Conditioning in Evidence Theory from the Perspective of Multidimensional Models

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Abstract. Conditioning belongs to the most important topics of any theory dealing with uncertainty. From the viewpoint of construction of Bayesian-network-like multidimensional models it seems to be inevitable. In evidence theory, in contrary to the probabilistic framework, various rules were proposed to define conditional beliefs and/or plausibilities (or basic assignments) from joint ones. Two of them — Dempster's conditioning rule and focusing (more precisely their versions for variables) — have recently been studied in connection with the relationship between conditional independence and irrelevance and it has been shown, that for none of them conditional irrelevance is implied by conditional independence, which seems to be extremely inconvenient. Therefore we suggest a new conditioning rule for variables, which seems to be more promising from the viewpoint of conditional irrelevance, prove its correctness and also study the relationship between conditional independence and irrelevance based on this conditioning rule.

Keywords: Evidence theory, conditioning, multidimensional models, conditional independence, conditional irrelevance.

1 Introduction

The most widely used models managing uncertainty and multidimensionality are, at present, so-called *probabilistic graphical Markov models*. The problem of multidimensionality is solved in these models with the help of the notion of conditional independence, which enables factorization of a multidimensional probability distribution into small parts (marginals, conditionals or just factors).

It is easy to realize that if we need efficient methods for representation of probability distributions (requiring an exponential number of parameters), so much greater is the need of an efficient tool for representation of belief functions, which cannot be represented by a distribution (but only by a set function), and therefore the space requirements for its representation are superexponential. To solve this problem many conditional independence concepts have been proposed [3,8,11].

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However, another problem appears when one tries to construct an evidential counterpart of Bayesian network: problem of conditioning, which is not sufficiently solved in evidence theory. There exist many conditioning rules [6], but is any of them compatible with our [8] conditional independence concept? In [16] we dealt with two conditioning rules and studied the relationship between conditional irrelevance based on them and our notion of conditional independence [8], but the results were not satisfactory. Therefore, in this paper we propose a new conditioning rule which seems to be more promising.

The contribution is organized as follows. After a short overview of necessary terminology and notation (Section 2), in Section 3 we recall two conditioning rules and introduce the new one. In Section 4 the above-mentioned concept of conditional independence is recalled, a new concept of conditional irrelevance is presented and the relationship between conditional independence and conditional irrelevance is studied.

2 Basic Concepts

In this section we briefly recall basic concepts from evidence theory [12] concerning sets and set functions.

2.1 Set Projections and Extensions

For an index set $N = \{1, 2, \ldots, n\}$ let $\{X_i\}_{i \in N}$ be a system of variables, each X_i having its values in a finite set $\mathbf{X}_i$. In this paper we will deal with a *multidimensional frame of discernment*

$$\mathbf{X}_N = \mathbf{X}_1 \times \mathbf{X}_2 \times \ldots \times \mathbf{X}_n,$$

and its *subframes* (for $K \subseteq N$) $\mathbf{X}_K = \bigtimes_{i \in K} \mathbf{X}_i$.

When dealing with groups of variables on these subframes, X_K will denote a group of variables $\{X_i\}_{i \in K}$ throughout the paper.

A *projection* of $x = (x_1, x_2, \ldots, x_n) \in \mathbf{X}_N$ into $\mathbf{X}_K$ will be denoted $x^{\downarrow K}$, i.e. for $K = \{i_1, i_2, \ldots, i_k\}$

$$x^{\downarrow K} = (x_{i_1}, x_{i_2}, \ldots, x_{i_k}) \in \mathbf{X}_K.$$

Analogously, for $M \subset K \subseteq N$ and $A \subset \mathbf{X}_K$, $A^{\downarrow M}$ will denote a *projection* of A into $\mathbf{X}_M$:[1]

$$A^{\downarrow M} = \{y \in \mathbf{X}_M \mid \exists x \in A : y = x^{\downarrow M}\}.$$

In addition to the projection, in this text we will also need an inverse operation usually called cylindrical extension. The *cylindrical extension* of $A \subset \mathbf{X}_K$ to $\mathbf{X}_L$ ($K \subset L$) is the set

$$A^{\uparrow L} = \{x \in \mathbf{X}_L : x^{\downarrow K} \in A\}.$$

Clearly, $A^{\uparrow L} = A \times \mathbf{X}_{L \setminus K}$.

