal distribution $N(0, \sigma^2)$. In this paper we use $N(0, 0.16)$ and $N(0, 0.09)$, respectively.² The memory parameter m_h of the stochastic trading strategy is a realization from the uniform distribution, specifically $U(1, 100)$. The memory parameter can be interpreted as the evaluation horizon for the trading strategy h .

3.2. Skilled traders

The simplest way how to implement this type of the market behavior into the heterogeneous agent model is using simple linear forecasting techniques. The skilled trading strategies thus use maximum likelihood estimation of an AR(1) process to estimate the trend parameter $g_{h,t}$ and the bias parameter $b_{h,t}$ for the next period. Skilled traders thus assume that the deviations x_t follow an AR(1) process, and they base their forecasts of x_{t+1} on the information set $F_t = \{x_t, x_{t-1}, \dots, x_{t-k-1}\}$. The length of information set k_h for every skilled trader h is a realization from the uniform distribution $k_h \sim U(5, 50)$. Then the trading strategies of the skilled traders are defined as follows

$$f_{h,t+1} = \hat{f}_{h,t} = \hat{g}_{h,t} x_{t-1} + \hat{b}_{h,t}, \quad (14)$$

where $\hat{g}_{h,t}$ and $\hat{b}_{h,t}$ are the parameter estimates of the trend $g_{h,t}$ and the bias $b_{h,t}$. In the simulations, we use various types of skilled traders with different lengths k of the information set F_t . Skilled traders have also the memory defined in a similar way as for the stochastic beliefs, i.e., $m_h \sim U(1, 100)$.

4. Simulation results

Altogether, we consider models with 40 trading strategies for each simulation. Intensity of choice, β is set to 500. We start all simulations with the model consisting from random trading strategies, i.e., there are only stochastic beliefs. Further, we gradually increase the ratio of skilled traders on the market up to the last model with only skilled traders present on the market. Moreover, we simulate three market settings with three different types of skilled traders (randomly generated bias, fixed bias and estimated bias). Fig. 1 shows an

² We chose this setting in order to assure stability of the model. We tried also other parameters sets, but the qualitative results remained the same. The results using the different settings are available from authors upon request.

