On Three Conditioning Rules in Evidence Theory

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Abstract. In evidence theory various rules were proposed to define conditional beliefs and/or plausibilities (or basic assignments). However, there exist no generally accepted criteria along which these rules can be compared. In this paper we concentrate to three of them (Dempster's conditioning rule, focusing and the approach based on lower and upper envelopes of sets of conditional probabilities) to study their mutual relationship. A new conditional rule for variables is presented afterwards and its correctness is proven.

Keywords: evidence theory, conditioning rules

1 Introduction

Conditioning belongs to the most important features of any model of uncertainty and therefore it is quite natural that it has also been studied within evidence theory from its very beginning. In evidence theory, in contrary to the probabilistic framework (and similarly to possibilistic one), various rules were proposed to define conditional beliefs and/or plausibilities (or conditional basic assignments) [1–3, 5]. However, there exist no generally accepted criteria along which these rules can be compared. In this paper we will concentrate to three of them: Dempster's conditioning rule (as it is the "classical" one), focusing (as it is, in a way, symmetric to Dempster's conditioning rule) and the approach based on lower and upper envelopes of set of probabilities (as it has a nice probabilistic interpretation).

From the viewpoint of multidimensional Bayesian-networks-like models the generalization of the conditioning rules from events to variables is inevitable. Conditioning rule is not only used to define conditional basic assignments (or conditional beliefs/plausibilities), but also conditional irrelevance based on it plays a principal role in this kind of models. In [8] we introduced generalizations of both Dempster's conditioning rule and focusing, however none of them seems to be appropriate for definition of conditional irrelevance. More precisely, conditional irrelevance based on these rules is not implied by conditional independence, which is not only unusual, but also weakens power of these models in evidence theory. Therefore the need for another conditioning rule for variables appeared. In this paper we introduce it and prove its correctness. Its relationship to conditional independence [6, 7] is behind the scope of this paper and will be studied in the future.

The paper is organized as follows. After a brief introduction of necessary concepts and notations (Section 2) in Section 3 mutual relationship among above mentioned conditioning rule for events is studied, while in Section 4 a new conditioning rule for variables is introduced and its correctness is proven.

2 Basic Concepts

In this section we will briefly recall basic concepts from evidence theory [5] concerning sets, set functions and marginalization.

For an index set $N = \{1, 2, \ldots, n\}$ let $\{X_i\}_{i \in N}$ be a system of variables, each X_i having its values in a finite set $\mathbf{X}_i$. In this paper we will deal with *multidimensional frame of discernment*

$$\mathbf{X}_N = \mathbf{X}_1 \times \mathbf{X}_2 \times \ldots \times \mathbf{X}_n,$$

and its *subframes* (for $K \subseteq N$)

$$\mathbf{X}_K = \times_{i \in K} \mathbf{X}_i.$$

When dealing with groups of variables on these subframes, X_K will denote a group of variables $\{X_i\}_{i \in K}$ throughout the paper.

A *projection* of $x = (x_1, x_2, \ldots, x_n) \in \mathbf{X}_N$ into $\mathbf{X}_K$ will be denoted $x^{\downarrow K}$, i.e. for $K = \{i_1, i_2, \ldots, i_k\}$

$$x^{\downarrow K} = (x_{i_1}, x_{i_2}, \ldots, x_{i_k}) \in \mathbf{X}_K.$$

Analogously, for $M \subset K \subseteq N$ and $A \subset \mathbf{X}_K$, $A^{\downarrow M}$ will denote a *projection* of A into $\mathbf{X}_M$:[1]

$$A^{\downarrow M} = \{y \in \mathbf{X}_M \mid \exists x \in A : y = x^{\downarrow M}\}.$$

In evidence theory [5] (or Dempster-Shafer theory) two measures are used to model the uncertainty: belief and plausibility measures. Both of them can be defined with the help of another set function called a *basic (probability* or *belief) assignment* m on $\mathbf{X}_N$, i.e.,

$$m : \mathcal{P}(\mathbf{X}_N) \longrightarrow [0, 1],$$

where $\mathcal{P}(\mathbf{X}_N)$ is power set of $\mathbf{X}_N$ and

$$\sum_{A \subseteq \mathbf{X}_N} m(A) = 1.$$

Furthermore, we assume that $m(\emptyset) = 0$. A set $A \in \mathcal{P}(\mathbf{X}_N)$ is a *focal element* if $m(A) > 0$, therefore, to characterize a basic assignment we may confine ourselves to focal elements and their values.