[1] Let us remark that we do not exclude situations where $M = \emptyset$. In this case $A^{\downarrow \emptyset} = \emptyset$.

A more complicated case is to make common extension of two sets, which will be called a join. By a *join*[2] of two sets $A \subseteq \mathbf{X}_K$ and $B \subseteq \mathbf{X}_L$ $(K, L \subseteq N)$ we will understand a set

$$A \bowtie B = \{x \in \mathbf{X}_{K\cup L} : x^{\downarrow K} \in A \ \ \& \ \ x^{\downarrow L} \in B\}.$$

Let us note that for any $C \subseteq \mathbf{X}_{K\cup L}$ naturally $C \subseteq C^{\downarrow K} \bowtie C^{\downarrow L}$, but generally $C \neq C^{\downarrow K} \bowtie C^{\downarrow L}$.

Let us also note that if K and L are disjoint, then the join of A and B is just their Cartesian product $A \bowtie B = A \times B$, if $K = L$ then $A \bowtie B = A \cap B$. If $K \cap L \neq \emptyset$ and $A^{\downarrow K\cap L} \cap B^{\downarrow K\cap L} = \emptyset$ then also $A \bowtie B = \emptyset$. Generally,

$$A \bowtie B = (A \times \mathbf{X}_{L\setminus K}) \cap (B \times \mathbf{X}_{K\setminus L}),$$

i.e. a join of two sets is the intersection of their cylindrical extensions.

2.2 Set Functions

In evidence theory [12] (or Dempster-Shafer theory) two dual measures are used to model uncertainty: belief and plausibility measures. Both of them can be defined with the help of another set function called a *basic (probability* or *belief) assignment* m on $\mathbf{X}_N$, i.e.,

$$m : \mathcal{P}(\mathbf{X}_N) \longrightarrow [0, 1],$$

where $\mathcal{P}(\mathbf{X}_N)$ is power set of $\mathbf{X}_N$ and $\sum_{A\subseteq \mathbf{X}_N} m(A) = 1$. Furthermore, we assume that $m(\emptyset) = 0$.

A set $A \in \mathcal{P}(\mathbf{X}_N)$ is a *focal element* if $m(A) > 0$. Let $\mathcal{F}$ denote the set of all focal elements, a focal element $A \in \mathcal{F}$ is called an $m-$*atom* if for any $B \subseteq A$ either $B = A$ or $B \notin \mathcal{F}$. In other words, an $m-$ atom is a setwise-minimal focal element.

Belief and *plausibility measures* are defined for any $A \subseteq \mathbf{X}_N$ by the equalities

$$Bel(A) = \sum_{B\subseteq A} m(B), \qquad Pl(A) = \sum_{B\cap A\neq\emptyset} m(B),$$

respectively.

For a basic assignment m on $\mathbf{X}_K$ and $M \subset K$, a *marginal basic assignment* of m on $\mathbf{X}_M$ is defined (for each $A \subseteq \mathbf{X}_M$) by the equality

$$m^{\downarrow M}(A) = \sum_{\substack{B\subseteq \mathbf{X}_K \\ B^{\downarrow M}=A}} m(B). \tag{1}$$

Analogously we will denote by $Bel^{\downarrow M}$ and $Pl^{\downarrow M}$ marginal belief and plausibility measures on $\mathbf{X}_M$, respectively.

[2] This term and notation are taken from the theory of relational databases [1].

3 Conditioning

Conditioning belongs to the most important topics of any theory dealing with uncertainty. From the viewpoint of the construction of Bayesian-network-like multidimensional models it seems to be inevitable.

3.1 Conditioning of Events

In evidence theory the "classical" conditioning rule is the so-called *Dempster's rule of conditioning* defined for any $\emptyset \neq A \subseteq \mathbf{X}_N$ and $B \subseteq \mathbf{X}_N$ such that $Pl(B) > 0$ by the formula

$$m(A|_D B) = \frac{\displaystyle\sum_{C \subseteq \mathbf{X}_N : C \cap B = A} m(C)}{Pl(B)}$$

and $m(\emptyset|_D B) = 0$.

