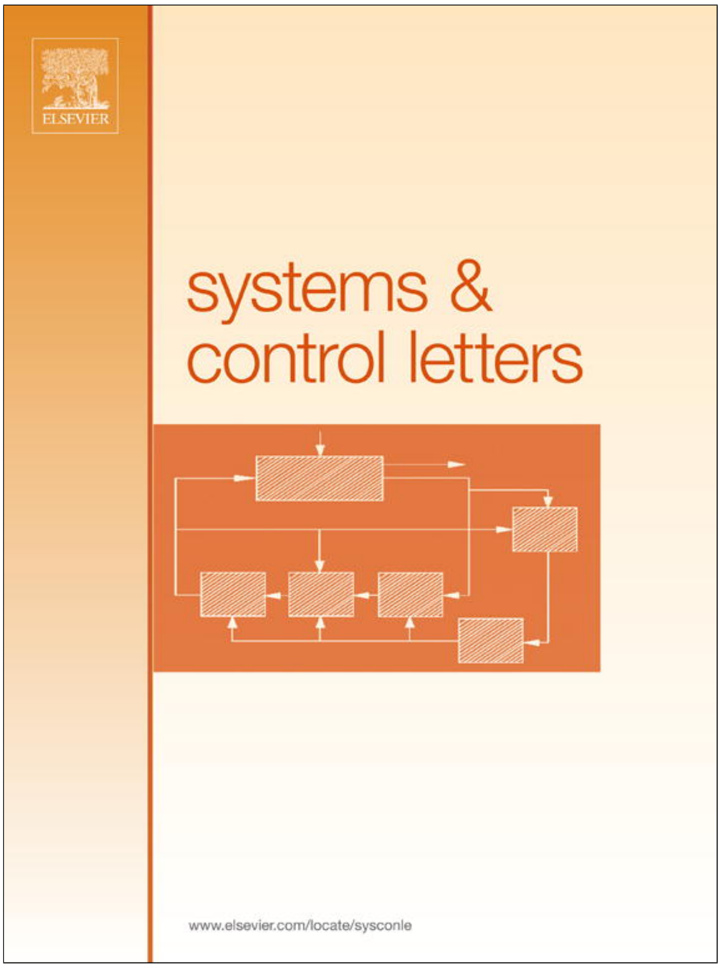
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Control of discrete linear repetitive processes using strong practical stability and $\mathcal{H}_\infty$ disturbance attenuation

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ABSTRACT

Repetitive processes are a distinct class of 2*D* systems of both theoretical and practical interest. This paper develops algorithms for control law design to ensure stabilization and a prescribed level of disturbance attenuation as measured by an $\mathcal{H}_\infty$ norm.

1. Introduction

Repetitive processes make a series of sweeps, termed passes, through a set of dynamics defined over a finite interval known as the pass duration or length. At the end of each pass the process is reset to the starting position and the next pass begins. The output of a repetitive process is known as the pass profile and the unique feature of these processes is that the previous pass profile acts as a forcing function on, and hence contributes to, the dynamics produced on the current pass [1]. The result can be oscillations in the sequence of pass profiles that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition, let $\alpha < \infty$ be a natural number that denote the pass length. Then in a repetitive process the pass profile $y_k(p),\ 0 \le p \le \alpha - 1$, where $k \in \mathbb{N}$, $p \in \mathbb{N}$ and $\mathbb{N}$ is the set of natural numbers, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(p),\ 0 \le p \le \alpha - 1$.

Repetitive processes have their origins in the coal mining and metal rolling industries [1]. Also there are areas where adopting a repetitive process setting for analysis can be used to effect. Examples include classes of iterative learning control schemes with experimental verification [2]. In the coal mining example, the pass profile is the height of the stone/coal interface above some datum line and the cutting machine rests on the previous pass profile during the production of the current one. The result can be severe undulations/oscillations in the pass profiles that have to be removed after only a few passes and the downtime involved is lost production.

Recognizing the unique control problem, the stability theory for linear repetitive processes is of the bounded-input bounded-output (BIBO) form, that is, a bounded initial pass profile is required to produce a bounded sequence of pass profiles. The stability theory [1] has been developed in terms of an abstract model in a Banach space setting that includes many examples as special cases. Asymptotic stability in the pass-to-pass direction demands this BIBO stability property over the finite and fixed pass length, that is, over $(k, p) \in [0, \infty] \times [0, \alpha]$, and if this property holds then the sequence of pass profiles generated converges strongly to the limit profile that in some cases, including the processes considered here, is described by a standard discrete linear systems state-space model. The finite pass length does, however, mean that this limit profile can have unacceptable along-the-pass dynamics, including the case when it is unstable.

The most obvious way to exclude this last possibility is to demand the BIBO stability property uniformly with respect to the pass length, that is, over $(k, p) \in [0, \infty] \times [0, \infty]$, and this is termed stability along-the-pass. Moreover, for the processes considered here, the abstract model based stability conditions can

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be transformed into conditions that can be checked by standard discrete linear systems stability tests. In the case of asymptotic stability in the pass-to-pass direction, the condition reduces to computing the eigenvalues of a matrix and for stability along-the-pass the extra conditions are an eigenvalue computation for a matrix plus, in the single-input single-output (SISO) case for simplicity, a condition interpreted in terms of the frequency content of the initial pass profile, that is, all of its frequency content must be attenuated from pass-to-pass. In control law design terms this is a very stringent condition to meet and in recent work [3] strong practical stability has been developed as an alternative where in the SISO case the frequency attenuation is no longer imposed over the complete frequency range. The approach used is to relax the requirement for the BIBO property to hold when $k \to \infty$ and $p \to \infty$ simultaneously, where this combination of k and p cannot arise in a physical application.