Belief and *plausibility measures* are defined for any $A \subseteq \mathbf{X}_N$ by the equalities

$$Bel(A) = \sum_{B \subseteq A} m(B), \tag{1}$$

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B), \tag{2}$$

respectively.

It is well-known (and evident from these formulae) that for any $A \in \mathcal{P}(\mathbf{X}_N)$

$$Bel(A) \leq Pl(A), \tag{3}$$

$$Pl(A) = 1 - Bel(A^C), \tag{4}$$

where A^C is the set complement of $A \in \mathcal{P}(\mathbf{X}_N)$. Furthermore, basic assignment can be computed from belief function via Möbius inversion:

$$m(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} Bel(B), \tag{5}$$

i.e. any of these three functions is sufficient to define values of the remaining two.

When dealing with multidimensional models, marginalization plays an important role. For a basic assignment m on $\mathbf{X}_K$ and $M \subset K$, a *marginal basic assignment* of m on $\mathbf{X}_M$ is defined (for each $A \subseteq \mathbf{X}_M$):

$$m^{\downarrow M}(A) = \sum_{\substack{B \subseteq \mathbf{X}_K \\ B^{\downarrow M} = A}} m(B). \tag{6}$$

3 Conditioning of Events

Conditioning belongs to the most important topics of any theory dealing with uncertainty. From the viewpoint of construction of Bayesian-network-like multidimensional models it seems to be inevitable. In this section we will confine ourselves to conditioning of events, in the next one we introduce the generalization of (some of) them to conditional variables.

[1] Let us remark that we do not exclude situations when $M = \emptyset$. In this case $A^{\downarrow \emptyset} = \emptyset$.

3.1 Dempster's Conditioning Rule and Focusing

In evidence theory the "classical" conditioning rule is so-called *Dempster's rule of conditioning* defined for any $\emptyset \neq A \subseteq \mathbf{X}_N$ and $B \subseteq \mathbf{X}_N$ such that $Pl(B) > 0$ by the formula

$$m(A|B) = \frac{\sum_{C \subseteq \mathbf{X}_N : C \cap B = A} m(C)}{Pl(B)} \tag{7}$$

and $m(\emptyset|B) = 0$.

Let us note that formula (7) is a special case of Dempster's rule of combination, when combining basic assignment m with another m_B such that $m_B(B) = 1$.

From this formula one can immediately obtain:

$$Bel(A|B) = \frac{Bel(A \cup B^C) - Bel(B^C)}{1 - Bel(B^C)},$$
$$Pl(A|B) = \frac{Pl(A \cap B)}{Pl(B)}.$$

This is not the only possibility how to make conditioning, another — in a way symmetric — conditioning rule is the following one called *focusing* defined for any $\emptyset \neq A \subseteq \mathbf{X}_N$ and $B \subseteq \mathbf{X}_N$ such that $Bel(B) > 0$ by the formula

$$m(A||B) = \begin{cases} \dfrac{m(A)}{Bel(B)} & \text{if} \quad A \subseteq B, \\ 0 & \text{otherwise.} \end{cases}$$

From the following two equalities one can see, in which sense are these two conditioning rules symmetric:

$$Bel(A||B) = \frac{Bel(A \cap B)}{Bel(B)},$$
$$Pl(A||B) = \frac{Pl(A \cup B^C) - Pl(B^C)}{1 - Pl(B^C)}.$$

These rules are based on different philosophies. Focusing assigns positive values only to those elements which are subsets of B, while Dempster's rule of conditioning to those which have nonempty intersection with it.

It is evident, that focusing is applicable in less cases than Dempster's rule, because of relation (3), hence from this point of view the latter seems to be more advantageous.