From this formula one can immediately obtain:

$$\begin{aligned} Bel(A|_D B) &= \frac{Bel(A \cup B^C) - Bel(B^C)}{1 - Bel(B^C)}, \\ Pl(A|_D B) &= \frac{Pl(A \cap B)}{Pl(B)}. \end{aligned} \tag{2}$$

This is not the only possibility how to perform conditioning, another — in a way symmetric — conditioning rule is the following one called *focusing* defined for any $\emptyset \neq A \subseteq \mathbf{X}_N$ and $B \subseteq \mathbf{X}_N$ such that $Bel(B) > 0$ by the formula

$$m(A|_F B) = \begin{cases} \dfrac{m(A)}{Bel(B)} & \text{if} \quad A \subseteq B, \\ 0 & \text{otherwise.} \end{cases}$$

From the following two equalities one can see, in which sense are these two conditioning rules symmetric:

$$Bel(A|_F B) = \frac{Bel(A \cap B)}{Bel(B)}, \tag{3}$$

$$Pl(A|_F B) = \frac{Pl(A \cup B^C) - Pl(B^C)}{1 - Pl(B^C)}.$$

Formulae (2) and (3) are, in a way, evidential counterparts of conditioning in probabilistic framework. Let us note that the seemingly "natural" way of conditioning

$$m(A|_P B) = \frac{m(A \cap B)}{m(B)} \tag{4}$$

is not possible, since $m(A|_P B)$ need not be a basic assignment. It is caused by a simple fact that m, in contrary to Bel and Pl is not monotonous with respect to set inclusion. A simple counterexample can be found in [16].

Nevertheless, in Bayesian-networks-like multidimensional models we need conditional basic assignments (or beliefs or plausibilities) for variables. This problem will be in the center of our attention in the next subsection.

3.2 Conditional Variables

In [16] we presented the following two definitions of conditioning by variables, based on Dempster conditioning rule and focusing.

Let X_K and X_L ($K \cap L = \emptyset$) be two groups of variables with values in $\mathbf{X}_K$ and $\mathbf{X}_L$, respectively. Then the *conditional basic assignment according to Dempster's conditioning rule* of X_K given $X_L \in B \subseteq \mathbf{X}_L$ (for B such that $Pl^{\downarrow L}(B) > 0$) is defined as follows:

$$m_{X_K|_D X_L}(A|_D B) = \frac{\displaystyle\sum_{C \subseteq \mathbf{X}_{K\cup L}: (C \cap B^{\uparrow K\cup L})^{\downarrow K} = A} m(C)}{Pl^{\downarrow L}(B)}$$

for $A \neq \emptyset$ and $m_{K|L}(\emptyset|B) = 0$. Similarly, the *conditional basic assignment according to focusing* of X_K given $X_L \in B \subseteq \mathbf{X}_L$ (for B such that $Bel^{\downarrow L}(B) > 0$) is defined by the equality

$$m_{X_K|_F X_L}(A|_F B) = \frac{\displaystyle\sum_{C \subseteq \mathbf{X}_{K\cup L}: C \subseteq B^{\uparrow K\cup L} \& C^{\downarrow K} = A} m(C)}{Bel^{\downarrow L}(B)}$$

for any $A \neq \emptyset$ and $m_{K|_F L}(\emptyset|_F B) = 0$.

In the above-mentioned paper we proved that these definitions are correct, i.e. these rules define (generally different) basic assignments. Nevertheless, their usefulness for multidimensional models is rather questionable, as we shall see in Section 4.3.

Therefore, in this paper we propose a new conditioning rule which is, in a way, a generalization of (4). Although we said above, that it makes little sense for conditioning events, it is sensible in conditioning of variables, as expressed by Theorem 1 below. The above-mentioned problem of non-monotonicity is avoided, because a marginal basic assignment is always greater (or equal) to the joint one.