Previous work [3] has shown that for discrete linear repetitive processes, necessary and sufficient conditions for this property can be formulated in terms of Linear Matrix Inequalities (LMIs) that also give algorithms for the design of a stabilizing control law. In this paper we develop substantial new LMI based results to design control laws for strong practical stability with $\mathcal{H}_\infty$ disturbance attenuation.

Throughout this paper, the null and identity matrices with the required dimensions are denoted by 0 and I respectively and $\rho(X)$ denotes a spectral radius of the matrix or operator X. Moreover, $M > 0$ (<0) denotes a real symmetric positive (negative) definite matrix.

2. Background

The state-space model of a discrete linear repetitive process [1] has the following form over $0 \le p \le \alpha - 1,\ k \ge 0$,

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p), \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p), \end{aligned} \tag{1}$$

where $\alpha < \infty$ denotes the number of samples along the pass and on pass k $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile vector, and $u_k(p) \in \mathbb{R}^r$ is the vector of control inputs. The boundary conditions are $x_{k+1}(0) = d_{k+1},\ k \ge 0$, where the entries in d_{k+1} are known constants and the initial pass profile vector $y_0(p) = f(p)$, where the entries in $f(p)$ are known functions of $p \in [0, \alpha - 1]$. Also it is assumed throughout this paper that the pair $\{A, B_0\}$ is controllable and the pair $\{C, A\}$ observable.

In this model the state updating is in the p direction and the pass profile updating is in the k direction. The terms $B_0 y_k(p)$ and $D_0 y_k(p)$, respectively, represent the contribution from the previous pass profile. See [1] for cases where other representations for the contribution from the previous pass profile are required. As expected, these previous pass terms are critical to the stability analysis for these processes.

The stability theory [1] for linear repetitive processes is based on an abstract model in a Banach space setting that includes a wide range of such processes as special cases, including those described by (1). Let E_α be a Banach space, W_α a linear subspace of E_α, and L_α a bounded linear operator mapping E_α into itself. Then the dynamics of a linear repetitive process with constant pass length are described by linear recursion relations of the form

$$y_{k+1} = L_\alpha y_k + b_{k+1}, \quad k \ge 0, \tag{2}$$

where y_k is the pass profile on pass k, and $b_{k+1} \in W_\alpha$, $k \ge 0$. The term $L_\alpha y_k$ represents the contribution of pass k to pass $k+1$ and b_{k+1} represents initial conditions, disturbances and control input effects that enter on pass $k+1$.

In the case of processes described by (1), take $E_\alpha = \ell_2^m[0, \alpha]$, that is, the space of real $m \times 1$ vectors $\{y_k(1), \ldots, y_k(\alpha)\}$ and then

$$(L_\alpha y)(p) = \sum_{i=0}^{p-1} CA^{p-1-i} B_0 y(i) + D_0 y(p),$$

and

$$b_{k+1} = CA^p d_{k+1} + \sum_{i=0}^{p-1} CA^{p-1-i} Bu_{k+1}(i) + Du_{k+1}(p).$$

Given the unique control problem, the natural approach to a definition of stability for these processes is to ask that given any initial profile y_0 and any disturbance sequence $\{b_{k+1}\}_{k\ge1}$ that converges strongly to b_∞ as $k \to \infty$, the sequence of pass profiles generated $\{y_k\}_{k\ge1}$ converges strongly to y_∞ as $k \to \infty$. This property is termed asymptotic stability in the pass-to-pass direction, or asymptotic stability for short, of (2) and for the given finite pass length α is equivalent [1] the existence of finite real scalars $M_\alpha > 0$ and $\lambda_\alpha \in (0, 1)$ such that $\|L_\alpha^k\| \le M_\alpha \lambda_\alpha^k$, where $\|\cdot\|$ denotes both the norm on E_α and the induced operator norm as appropriate. This property holds [1] if and only if the spectral radius of L_α, $\rho(L_\alpha)$, satisfies $\rho(L_\alpha) < 1$ and also y_∞, termed the limit profile, is given by

$$y_\infty = (I - L_\alpha)^{-1} b_\infty,$$

where I denotes the identity operator in E_α.

For processes described by (1) it has been shown [1] that asymptotic stability holds if and only if $\rho(D_0) < 1$. Also if (1) is asymptotically stable and the input sequence applied $\{u_k\}_{k+1}$ converges strongly as $k \to \infty$ to u_∞, the resulting limit profile is described by a discrete linear systems state-space model with state matrix $A_{lp} = (A + B_0(I - D_0)^{-1}C)$.

Asymptotic stability does not guarantee that the limit profile has acceptable along-the-pass dynamics. A simple example is $A = -0.5$, $B = 1$, $B_0 = 0.5 + \beta$, $C = 1$, $D = 0$, $D_0 = 0$, where β is a real scalar. In this case $A_{lp} = \beta$ and $\rho(A_{lp}) \ge 1$ for $|\beta| \ge 1$.