On the other hand, from the computational viewpoint the latter is more suitable, as it produces less focal elements (and in any of them a bigger "mass" is contained; cf. also Example 2). Due to this fact it may seem that focusing produces bigger intervals than Dempster's rule, more precisely that

$$Bel(A||B) \leq Bel(A|B) \leq Pl(A|B) \leq Pl(A||B). \tag{8}$$

It can also be seen from the following example demonstrating the difference between Dempster's rule of conditioning and focusing.

Example 1. Let $\mathbf{X} = \{a, b, c\}$ and m on $\mathbf{X}$ be defined as follows:

$$m(\{a\}) = m(\{b\}) = m(\{c\}) = \frac{1}{6}, \tag{9}$$
$$m(\{a, b\}) = m(\{b, c\}) = m(\{a, c\}) = \frac{1}{6}.$$

Let $A = \{b\}$ and $B = \{b, c\}$. First let us compute belief and plausibility of conditioning set $\{b, c\}$.

$$Bel(\{b, c\}) = \frac{1}{2} \quad \text{and} \quad Pl(\{b, c\}) = \frac{5}{6}.$$

Then we have

$$m(\{b\}|\{b,c\}) = \frac{m(\{b\}) + m(\{a,b\})}{Pl(\{b,c\})} = \frac{2}{5},$$
$$m(\{c\}|\{b,c\}) = \frac{m(\{c\}) + m(\{a,c\})}{Pl(\{b,c\})} = \frac{2}{5},$$
$$m(\{b,c\}|\{b,c\}) = \frac{m(\{b,c\})}{Pl(\{b,c\})} = \frac{1}{5},$$

as $\{a,b\} \cap \{b,c\} = \{b\}$ and $\{a,c\} \cap \{b,c\} = \{c\}$, while

$$m(\{b\}||\{b,c\}) = \frac{m(\{b\})}{Bel(\{b,c\})} = \frac{1}{3},$$
$$m(\{c\}||\{b,c\}) = \frac{m(\{c\})}{Bel(\{b,c\})} = \frac{1}{3},$$
$$m(\{b,c\}||\{b,c\}) = \frac{m(\{b,c\})}{Bel(\{b,c\})} = \frac{1}{3},$$

as $\{b\}$ and $\{c\}$ are the only proper subsets of $\{b,c\}$.

Using these results we obtain that

$$Bel(\{b\}|\{b,c\}) = \frac{2}{5} \quad \text{and} \quad Pl(\{b\}|\{b,c\}) = \frac{3}{5},$$
$$Bel(\{c\}|\{b,c\}) = \frac{2}{5} \quad \text{and} \quad Pl(\{b\}|\{b,c\}) = \frac{3}{5},$$
$$Bel(\{b\}||\{b,c\}) = \frac{1}{3} \quad \text{and} \quad Pl(\{b\}||\{b,c\}) = \frac{2}{3},$$
$$Bel(\{c\}||\{b,c\}) = \frac{1}{3} \quad \text{and} \quad Pl(\{c\}||\{b,c\}) = \frac{2}{3},$$

as expected. $\diamond$

Nevertheless, inequality (8) does not hold in general, as can be seen from the following simple example.

Example 2. Let $\mathbf{X} = \{a,b,c\}$ and m on $\mathbf{X}$ be defined as follows:

$$m(\{a\}) = m(\{b\}) = m(\{c\}) = \frac{1}{4}, \qquad (10)$$
$$m(\{a,b\}) = m(\mathbf{X}) = \frac{1}{8}.$$

To show that (8) need not hold let us compute

$$Bel(\{b,c\}) = \frac{1}{2} \quad \text{and} \quad Pl(\{b,c\}) = \frac{3}{4}.$$

Then we have

$$m(\{b\}|\{b,c\}) = \frac{m(\{b\}) + m(\{a,b\})}{Pl(\{b,c\})} = \frac{1}{2},$$
$$m(\{c\}|\{b,c\}) = \frac{m(\{c\})}{Pl(\{b,c\})} = \frac{1}{3},$$
$$m(\{b,c\}|\{b,c\}) = \frac{m(\mathbf{X})}{Pl(\{b,c\})} = \frac{1}{6},$$