Definition 1. *Let X_K and X_L ($K \cap L = \emptyset$) be two groups of variables with values in $\mathbf{X}_K$ and $\mathbf{X}_L$, respectively. Then the conditional basic assignment of X_K given $X_L \in B \subseteq \mathbf{X}_L$ (for B such that $m^{\downarrow L}(B) > 0$) is defined as follows:*

$$m_{X_K|_P X_L}(A|_P B) = \frac{\displaystyle\sum_{\substack{C \subseteq \mathbf{X}_{K\cup L}: \\ C^{\downarrow K} = A \& C^{\downarrow L} = B}} m(C)}{m^{\downarrow L}(B)} \qquad (5)$$

for any $A \subseteq \mathbf{X}_K$.

Now, let us prove that this definition is makes sense.

Theorem 1. *The set function* $m_{X_K|_P X_L}$ *defined for any fixed* $B \subseteq \mathbf{X}_L$*, such that* $m^{\downarrow L}(B) > 0$ *by Definition 1 is a basic assignment on* $\mathbf{X}_K$*.*

Proof. Let $B \subseteq \mathbf{X}_L$ be such that $m^{\downarrow L}(B) > 0$. As nonnegativity of $m_{X_K|_p X_L}(A|_p B)$ for any $A \subseteq \mathbf{X}_K$ and the fact that $m_{X_K|_p X_L}(\emptyset|_p B) = 0$ follow directly from the definition, to prove that $m_{X_K|_p X_L}$ is a basic assignment it is enough to show that

$$\sum_{A \subseteq \mathbf{X}_K} m_{X_K|_P X_L}(A|_P B) = 1.$$

To check it, let us sum the values of the numerator in (5)

$$\begin{aligned}\sum_{A \subseteq \mathbf{X}_K} \sum_{\substack{C \subseteq \mathbf{X}_{K\cup L}: \\ C^{\downarrow K} = A \,\&\, C^{\downarrow L} = B}} m(C) &= \sum_{\substack{C \subseteq \mathbf{X}_{K\cup L} \\ C^{\downarrow L} = B}} m(C) \\ &= m^{\downarrow L}(B),\end{aligned}$$

where the last equality follows directly from (1). □

4 Conditional Independence and Irrelevance

Independence and irrelevance need not be (and usually are not) distinguished in the probabilistic framework, as they are almost equivalent to each other. Similarly, in possibilistic framework adopting De Cooman's measure-theoretical approach [7] (particularly his notion of almost everywhere equality) we proved that the analogous concepts are equivalent (for more details see [13]).

4.1 Independence

In evidence theory the most common notion of independence is that of random set independence [5].[3] It has already been proven [14,15] that it is also the only sensible one, as e.g. application of strong independence to two bodies of evidence may generally lead to a model which is beyond the framework of evidence theory.

Definition 2. Let m be a basic assignment on $\mathbf{X}_N$ and $K, L \subset N$ be disjoint. We say that groups of variables X_K and X_L are *independent with respect to a basic assignment* m (in notation $K \perp\!\!\!\perp L\ [m]$) if

$$m^{\downarrow K\cup L}(A) = m^{\downarrow K}(A^{\downarrow K}) \cdot m^{\downarrow L}(A^{\downarrow L})$$

for all $A \subseteq \mathbf{X}_{K\cup L}$ for which $A = A^{\downarrow K} \times A^{\downarrow L}$, and $m(A) = 0$ otherwise.

This notion can be generalized in various ways [3,11,15]; the concept of conditional non-interactivity from [3], based on conjunction combination rule, is used for construction of directed evidential networks in [4]. In this paper we will use

[3] Klir [9] calls it *non-interactivity.*

the concept introduced in [8,15], as we consider it more suitable: in contrary to other conditional independence concepts [3,11] it is *consistent with marginalization*, in other words, the multidimensional model of conditionally independent variables keeps the original marginals (for more details see [15]).

Definition 3. Let m be a basic assignment on $\mathbf{X}_N$ and $K, L, M \subset N$ be disjoint, $K \neq \emptyset \neq L$. We say that groups of variables X_K and X_L are *conditionally independent given X_M with respect to m* (and denote it by $K \perp\!\!\!\perp L|M\ [m]$), if the equality

$$m^{\downarrow K \cup L \cup M}(A) \cdot m^{\downarrow M}(A^{\downarrow M}) = m^{\downarrow K \cup M}(A^{\downarrow K \cup M}) \cdot m^{\downarrow L \cup M}(A^{\downarrow L \cup M})$$

holds for any $A \subseteq \mathbf{X}_{K \cup L \cup M}$ such that $A = A^{\downarrow K \cup M} \bowtie A^{\downarrow L \cup M}$, and $m(A) = 0$ otherwise.