The problem highlighted by this example can be overcome by demanding the BIBO property for any possible value of the pass length, where mathematically this can be analyzed by letting $\alpha \to \infty$. This is the stability along-the-pass property which [1] is equivalent to the existence of finite real scalars $M_\infty > 0$ and $\lambda_\infty \in (0, 1)$, which are independent α, such that $\|L_\alpha^k\| \le M_\infty \lambda_\infty^k,\ k \ge 0$, For the processes described by (1), with the controllability and observability assumptions stated above, this property [1] holds if and only if (i) $\rho(D_0) < 1$ (asymptotic stability), (ii) $\rho(A) < 1$, and (iii) all eigenvalues of the transfer-function matrix $G(z) = C(zI - A)^{-1}B_0 + D_0$ must lie inside the unit circle in the complex plane for all $|z| = 1$. In the case of the example above it is this last condition which fails when $|\beta| \ge 1$.

To give a physically based interpretation of asymptotic stability and stability along the pass and the differences between them, suppose that no control input term is present and $x_{k+1}(0) = 0,\ k \ge 0$. Then $y_k(0) = D_0^k y_0,\ k \ge 1$ and in the SISO case for simplicity, asymptotic stability ensures that the sequence $\{y_k(0)\},\ k \ge 0$, does not become unbounded as $k \to \infty$. This places no restriction on the along the pass (state) dynamics and condition (ii) is to be expected but as the example above shows this is not enough to ensure stability along the pass. In the case of condition (iii), again in the SISO case for simplicity, the transfer-function involved describes the contribution of the previous pass profile to the current one [1] and reduces to $|G(z)| < 1$ for all $|z| = 1$. Hence this condition is equivalent to requiring that each frequency component of the initial pass profile is attenuated from pass-to-pass, whereas asymptotic stability alone restricts this requirement to $p = 0$.

Condition (iii) for processes described by (1) is very strict and, by analogy with the standard linear systems case, this would make control law design to force the pass profile vector to track a reference very difficult. Motivated by this last fact, strong practical stability [3] relaxes this BIBO stability property by removing the uniform boundedness requirement as both $k \to \infty$ and $\alpha \to \infty$ but still demands it when (a) both k and α are finite, (b) the pass index $k \to \infty$ and the pass length α is finite and (c) the pass index k is finite and the pass length $\alpha \to \infty$. Cases (a) and (b) have obvious practical motivation and Case (c) is the mathematical formulation of an application where the process completes a finite number of passes but the pass length is very long, and there is a requirement to control the along the pass dynamics. In particular, in control law design to force the pass profile to track a reference attenuation at each frequency component is relaxed to that of requiring this property at a subset of frequencies.

In line with the developments in [3], the repetitive process model (1) is said to have the strong practical stability property if the following conditions hold: [a] $\rho(D_0) < 1$; [b] $\rho(A) < 1$; [c] $\rho(A + B_0(I - D_0)^{-1}C) < 1$; and [d] $\rho(C(I - A)^{-1}B_0 + D_0) < 1$. The matrix inverse in [c], respectively, [d] holds whenever [a], respectively, [b] holds. A numerical example of a process that has the strong practical stability property but is not stable along the pass can be found in the literature [3]. The implications of this property are explored in the subsequent section, from the perspective of limit profile existence and corresponding measures of performance.

3. Strong practical stability and $\mathcal{H}_\infty$ disturbance attenuation

Previous research [3] on strong practical stability for discrete linear repetitive processes assumed that the example under consideration was disturbance free. This paper starts from the following process state-space model over $0 \le p \le \alpha - 1,\ k \ge 0$,

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) \\ &\quad + B_0 y_k(p) + B_1\omega_{k+1}(p), \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) \\ &\quad + D_0 y_k(p) + D_1\omega_{k+1}(p), \end{aligned} \tag{3}$$

where, on pass k, $\omega_k(p) \in \mathbb{R}^s$ is a disturbance vector on both the state and pass profile vectors. The remainder of the notation is the same as the disturbance-free model.

To formulate H_∞ disturbance attenuation for this model, one way would be to use stability along the pass and the norm on 2-D signals defined over $(k, p) \in [0, \infty] \times [0, \infty]$, considered in, for example, [4] for 2-D discrete linear systems. This approach for control law design would have the same implications in terms of frequency attenuation detailed in the previous section for stability along the pass. Instead, this paper uses strong practical stability and measures of disturbance attenuation defined in terms of the evolution over one independent variable with the other fixed; that is, with p fixed and k ranging over $[0, \infty]$ and also k fixed and p ranging over $[0, \infty]$.

Consider an asymptotically stable process described by (3) with zero input vector ($u_k(p) = 0$) with a sequence $\{\omega_k\}_{k\ge1}$ that converges strongly to ω_∞ as $k \to \infty$. Then (following the analysis for the input only case in [1]) the repetitive process dynamics converge as $k \to \infty$ to the limit profile

$$\begin{aligned} x_\infty(p+1) &= (A + B_0(I - D_0)^{-1}C)x_\infty(p) + (B_1 \\ &\quad + B_0(I - D_0)^{-1}D_1)\omega_\infty(p), \end{aligned}$$

$$y_\infty(p) = (I - D_0)^{-1}Cx_\infty(p) + (I - D_0)^{-1}D_1\omega_\infty(p), \tag{4}$$