as $\{a,b\} \cap \{b,c\} = \{b\}$ and $\mathbf{X} \cap \{b,c\} = \{b,c\}$, while

$$m(\{b\}||\{b,c\}) = \frac{m(\{b\})}{Bel(\{b,c\})} = \frac{1}{2},$$
$$m(\{c\}||\{b,c\}) = \frac{m(\{c\})}{Bel(\{b,c\})} = \frac{1}{2},$$

as $\{b\}$ and $\{c\}$ are the only subsets of $\{b.c\}$. From these conditional basic assignments we obtain:

$$\begin{aligned} Bel(\{b\}|\{b,c\}) &= \frac{1}{2} \quad \text{and} \quad Pl(\{b\}|\{b,c\}) = \frac{2}{3}, \\ Bel(\{c\}|\{b,c\}) &= \frac{1}{3} \quad \text{and} \quad Pl(\{b\}|\{b,c\}) = \frac{1}{2}, \\ Bel(\{b\}||\{b,c\}) &= \frac{1}{2} \quad \text{and} \quad Pl(\{b\}||\{b,c\}) = \frac{1}{2}, \\ Bel(\{c\}||\{b,c\}) &= \frac{1}{2} \quad \text{and} \quad Pl(\{c\}||\{b,c\}) = \frac{1}{2}, \end{aligned}$$

i.e. inequality (8) is not satisfied. Furthermore, in this case focusing produces precise conditionals. ◇

3.2 Lower and Upper Envelopes

Another conditioning rule, based on lower and upper envelopes of sets probabilities, can be found in [3]. Unfortunately, this conditioning rule does not exist in a closed form, but must be computed from beliefs and plausibilities

$$\begin{aligned} Bel(A|_eB) &= \frac{Bel(A\cap B)}{Bel(A\cap B)+Pl(A^C\cap B)}, \\ Pl(A|_eB) &= \frac{Pl(A\cap B)}{Pl(A\cap B)+Bel(A^C\cap B)} \end{aligned}$$

via Möbius inverse (5). From this point of view it does not seem to be very useful either for generalization to conditional variables or even for a definition of conditional irrelevance.

The following theorem reveals the relationship between this way of conditioning and the preceding two.

Theorem 1. *Let $A, B \subseteq \mathbf{X}$ be two events. Then:*

(i)

$$Bel(A|_eB) \le Bel(A|B) \le Pl(A|B) \le Pl(A|_eB), \tag{11}$$

(ii) if furthermore $A \subseteq B$ then also

$$Bel(A|_eB) \le Bel(A||B) \le Pl(A||B) \le Pl(A|_eB). \tag{12}$$

Proof. (i) First let us prove the last inequality:

$$\frac{Pl(A\cap B)}{Pl(B)} \le \frac{Pl(A\cap B)}{Pl(A\cap B)+Bel(A^C\cap B)} \tag{13}$$

To do so, it is enough to prove that

$$Pl(B) \ge Pl(A\cap B)+Bel(A^C\cap B), \tag{14}$$

as the numerators on both sides of (13) are the same. Let us rewrite the right-hand side of this inequality by formulae (2) and (1) and we will obtain

$$\sum_{C:C\cap(A\cap B)\neq\emptyset} m(C) + \sum_{C\subseteq A^C\cap B} m(C). \tag{15}$$

If $C\cap(A\cap B)\neq\emptyset$, then $C \not\subseteq (A^C\cap B)$, hence the two sums in (15) are over different elements. Since

$$(\{C : C\cap(A\cap B)\neq\emptyset\} \cup \{C \subseteq (A^C\cap B)\}) \subseteq \{C : C\cap B\neq\emptyset\}$$

it follows that (15) is not greater than

$$\sum_{C:C\cap B\neq\emptyset} m(C) = Pl(B),$$

i.e. (14) is satisfied.

The first inequality in (11) follows from the last one due to (4) and the second one is generally valid (cf. (3)).