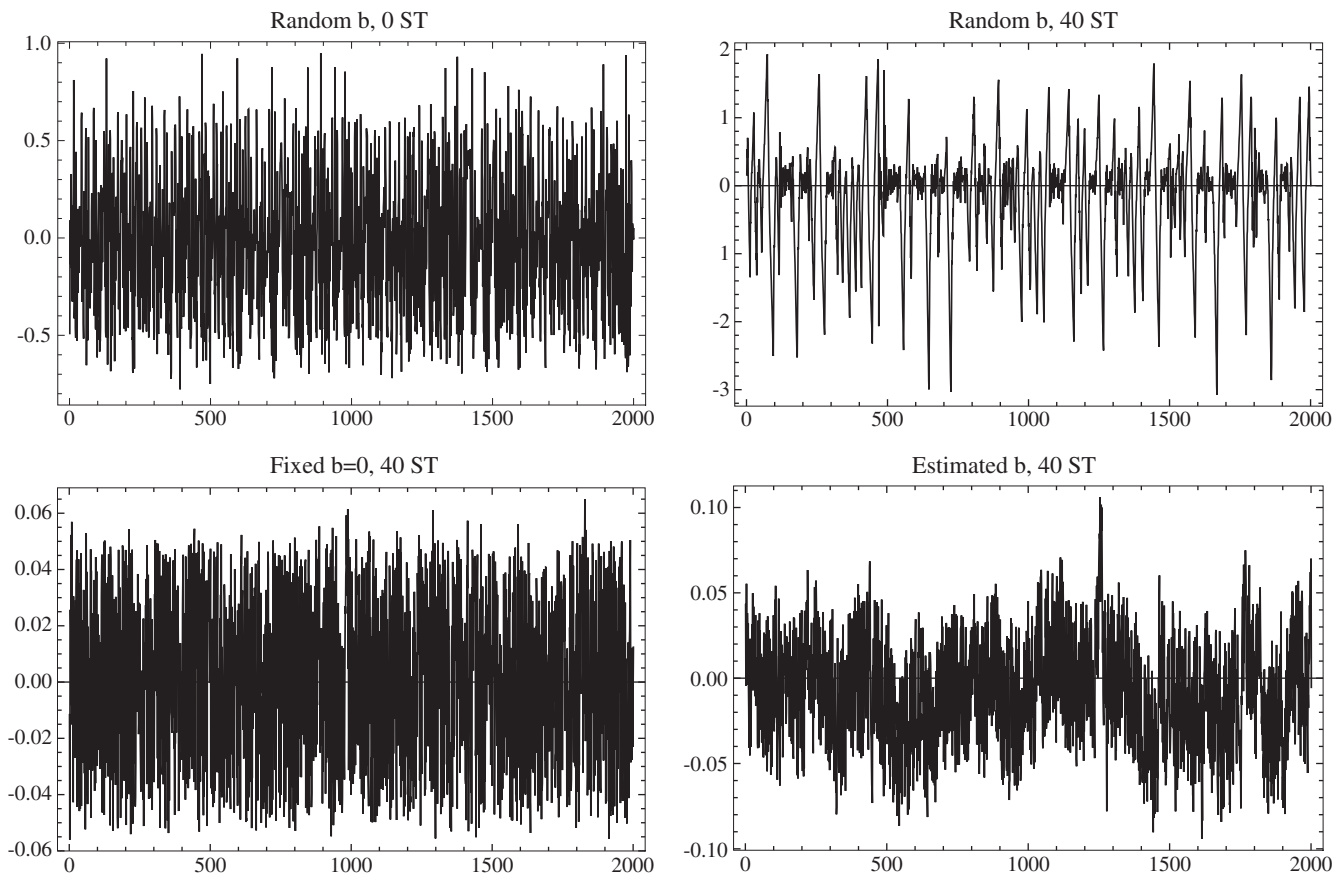


Fig. 1. Simulated market returns x_t . Market with no skilled traders (upper left). Market with 40 skilled traders: random bias (upper right), zero bias (lower left), estimated bias (lower right).

example of the simulated returns with no skilled traders and the three cases where only skilled traders are on the market. Following section discuss the results³ in detail.

4.1. Skilled traders with random bias

There are two main characteristics of the generated time series that we study; variance and the Hurst exponent. The first model setting we studied is the one with the skilled traders with a random bias. Volatility of the simulated market returns measured by standard deviation grows rapidly with growing number of skilled traders in this market. In simulations where the ratio of skilled traders is greater than 3/4, we observe an enormous variance increase, maximum variance is reached when market consists only of skilled traders. Fig. 2 shows the growing variance of simulated returns x_t .

It is interesting also to study the Hurst exponent of the simulated time series as it measures the persistence of returns. When the Hurst exponent is significantly higher than 1/2 the market returns are not independent, indicating that the skilled traders create a (predictable) structure in the market. Similarly as with the variance, the Hurst exponent starts rising when the ratio of skilled traders is approaching one. In the limiting case when the market consists of skilled traders only, then the value of the Hurst exponent is close to 0.8 which indicates persistent process of prices,⁴ see Fig. 3.

³ We repeat each simulation 100 times and report arithmetic averages for all statistics.

⁴ As the persistence measure, we use Generalized Hurst exponent, for details see (Di Matteo, 2007), (Barunik & Kristoufek, 2010).

Kurtosis of returns is increasing when the proportion of skilled traders increases. When the number of skilled traders is greater than fifteen, the returns are leptokurtic, reaching very high values for cases with 35 and 40 skilled traders on the market (Table 1). Thus we can expect fatter tails in the distribution of returns when skilled traders dominate the market.