(where $(I - D_0)^{-1}$ exists since $\rho(D_0) < 1$). This is a discrete linear systems state-space model with indeterminate p and condition [c] is the requirement for this model to be stable. Furthermore, under condition [b] for strong practical stability the dynamics as $p \to \infty$ for any finite k are (again following the analysis for the input only case in [1]) described by

$$\begin{aligned} y_{k+1}(\infty) &= \left(C(I - A)^{-1}B_0 + D_0\right)y_k(\infty) \\ &\quad + \left(C(I - A)^{-1}B_1 + D_1\right)\omega_{k+1}(\infty) \end{aligned}$$

$$x_{k+1}(\infty) = (I - A)^{-1}B_0 y_k(\infty) + (I - A)^{-1}B_1\omega_{k+1}(\infty) \tag{5}$$

where $(I-A)^{-1}$ exists since $\rho(A) < 1$. This a discrete linear systems state-space model with indeterminate k and condition [d] is the requirement for this model to be stable. In what follows, the initial pass profile at $p = 0$ with zero state initial vector sequence and control input vector, respectively, is used, that is,

$$y_{k+1}(0) = D_0 y_k(0) + D_1\omega_{k+1}(0). \tag{6}$$

Consider again H_∞ disturbance attenuation for these processes, where the 2-D systems formulation would consider this property over $(k, p) \in [0, \infty] \times [0, \infty]$. Under strong practical stability, this requirement is relaxed to the 'boundaries' of this domain, that is, is imposed on (a) $y_k(0)$ for all values of the variable k, (b) $y_k(\infty)$ for all values of k and (c) $y_\infty(p)$ for all values of p. These disturbance attenuation measures are practically relevant since control design for applications should aim to reach a limit profile, that is, produce the same output of each pass with acceptable along the pass dynamics and ensure that each pass completed also has acceptable along the pass dynamics. The required conditions can be formulated in terms of the standard linear systems H_∞ norm as detailed next.

Definition 1 (*Performance*)**.** Suppose that the repetitive process (3) with zero boundary conditions $x_k(0) = 0$ for $k \ge 1$ and zero input $u_k(p) = 0$ for $k, p \ge 0$, has the strong practical stability property. Given any 2-D disturbance sequence $w = \{w_k\}_{k\ge0}$, of p-indexed sequences $\{w_k(p)\}_{p\ge0}$, that has the 1-D strong limit $\bar{w} := \{w_\infty(p)\}_{p\ge0}$ in k and/or 1-D strong limit $\hat{w} := \{w_k(\infty)\}_{k\ge0}$ in p, define the corresponding 1-D sequences $\bar{y} := \{y_\infty(p)\}_{p\ge0}$ and/or $\hat{y} := \{y_k(\infty)\}_{k\ge0}$ according to (4) and/or (5). Moreover, let $\tilde{w} := \{w_k(0)\}_{k\ge0}$ and define the corresponding 1-D sequence $\tilde{y} := \{y_k(0)\}_{k\ge0}$ according to (6). Then the process (3) is said to achieve H_∞ strong practical performance at the level of $\gamma_1 > 0$ in k if

$$\sup_{0\ne\tilde{w}\in\ell_2} \frac{\|\tilde{y}\|_2}{\|\tilde{w}\|_2} < \gamma_1 \quad \text{and} \quad \sup_{0\ne\hat{w}\in\ell_2} \frac{\|\hat{y}\|_2}{\|\hat{w}\|_2} < \gamma_1$$

and/or at the level of $\gamma_2 > 0$ in p if

$$\sup_{0\ne\bar{w}\in\ell_2} \frac{\|\bar{y}\|_2}{\|\bar{w}\|_2} < \gamma_2$$

where ℓ_2 denotes the Hilbert space of square summable 1-D sequences with norm $\|\{u_k\}_{k\ge0}\|_2 = \sqrt{\sum_{k\ge0} u_k^T u_k}$ (or $\|\{u(p)\}_{p\ge0}\|_2 = \sqrt{\sum_{p\ge0} u^T(p)u(p)}$).

Define the along the pass and pass-to-pass shift operators as z_1 and z_2 respectively, applied, for example, to $x_k(p)$ and $y_k(p)$ as $z_1 x_k(p) = x_k(p+1)$ and $z_2 y_k(p) = y_{k+1}(p)$, respectively. Also let $\|T(z)\|_\infty$ denote the H_∞ norm of the transfer-function matrix $T(z)$ in one indeterminate z.

Next, the conditions for $\mathcal{H}_\infty$ disturbance attenuation under strong practical stability, that is, conditions [a]–[d] in the previous section hold, are developed, where disturbance attenuation in the sense of Definition 1 for $p = 0$ in direction k holds when

$$\|G_a(z_2)\|_\infty < \gamma_1,$$

using condition [a] for strong practical stability and $G_a(z_2)$ is obtained by applying the z_2 transform to (6).

By condition [c] for strong practical stability, (4) is a stable discrete linear system and disturbance attenuation in the sense of Definition 1 in the direction p holds when

$$\|G_b(z_1)\|_\infty < \gamma_2,$$

where $G_b(z_1)$ is obtained by applying the z_1 transform to (4).

By condition [d] for strong practical stability, (5) is a stable discrete linear system and hence disturbance attenuation in the sense of Definition 1 for $p = \infty$ in direction k holds when $\|G_c(z_2)\|_\infty < \gamma_1$ where $G_c(z_2)$ is obtained by applying the z_2 transform to (5).

Finally, strong practical stability with $\mathcal{H}_\infty$ disturbance attenuation for processes described by (1) holds when

[e] $\rho(D_0) < 1$ with $\|G_a(z_2)\|_\infty < \gamma_1$,
[f] $\rho(A) < 1$,
[g] $\rho(A + B_0(I - D_0)^{-1}C) < 1$ with $\|G_b(z_1)\|_\infty < \gamma_2$ and
[h] $\rho(C(I - A)^{-1}B_0 + D_0) < 1$ with $\|G_c(z_2)\|_\infty < \gamma_1$.