(ii) Here we will first prove the first inequality:

$$\frac{Bel(A \cap B)}{Bel(A \cap B) + Pl(A^C \cap B)} \leq \frac{Bel(A \cap B)}{Bel(B)}. \tag{16}$$

Analogous to the previous case, it is enough to prove that

$$Bel(B) \leq Bel(A \cap B) + Pl(A^C \cap B), \tag{17}$$

as the numerators on both sides of (16) are the same. Let us rewrite the right-hand side of this inequality by formulae (1) and (2) and we will obtain

$$\sum_{C \subseteq A \cap B} m(C) + \sum_{C: C \cap (A^C \cap B) \neq \emptyset} m(C). \tag{18}$$

To prove that (18) is not less than

$$Bel(B) = \sum_{C \subseteq B} m(C),$$

i.e. that (17) is satisfied, it is enough to realize that

$$(\{C : C \subseteq (A \cap B)\} \cup \{C \cup (A^C \cap B) \neq \emptyset\}) \supseteq \{C : C \subseteq B\}.$$

The second inequality in (12) follows directly from (3) and the last one due to (4) from the first one. □

Let us present an example demonstrating the difference in imprecision of mentioned conditioning rules. For this purpose let let us recall Example 1.

Example 1. (Continued) Let us recall that $A = \{b\}$ and $B = \{b, c\}$. From (9) we can easily compute

$$Bel(\{b\}) = \frac{1}{6} \quad \text{and} \quad Pl(\{b\}) = \frac{1}{2},$$
$$Bel(\{c\}) = \frac{1}{6} \quad \text{and} \quad Pl(\{c\}) = \frac{1}{2}$$

and then we obtain that

$$Bel(\{b\}|_e\{b,c\}) = \frac{Bel(\{b\})}{Bel(\{b\}) + Pl(\{c\})} = \frac{1}{4},$$
$$Bel(\{c\}|_e\{b,c\}) = \frac{Bel(\{c\})}{Bel(\{c\}) + Pl(\{b\})} = \frac{1}{4},$$
$$Pl(\{b\}|_e\{b,c\}) = \frac{Pl(\{b\})}{Pl(\{b\}) + Bel(\{c\})} = \frac{3}{4},$$
$$Pl(\{c\}|_e\{b,c\}) = \frac{Pl(\{c\})}{Pl(\{c\}) + Bel(\{b\})} = \frac{3}{4}.$$

Therefore, in this example

$$Bel(\{x\}|_e\{b,c\}) < Bel(\{x\}||\{b,c\}) < Bel(\{x\}|\{b,c\}) < Pl(\{x\}|\{b,c\}) < Pl(\{x\}||\{b,c\}) < Pl(\{x\}|_e\{b,c\}),$$

for any $x = b, c$, as expected. ◇

3.3 Other Possibilities

Formulae (8) and (11) are, in a way, evidential counterparts of conditioning in probabilistic framework. Let us note that seemingly "natural" way of conditioning

$$m(A|_p B) = \frac{m(A \cap B)}{m(B)} \tag{19}$$

is not possible, since $m(A|_p B)$ need not be a basic assignment, as can be seen from the following simple example. It is caused by a simple fact that m, in contrary to Bel and Pl is not monotonous with respect to set inclusion.

Example 2. (Continued) For above define basic assignment (10) using (19) one would obtain

$$m(\{a\}|_p\{a,b\}) = m(\{b\}|_p\{a,b\}) = 2,$$

which is out of the framework of evidence theory. $\diamondsuit$

Nevertheless, rather than in conditional beliefs and plausibilities of events we are interested in conditioning by variables, therefore we will not study other conditioning rules ([2]), because their generalization for variables is rather questionable. The problem of a proper definition of conditioning rule for variables will be in the center of our attention in the next section.

4 Conditioning of Variables

In [8] we presented the following two conditioning rules for variables — generalizations of Dempster's conditioning rule and focusing.