Mean of the trend parameter $g_{h,t}$, measured for all strategies on the simulated market, is increasing with the increasing number of skilled traders (see Fig. 4). The increasing mean of the trend parameter $g_{h,t}$ can be attributed to an inefficiency caused by growing number of trading strategies with similar prediction tools. In a consequence,

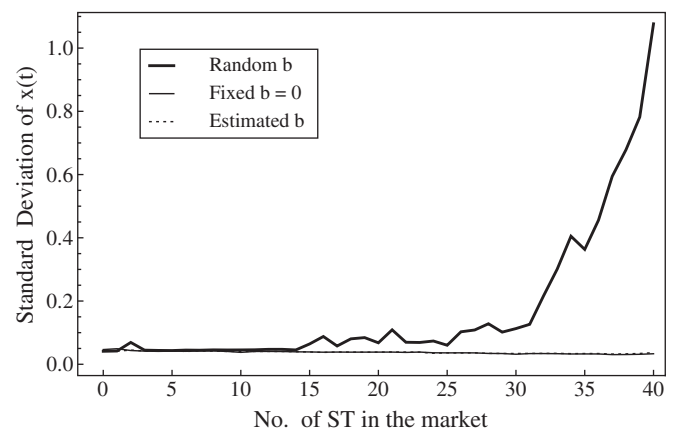


Fig. 2. Average standard deviation of simulated stock market returns x_t . Comparison of simulations with random bias b_h , fixed bias $b_h=0$ and estimated bias $b_{h,t}$.

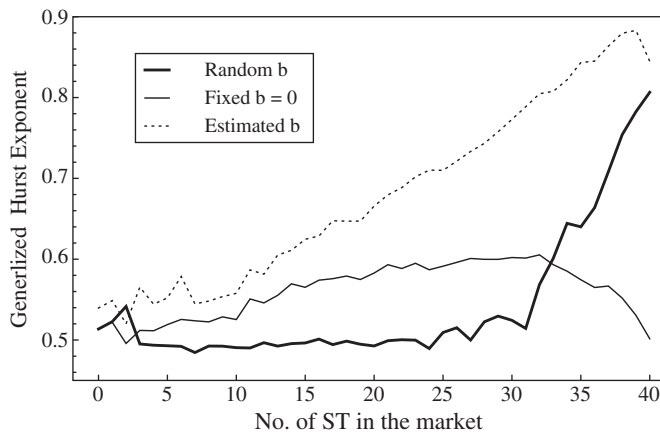


Fig. 3. Average Hurst Exponent. Comparison of simulations with random bias b_h , fixed bias $b_h = 0$, estimated bias $b_{h,t}$.

skilled traders exploit strong autoregressive component in the stock market leading to highly persistent market with frequent extreme values. In this type of market there are periods where the most profitable strategies are the ones with the high trend parameter. Such periodic behavior is in line with the increasing persistence.

We can think of the first type of traders as an extreme case and among the three cases we studied, this one constitutes the most unlikely situation on the market. However, it may be viewed as a simplification to situations where it is not clear what is the fundamental price on the market. The two other groups do not have such volatile bias, so we cannot observe such extreme market behavior.

4.2. Skilled traders without bias

Contrary to the previous setting, with increasing proportion of skilled traders in the model, volatility of the simulated market returns is slowly decreasing. This feature can be explained by decreasing heterogeneity in the model, which is a consequence of rising number of strategies with zero bias. Even in the limiting case of all skilled traders with no bias on the market, the variance is still smaller than in the case with stochastic predictors only. This situation is similar to the third case with skilled traders that can estimate the bias. This is due to the fact that the bias estimates are close to zero, so we can again observe the decrease of heterogeneity.

An interesting feature, also similar with the third case, is the changing mean of the trend parameter $g_{h,t}$ in the model. With increasing proportion of skilled traders, the mean is increasing slowly to positive values, but when number of skilled traders reaches 35, the mean value of the trend parameter drops close to zero, which means that skilled traders predict very low values of trend parameter $g_{h,t}$. In other words, we have market, where all participants are close to

Table 1 Descriptive statistics of the simulated returns x_t for the simulations with random bias b_h , with the fixed bias $b_h = 0$ and estimated bias $b_{h,t}$.