Conditions [e] and [f] are standard in the theory of standard discrete linear systems where [e] is the stability with $\mathcal{H}_\infty$ disturbance attenuation problem. The remaining two conditions are somewhat more involved, especially in terms of control law design and, as a preliminary step in the derivation of computable control law design algorithms. The next section develops them into LMI based conditions using results from the analysis of standard discrete linear nonsingular descriptor systems.

3.1. Stability tests with disturbance attenuation

Suppose that the example under consideration is asymptotically stable and hence $\rho(D_0) < 1$. Then as $k \to \infty$ the resulting limit profile state-space model (4) can be rewritten as

$$x_\infty(p+1) - B_0 y_\infty(p) = A x_\infty(p) + B_1 \omega_\infty(p),$$
$$(I - D_0) y_\infty(p) = C x_\infty(p) + D_1 \omega_\infty(p).$$

Moreover, condition [g] is equivalent to the requirement that the standard discrete linear nonsingular descriptor system

$$\begin{aligned} \bar{E}_1 z(h+1) &= \bar{A}_1 z(h) + \widetilde{B}\omega(h), \\ y(h) &= \bar{C} z(h), \end{aligned} \tag{7}$$

where

$$z(h) = \begin{bmatrix} x_\infty(h) \\ y_\infty(h-1) \end{bmatrix}, \qquad \bar{A}_1 = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix},$$

$$\bar{E}_1 = \begin{bmatrix} I & -B_0 \\ 0 & I - D_0 \end{bmatrix} \qquad \widetilde{B} = \begin{bmatrix} B_1 \\ D_1 \end{bmatrix}, \qquad \bar{C} = \begin{bmatrix} 0 & I \end{bmatrix},$$

is stable and the $\mathcal{H}_\infty$-norm of the transfer-function matrix coupling $\omega(h)$ to $y(h)$ has value less than γ_2.

In the case when $\rho(A) < 1$, k finite, and $p \to \infty$, rewrite (5) as

$$(I - A)x_{k+1}(\infty) = B_0 y_k(\infty) + B_1 \omega_{k+1}(\infty),$$
$$-C x_{k+1}(\infty) + y_{k+1}(\infty) = D_0 y_k(\infty) + D_1 \omega_{k+1}(\infty).$$

Then condition [h] is equivalent to the requirement that the standard discrete linear nonsingular descriptor system

$$\bar{E}_2 \hat{z}(o+1) = \bar{A}_2 \hat{z}(o) + \widetilde{B}\widehat{\omega}(o),$$
$$\hat{y}(o) = \bar{C}\hat{z}(o),$$

where

$$\widehat{\omega}(o) = \omega(o+1), \qquad \hat{z}(o) = \begin{bmatrix} x_{o-1}(\infty) \\ y_o(\infty) \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} 0 & B_0 \\ 0 & D_0 \end{bmatrix},$$

$$\bar{E}_2 = \begin{bmatrix} I - A & 0 \\ -C & I \end{bmatrix}, \qquad \bar{C} = \begin{bmatrix} 0 & I \end{bmatrix},$$

is stable and the $\mathcal{H}_\infty$-norm of the transfer-function matrix coupling $\hat{\omega}(o)$ to $\hat{y}(o)$ has value less than γ_1.

The following from the analysis of standard discrete linear systems are used in the proofs of the new results in this paper.

Lemma 1 (*[5]*). *Given a scalar $\gamma > 0$, the standard discrete linear system*

$$\begin{aligned} x(t+1) &= \hat{A}x(t) + \hat{B}_1\omega(t), \\ z(t) &= \hat{C}x(t) + \hat{D}_1\omega(t), \end{aligned} \tag{8}$$

where the pair $\{\hat{A}, \hat{B}\}$ is controllable and the pair $\{\hat{A}, \hat{C}\}$ observable is stable and its transfer-function matrix $G(z)$ satisfies $\|G(z)\|_\infty < \gamma$ if and only if there exists a matrix $P > 0$ such that following LMI holds:

$$\begin{bmatrix} \hat{A}^T P \hat{A} - P + \hat{C}^T \hat{C} & \hat{A}^T P \hat{B}_1 + \hat{C}^T \hat{D}_1 \\ \hat{B}_1^T P \hat{A} + \hat{D}_1^T \hat{C} & \hat{B}_1^T P \hat{B}_1 + \hat{D}_1^T \hat{D}_1 - \gamma^2 I \end{bmatrix} < 0.$$

Lemma 2 (*[5]*). *Given compatibly dimensioned real matrices $\mathbf{B}, \mathbf{C}$, and $\mathbf{Q} = \mathbf{Q}^T$, the following statements are equivalent*

(i) *There exists a compatibly dimensioned matrix $\mathbf{X}$ satisfying*

$$\mathbf{BXC} + (\mathbf{BXC})^T + \mathbf{Q} < 0,$$

(ii) *the following two conditions hold*
 (a) $\mathbf{B}_\perp \mathbf{Q} (\mathbf{B}_\perp)^T < 0$ *or* $\mathbf{BB}^T > 0$,
 (b) $(\mathbf{C}^T)_\perp \mathbf{Q}((\mathbf{C}^T)_\perp)^T < 0$ *or* $\mathbf{C}^T\mathbf{C} > 0$,

where $\mathbf{M}_\perp$ denotes the orthogonal complement, that is, a full rank matrix such that $\mathbf{MM}_\perp = 0$ or $\mathrm{span}(\mathbf{M}_\perp) = \mathrm{Ker}(\mathbf{M})$.