Definition 1. Let X_K and X_L ($K \cap L = \emptyset$) be two groups of variables with values in $\mathbf{X}_K$ and $\mathbf{X}_L$, respectively. Then the *conditional basic assignment according to Dempster's conditioning rule* of X_K given $X_L \in B \subseteq \mathbf{X}_L$ (for B such that $Pl(B) > 0$) is defined as follows:

$$m_{X_K|X_L}(A|B) = \frac{\displaystyle\sum_{\substack{C \subseteq \mathbf{X}_{K\cup L}: \\ (C \cap B^{\uparrow K\cup L})^{\downarrow K} = A}} m(C)}{Pl(B)}$$

for $A \neq \emptyset$ and $m_{K|L}(\emptyset|B) = 0$. Similarly, the *conditional basic assignment according to focusing* of X_K given $X_L \in B \subseteq \mathbf{X}_L$ (for B such that $Bel(B) > 0$) is defined by the equality

$$m_{X_K||X_L}(A||B) = \frac{\displaystyle\sum_{\substack{C \subseteq \mathbf{X}_{K\cup L}: \\ C \subseteq B^{\uparrow K\cup L} \& C^{\downarrow K} = A}} m(C)}{Bel(B)}$$

for any $A \neq \emptyset$ and $m_{K||L}(\emptyset||B) = 0$.

Although we proved [8] that both of these conditioning rules are correct, we simultaneously showed that none of them is appropriate for definition of conditional irrelevance, as none of them is implied by conditional independence [4,6]. This fact substantially decreases possibility of construction and application of Bayesian-network-like models in the framework of evidence theory. Therefore we propose the following one, although its pre-image for conditioning of events is senseless (cf. Example 2).

Definition 2. *Let X_K and X_L ($K \cap L = \emptyset$) be two groups of variables with values in $\mathbf{X}_K$ and $\mathbf{X}_L$, respectively. Then the conditional basic assignment of X_K given $X_L \in B \subseteq \mathbf{X}_L$ (for B such that $m(B) > 0$) is defined as follows:*

$$m_{X_K|_pX_L}(A|_pB) = \frac{\displaystyle\sum_{\substack{C \subseteq \mathbf{X}_{K\cup L}: \\ C^{\downarrow L} = B \& C^{\downarrow K} = A}} m(C)}{m(B)} \tag{20}$$

for any $A \subseteq \mathbf{X}_K$.

It is evident that the conditioning is defined only for focal elements of the marginal basic assignment, but we do not consider it a substantial disadvantage, because all the information about a basic assignment is concentrated in focal elements.

Now, let us prove that this definition is correct.

Theorem 2. *Set function $m_{X_K|_pX_L}$ defined for any fixed $B \subseteq \mathbf{X}_L$, such that $m(B) > 0$ by Definition 2 is a basic assignment on $\mathbf{X}_K$.*

Proof. Let $B \subseteq \mathbf{X}_L$ be such that $m(B) > 0$. As nonnegativity of $m_{X_K|_p X_L}(A|_p B)$ for any $A \subseteq \mathbf{X}_K$ and the fact that $m_{X_K|_p X_L}(\emptyset|_p B) = 0$ follow directly from the definition, to prove that $m_{X_K|_p X_L}$ is a basic assignment it is enough to show that

$$\sum_{A \subseteq \mathbf{X}_K} m_{X_K|_p X_L}(A|_p B) = 1.$$

To check it, let us sum the values of the numerator in (20)

$$\sum_{A \subseteq \mathbf{X}_K} \sum_{\substack{C \subseteq \mathbf{X}_{K \cup L}: \\ C^{\downarrow K} = A \,\&\, C^{\downarrow L} = B}} m(C) = \sum_{\substack{C \subseteq \mathbf{X}_{K \cup L} \\ C^{\downarrow L} = B}} m(C)$$

$$= m^{\downarrow L}(B),$$

where the last equality follows directly from (6). □

5 Conclusions

The aim of this paper was to clarify the relationship among three conditioning rules (for events) in evidence theory. We proved that conditioning based on upper and lower envelopes of set of probabilities covers both the intervals produced by Dempster's conditioning rule and those produced by focusing. Furthermore we showed that there is no inclusion between Dempster's rule of conditioning and focusing.

We also suggested a new conditioning rule for variables, which seems to be more promising from the viewpoint of conditional irrelevance (and its relationship to conditional independence [6, 7]), and proved its correctness. It remains to study the relationship between conditional independence and irrelevance based on this conditioning rule, which will be the most important topic of our future research.

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