No. of ST	Random b_h		Fixed b_h		Estimated $b_{h,t}$	
	St. dev.	Kurtosis	St. dev.	Kurtosis	St. dev.	Kurtosis
0 ^a	0.0411	2.4080	0.0466	2.7670	0.0426	2.8050
5 ^a	0.0434	2.7300	0.0416	2.4970	0.0429	2.9000
10 ^a	0.0454	2.7280	0.0376	2.3260	0.0406	2.3760
15 ^a	0.0643	4.5940	0.0390	2.3770	0.0389	2.3950
20 ^a	0.0679	4.4690	0.0385	2.3140	0.0384	2.3420
25 ^a	0.0602	4.9590	0.0361	2.2240	0.0362	2.2540
30 ^a	0.1129	7.1930	0.0318	2.1420	0.0340	2.2320
35 ^a	0.3632	23.2300	0.0327	2.0930	0.0340	2.2420
40 ^a	1.0770	17.0700	0.0333	2.5630	0.0366	2.8750

^a Total number of traders in all simulations is 40

fundamentalists. For the number of skilled traders greater than 35, we observe negative value of the mean, indicating that the market consist, in average, of contrarians.

4.3. Skilled traders with estimated bias

The dynamics of the model output in the third setting with skilled traders who estimate both trend a bias is similar to the second case, except the evolution of the Hurst exponent. While average values of trend parameter, standard deviation and kurtosis of simulated series are similar to the second model with fixed bias, Fig. 3 depicts the evolution of the Hurst exponent for the third case as an almost linear increase of the market returns persistence, indicating formation of predictable (inefficient) market structure as we add more skilled traders into the model. Thus while estimates of $b_{h,t}$ are close to zero, they have significant impact on the market structure.

It is important to note that skilled traders have similar procedure for prediction, but differ in length of information set used for prediction.

5. Conclusion

In this work we extend the original heterogeneous agent model by introducing the skilled traders. Preliminary results show that presence of skilled traders in the model changes the behavior of the simulated market significantly. We increase the proportion of the skilled traders who are able to predict trend parameter in the market and study the changes in the simulated market behavior.

The main result is that with growing proportion of the skilled traders in the model with random bias, simulated market returns become more persistent and volatile. It is interesting to study the limit cases when there are only skilled traders present in the market. In the first case, when we consider only traders with random bias, market is still surprisingly heterogeneous, skilled traders are forecasting positive trend which results in high persistence of returns. In the case of skilled traders without bias in beliefs, all skilled traders become almost fundamentalists, while in the limit case when we have only skilled traders on the market, they become contrarians. As a result, market exhibits low heterogeneity and thus has very low volatility. When skilled traders are also allowed to forecast bias parameter, they forecast bias close to zero, but simulated returns show high persistence.

The most interesting result from simulations is that for all model settings the stock market changes its structure at some point with a growing number of skilled traders. It will be interesting to study this behavior with larger number of traders in simulations.

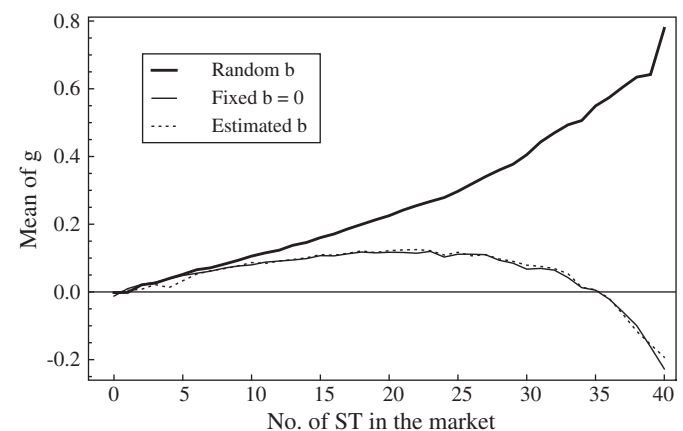


Fig. 4. Average parameter $g_{h,t}$. Comparison of simulations with random bias b_h , fixed bias $b_h = 0$, estimated bias $b_{h,t}$.

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