The following is the first major result of this paper and gives LMI based conditions for the existence of strong practical stability with $\mathcal{H}_\infty$ disturbance attenuation.

Theorem 1. *A discrete linear repetitive process described by (3) is strongly practically stable and has $\mathcal{H}_\infty$ disturbance attenuation in the sense of Definition 1 if and only if there exist compatibly dimensioned matrices $W_1 > 0$, $W_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, and nonsingular matrices G_1, G_2, and S_2, such that the following set of LMIs is feasible*

$$\begin{bmatrix} -W_2 & 0 & S_2^T D_0^T & S_2^T D_0^T \\ 0 & -\gamma_1^2 I & D_1^T & D_1^T \\ D_0 S_2 & D_1 & -I & 0 \\ D_0 S_2 & D_1 & 0 & W_2 - S_2 - S_2^T \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} -W_1 & W_1 A^T \\ A W_1 & -W_1 \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} -Q_1 & 0 & G_1^T \bar{A}_1^T & G_1^T \bar{C}^T \\ 0 & -\gamma_2^2 & \widetilde{B}^T & 0 \\ \bar{A}_1 G_1 & \widetilde{B} & Q_1 - \bar{E}_1 G_1 - G_1^T \bar{E}_1^T & 0 \\ \bar{C} G_1 & 0 & 0 & -I \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} -Q_2 & 0 & G_2^T \bar{A}_2^T & G_2^T \bar{C}^T \\ 0 & -\gamma_1^2 & \widetilde{B}^T & 0 \\ \bar{A}_2 G_2 & \widetilde{B} & Q_2 - \bar{E}_2 G_2 - G_2^T \bar{E}_2^T & 0 \\ \bar{C} G_2 & 0 & 0 & -I \end{bmatrix} < 0, \tag{12}$$

$$\bar{C} = \begin{bmatrix} 0 & I \end{bmatrix}.$$

Proof. • The first part of the proof is to show that the LMI (9) is equivalent to the requirement [e], that is, asymptotic stability with $\mathcal{H}_\infty$ attenuation γ_1.

Introduce the substitution $\zeta_{k+1} = y_k$ to write (6) in the form (8) as

$$\zeta_{k+2} = D_0\zeta_{k+1} + D_1\omega_{k+1},$$
$$y_{k+1} = D_0\zeta_{k+1} + D_1\omega_{k+1},$$

with $\hat{A} = \hat{C} = D_0$ and $\hat{B}_1 = \hat{D}_1 = D_1$. Applying the result of Lemma 1 to this last state-space model gives

$$\begin{bmatrix} D_0^T\hat{P}_2D_0 - \hat{P}_2 + D_0^TD_0 & D_0^T\hat{P}_2D_1 + D_0^TD_1 \\ D_1^T\hat{P}_2D_0 + D_1^TD_0 & D_1^T\hat{P}_2D_1 + D_1^TD_1 - \gamma_1^2I \end{bmatrix} < 0,$$

or, on application of the Schur's complement formula,

$$\begin{bmatrix} D_0^T\hat{P}_2D_0 - \hat{P}_2 & D_0^T\hat{P}_2D_1 & D_0^T \\ D_1^T\hat{P}_2D_0 & D_1^T\hat{P}_2D_1 - \gamma_1^2I & D_1^T \\ D_0 & D_1 & -I \end{bmatrix} < 0. \tag{13}$$

The LMI (13) is a special case of condition (ii)(b) of Lemma 2 with $(\mathbf{C}^T)_\perp = \begin{bmatrix} I & 0 & 0 & D_0^T \\ 0 & I & 0 & D_1^T \\ 0 & 0 & I & 0 \end{bmatrix}$ and $\mathbf{Q} = \begin{bmatrix} -\hat{P}_2 & 0 & D_0^T & 0 \\ 0 & -\gamma_1^2I & D_1^T & 0 \\ D_0 & D_1 & -I & 0 \\ 0 & 0 & 0 & \hat{P}_2 \end{bmatrix}$.

Also for $\mathbf{B}_\perp = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}^T$ and this $\mathbf{Q}$, application of (13) gives

$$\begin{bmatrix} -\hat{P}_2 & 0 & D_0^T \\ 0 & -\gamma_1^2I & D_1^T \\ D_0 & D_1 & -I \end{bmatrix} < 0.$$

Hence (ii)(a) of Lemma 2 holds and by (i) of Lemma 2 there exist $\mathbf{X}$ satisfying $\mathbf{BXC} + (\mathbf{BXC})^T + \mathbf{Q} < 0$ with $\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I \end{bmatrix}^T$, $\mathbf{C} = \begin{bmatrix} D_0 & D_1 & 0 & -I \end{bmatrix}$, or

$$\begin{bmatrix} -\hat{P}_2 & 0 & D_0^T & D_0^T\mathbf{X} \\ 0 & -\gamma_1^2I & D_1^T & D_1^T\mathbf{X} \\ D_0 & D_1 & -I & 0 \\ \mathbf{X}^TD_0 & \mathbf{X}^TD_1 & 0 & \hat{P}_2 - \mathbf{X} - \mathbf{X}^T \end{bmatrix} < 0. \tag{14}$$

Also $\hat{P}_2 - \mathbf{X} - \mathbf{X}^T < 0$, which implies that $\mathbf{X}$ is full rank. (If this claim were not true then there exists a vector $q \neq 0$ such that $\mathbf{X}q = 0$ and $q^T(\hat{P}_2 - \mathbf{X} - \mathbf{X}^T)q > 0$.) Hence $\mathbf{X}^{-1}$ exists and is denoted by S_2.