It has been proven in [15] that this conditional independence concept satisfies so-called the semi-graphoid properties taken as reasonable to be valid for any conditional independence concept (see e.g. [10]) and it has been shown in which sense this conditional independence concept is superior to previously introduced ones [3,11].

4.2 Irrelevance

Irrelevance is usually considered to be a weaker notion than independence (see e.g. [5]). It expresses the fact that a new piece of evidence concerning one variable cannot influence the evidence concerning the other variable, in other words is irrelevant to it.

More formally: a group of variables X_L is *irrelevant* to X_K ($K \cap L = \emptyset$) if for any $B \subseteq \mathbf{X}_L$ such that $Pl^{\downarrow L}(B) > 0$ (or $Bel^{\downarrow L}(B) > 0$ or $m^{\downarrow L}(B) > 0$)

$$m_{X_K|X_L}(A|B) = m(A) \tag{6}$$

for any $A \subseteq \mathbf{X}_K$.[4]

It follows from the definition of irrelevance that it need not be a symmetric relation. Its symmetrized version is sometimes taken as a definition of independence. Let us note, that in the framework of evidence theory neither irrelevance based on Dempster conditioning rule nor that based on focusing even in cases when the relation is symmetric, imply independence, as can be seen from examples in [16].

Generalization of this notion to conditional irrelevance may be done as follows. A group of variables X_L is *conditionally irrelevant* to X_K given X_M (K, L, M disjoint, $K \neq \emptyset \neq L$) if

$$m_{X_K|X_L X_M}(A|B) = m_{X_K|X_M}(A|B^{\downarrow M}) \tag{7}$$

is satisfied for any $A \subseteq \mathbf{X}_K$ and $B \subseteq \mathbf{X}_{L \cup M}$.

[4] Let us note that somewhat weaker definition of irrelevance one can found in [2], where equality is substituted by proportionality. This notion has been later generalized using conjunctive combination rule [3].

Let us note that the conditioning in equalities (6) and (7) stands for an abstract conditioning rule (any of those mentioned in the previous section or some other [6]). Nevertheless, the validity of (6) and (7) may depend on the choice of the conditioning rule, as we showed in [16] — more precisely irrelevance with respect to one conditioning rule need not imply irrelevance with respect to the other.

4.3 Relationship between Independence and Irrelevance

As mentioned at the end of preceding section, different conditioning rules lead to different irrelevance concepts. Nevertheless, when studying the relationship between (conditional) independence and irrelevance based on Dempster conditioning rule and focusing we realized that they do not differ too much from each other, as suggested by the following summary.

For both conditioning rules:

- Irrelevance is implied by independence.
- Irrelevance does not imply independence.
- Irrelevance is not symmetric, in general.
- Even in case of symmetry it does not imply independence.
- Conditional independence does not imply conditional irrelevance.

The only difference between these conditioning rules is expressed by the following theorem proven in [16]

Theorem 2. *Let X_K and X_L be conditionally independent groups of variables given X_M under joint basic assignment m on $\mathbf{X}_{K\cup L\cup M}$ (K, L, M disjoint, $K \neq \emptyset \neq L$). Then*

$$m_{X_K|_F X_L X_M}(A|_F B) = m_{X_K|_F X_M}(A|_F B^{\downarrow M}) \tag{8}$$

for any $m^{\downarrow L\cup M}$-atom $B \subseteq \mathbf{X}_{L\cup M}$ such that $B^{\downarrow M}$ is $m^{\downarrow M}$-atom and $A \subseteq \mathbf{X}_K$.

From this point of view focusing seems to be slightly superior to Dempster conditioning rule, but still it is not satisfactory.

Now, let us make an analogous investigation for irrelevance based on the new conditioning rule introduced by Definition 1.