Introduce the matrix $Z = \mathrm{diag}(S_2^T, I, I, S_2)$ and left and right multiply (14) by Z and Z^T, respectively, to obtain (9) and this part of the proof is complete.

- The LMI (10) is a well known condition for stability of 1D discrete linear systems and corresponds to condition [f].
- This part of the proof shows that the LMI (11) is equivalent to condition [g]. As a preliminary step, this condition can be equivalently expressed in terms of the standard discrete linear system (7). By Lemma 1 applied to the resulting standard discrete linear system, there exists a $P_1 > 0$ such that

$$\left[\begin{array}{c|c} -P_1 + \bar{C}^T\bar{C} + \bar{A}_1^T\bar{E}_1^{-T}P_1\bar{E}_1^{-1}\bar{A}_1 & \bar{A}_1^T\bar{E}_1^{-T}P_1\bar{E}_1^{-1}\tilde{B} \\ \hline \tilde{B}^T\bar{E}_1^{-T}P_1\bar{E}_1^{-1}\bar{A}_1 & \tilde{B}^T\bar{E}_1^{-T}P_1\bar{E}_1^{-1}\tilde{B} - \gamma_2^2I \end{array}\right] < 0, \tag{15}$$

with $\hat{A} = \bar{E}_1^{-1}\bar{A}_1, \hat{B}_1 = \bar{E}_1^{-1}\tilde{B}, \hat{C} = \bar{C} = \begin{bmatrix} 0 & I \end{bmatrix}$ and $\hat{D}_1 = 0$. Hence it is now required to prove that the LMIs of (15) and (11) are equivalent.

To prove sufficiency, consider the (3, 3)-block of (11), that is,

$$Q_1 - \bar{E}_1G_1 - (\bar{E}_1G_1)^T < 0,$$

or, since $Q_1 > 0$,

$$H = -\bar{E}_1G_1 - (\bar{E}_1G_1)^T < 0.$$

Assume also that G_1 is singular, and hence there exists a non-zero vector z such that $G_1z = 0$. However, $z^THz = 0$, which contradicts the previous inequality and therefore G_1 is nonsingular. Left and right-multiplying the LMI of (11) by $\mathrm{diag}(G_1^{-T}, I, G_1^{-T}, I)$ and its transpose, respectively, gives

$$\left[\begin{array}{cc|cc} -G_1^{-T}Q_1G_1^{-1} & 0 & \bar{A}_1^TG_1^{-1} & \bar{C}^T \\ 0 & -\gamma_2^2I & \tilde{B}^TG_1^{-1} & 0 \\ \hline G_1^{-T}\bar{A}_1 & G_1^{-T}\tilde{B} & G_1^{-T}Q_1G_1^{-1} - G_1^{-T}\bar{E}_1 - \bar{E}_1^TG_1^{-1} & 0 \\ \bar{C} & 0 & 0 & -I \end{array}\right] < 0.$$

Also introducing $P_1 = G_1^{-T}Q_1G_1^{-1} > 0$ and applying the Schur's complement formula to this last inequality gives

$$\left[\begin{array}{cc|c} -P_1 + \bar{C}^T\bar{C} & 0 & \bar{A}_1^TG_1^{-1} \\ 0 & -\gamma_2^2I & \tilde{B}^TG_1^{-1} \\ \hline G_1^{-T}\bar{A}_1 & G_1^{-T}\tilde{B} & P_1 - G_1^{-T}\bar{E}_1 - \bar{E}_1^TG_1^{-1} \end{array}\right] < 0, \tag{16}$$

which is condition (i) of Lemma 2 for this case with $\mathbf{X} = G_1^{-1}$, $\mathbf{B} = \begin{bmatrix} \bar{A}_1 & \tilde{B} & -\bar{E}_1 \end{bmatrix}^T$, $\mathbf{C} = \begin{bmatrix} 0 & 0 & I \end{bmatrix}$ and $\mathbf{Q} = \mathrm{diag}(-P_1 + \bar{C}^T\bar{C}, -\gamma_2^2I, P_1)$.

Equivalently, applying conditions (ii)(a) and (b) of Lemma 2 gives

$$\left[\begin{array}{c|c} -P_1 + \bar{C}^T\bar{C} + \bar{A}_1^T\bar{E}_1^{-T}P_1\bar{E}_1^{-1}\bar{A}_1 & \bar{A}_1^T\bar{E}_1^{-T}P_1\bar{E}_1^{-1}\tilde{B} \\ \hline \tilde{B}^T\bar{E}_1^{-T}P_1\bar{E}_1^{-1}\bar{A}_1 & \tilde{B}^T\bar{E}_1^{-T}P_1\bar{E}_1^{-1}\tilde{B} - \gamma_2^2I \end{array}\right] < 0, \tag{17}$$

$$\mathbf{B}_\perp = \begin{bmatrix} I & 0 & \bar{A}_1^T\bar{E}_1^{-T} \\ 0 & I & \tilde{B}^T\bar{E}_1^{-T} \end{bmatrix},$$

and

$$\left[\begin{array}{c|c} -P_1 + \bar{C}^T\bar{C} & 0 \\ \hline 0 & -\gamma_2^2I \end{array}\right] < 0, \qquad \mathbf{C}_\perp = \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}, \tag{18}$$

respectively. Also the condition of (18) is redundant in this case since it is the requirement that block (1, 1) of (17) is negative definite, and this part of the proof is complete.