Theorem 3. *Let K, LM be disjoint subsets of N such that $K, L \neq \emptyset$. It X_K and X_L are independent given X_M (with respect to a joint basic assignment m defined on $\mathbf{X}_{K\cup L\cup M}$), then X_L is irrelevant to X_K given X_M under the conditioning rule given by Definition 1.*

Proof. Let X_K and X_L be conditionally independent given X_M then for any $A \subseteq \mathbf{X}_{K\cup L\cup M}$ such that $A = A^{\downarrow K\cup M} \bowtie A^{\downarrow L\cup M}$

$$m(A) \cdot m^{\downarrow M}(A^{\downarrow M}) = m^{\downarrow K\cup M}(A^{\downarrow K\cup M}) \cdot m^{\downarrow L\cup M}(A^{\downarrow L\cup M})$$

and $m(A) = 0$ otherwise. From this equality we immediately obtain that for all A such that $m^{\downarrow L}(A^{\downarrow L \cup M}) > 0$ (it implies that also $m^{\downarrow M}(A^{\downarrow M}) > 0$) equality

$$\frac{m(A)}{m^{\downarrow L \cup M}(A^{\downarrow L \cup M})} = \frac{m^{\downarrow K \cup M}(A^{\downarrow K \cup M})}{m^{\downarrow M}(A^{\downarrow M})}$$

is satisfied. Let us note that the left-hand side of the equality is equal to $m_{X_K|X_{L\cup M}}(A^{\downarrow K}|A^{\downarrow L \cup M})$, while the right-hand side equals $m_{X_K|X_M}(A^{\downarrow K}|A^{\downarrow L})$, which means, that X_L is irrelevant to X_K. □

The reverse implication is not valid, as can be seen from the next example.

Example 1. Let X_1 and X_2 be two binary variables (with values in $\mathbf{X}_i = \{a_i, \bar{a}_i\}$) with joint basic assignment m defined as follows:

$$\begin{aligned} m(\{(a_1, a_2)\}) &= \frac{1}{4}, \\ m(\{a_1\} \times \mathbf{X}_2) &= \frac{1}{4}, \\ m(\mathbf{X}_1 \times \{a_2\}) &= \frac{1}{4}, \\ m(\mathbf{X}_1 \times \mathbf{X}_2 \setminus \{(\bar{a}_1, \bar{a}_2)\}) &= \frac{1}{4}. \end{aligned}$$

From these values one can obtain

$$m^{\downarrow 2}(\{a_2\}) = m^{\downarrow 2}(\mathbf{X}_2) = \frac{1}{2}.$$

Evidently, it is not possible to condition by $\{\bar{a}_2\}$ and we have to confine ourselves to conditioning by $\{a_2\}$:

$$\begin{aligned} m_{X_1|_P X_2}(\{a_1\}|_P\{a_2\}) &= \tfrac{1}{2} = m^{\downarrow 1}(\{a_1\}), \\ m_{X_1|_P X_2}(\{\bar{a}_1\}|_P\{a_2\}) &= 0 = m^{\downarrow 1}(\{\bar{a}_1\}), \\ m_{X_1|_P X_2}(\mathbf{X}_1|_P\{a_2\}) &= \tfrac{1}{2} = m^{\downarrow 1}(\mathbf{X}_1), \end{aligned}$$

i.e. X_2 is irrelevant to X_1,[5] but X_1 and X_2 are not independent, as the focal element $\mathbf{X}_1 \times \mathbf{X}_2 \setminus \{(\bar{a}_1, \bar{a}_2)\}$ is not a rectangle. ♢

Theorem 3 and Example 1 express the expected property: conditional independence is stronger than conditional irrelevance. Nevertheless, it is evident from the example, that irrelevance (with respect to this conditioning rule) does not imply independence even in case of symmetry.

5 Conclusions

We introduced a new conditioning rule for variables in evidence theory, proved its correctness and showed that conditional irrelevance based on this conditioning rule is implied by recently introduced conditional independence. From this

[5] Since we can interchange X_1 and X_2, it is evident that also X_1 is irrelevant to X_2.

viewpoint, it is superior to previously suggested conditioning rules. It will enable us to decompose multidimensional models in evidential framework into conditional basic assignments in a way analogous to Bayesian networks in probabilistic framework.

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