To prove necessity, it follows immediately that, with the notation used, (15) is the result of interpreting (ii)(a) and (ii)(b) of Lemma 2 for this particular case. Hence by (i) of this result, there exists $\mathbf{X}$ such that

$$\mathbf{Q} + \mathbf{BXC} + (\mathbf{BXC})^T < 0, \tag{19}$$

where the matrices $\mathbf{Q}$, $\mathbf{B}$, and $\mathbf{C}$ are defined in the previous part of the proof. Moreover, the LMI (19) can also be written as

$$\left[\begin{array}{cc|c} -P_1 + \bar{C}^T\bar{C} & 0 & \bar{A}_1^T\mathbf{X} \\ 0 & -\gamma_2^2 & \tilde{B}^T\mathbf{X} \\ \hline \mathbf{X}^T\bar{A}_1 & \mathbf{X}^T\tilde{B} & P_1 - \mathbf{X}^T\bar{E}_1 - \bar{E}_1^T\mathbf{X} \end{array}\right] < 0, \tag{20}$$

and from block (3, 3)

$$P_1 - \mathbf{X}^T\bar{E}_1 - \bar{E}_1^T\mathbf{X} < 0.$$

Hence, since $P_1 > 0$,

$$H = -\mathbf{X}^T\bar{E}_1 - \bar{E}_1^T\mathbf{X} < 0.$$

Also $\mathbf{X}$ is nonsingular since if this were not the case then there would exist a non-zero vector z such that $\mathbf{X}z = 0$ and $z^THz = 0$, which contradicts the previous inequality. Replacing $\mathbf{X}$ by G_1^{-1} in (20) gives (16), and hence the equivalence of (11) and (16) has been shown.

- The proof of the equivalence between the LMI of (12) and condition [f] follows a similar reasoning to that of the previous part and hence the details are omitted. □

4. Stabilization with disturbance attenuation

The problem considered in this section is stabilization with $\mathcal{H}_\infty$ disturbance attenuation in the sense of Definition 1 for processes described by (3) through application of the control law

$$u_{k+1}(p) = K_1 x_{k+1}(p) + K_2 y_k(p), \tag{21}$$

which is the weighted sum of current pass state feedback ($x_{k+1}(p)$) and feedforward (in the k direction) from the previous pass profile. Moreover, the current pass state vector in this control law must be estimated using an observer if not all entries are available for measurement. Also a reference vector can be added to define a tracking problem.

Iterative learning control is an application area for repetitive process systems theory where once a pass or trial is completed all data is available and hence available for control law design. In particular, the control law (21) can be augmented by non-causal terms to, for example,

$$u_{k+1}(p) = K_1 x_{k+1}(p) + K_2 y_k(p) + K_3 y_k(p+1),$$

or, more generally, have contributions for more than one or all sampled points along the previous pass and this is a very strong feature in some applications. The results in [2], with experimental verification, show that the repetitive process model considered in this paper can be used to design non-causal iterative learning control laws when there are no disturbances present. The results in this paper can therefore be extended to the design of such control laws in the presence of disturbances.

On applying (21) to (3) the matrices A, B_0, C, D_0 are mapped to $A + BK_1,\ \ B_0 + BK_2,\ \ C + DK_1$, and $D_0 + DK_2$, respectively. Strong practical stability with $\mathcal{H}_\infty$ disturbance attenuation for the controlled process holds when

[i] $\rho(D_0 + DK_2) < 1$ with $\|\hat{G}_a(z_2)\|_\infty < \gamma_1$,
[j] $\rho(A + BK_1) < 1$,
[k] $\rho[(B_0 + BK_2)(I_m - D_0 - DK_2)^{-1}(C + DK_1) + (A + BK_1)] < 1$ with $\|\hat{G}_b(z_1)\|_\infty < \gamma_2$ and
[l] $\rho[(C + DK_1)(I_n - A - BK_1)^{-1}(B_0 + BK_2) + (D_0 + DK_2)] < 1$ with $\|\hat{G}_c(z_2)\|_\infty < \gamma_1$,

where $\hat{G}_a(z_2)$, $\hat{G}_b(z_1)$ and $\hat{G}_c(z_2)$ denote the transfer-function matrices $G_a(z_2)$, $G_b(z_1)$ and $G_c(z_2)$, respectively, when applied to the controlled process. The next result gives sufficient conditions to solve this problem with LMI based computation of the control law matrices.

Theorem 2. *Suppose that a control law of the form (21) is applied to a discrete linear repetitive process described by (3). Then the resulting controlled process is strongly practically stable and has $\mathcal{H}_\infty$ disturbance attenuation in the sense of Definition 1 if there exist compatibly dimensioned matrices $W_1 > 0, W_2 > 0, Q_1 > 0, Q_2 > 0$, a nonsingular matrix $G = \operatorname{diag}(\bar{G}_1, \bar{G}_2)$ and rectangular matrices $\widetilde{N}_1 = \begin{bmatrix} N_1 & 0 \end{bmatrix}$ and $\widetilde{N}_2 = \begin{bmatrix} 0 & N_2 \end{bmatrix}$ such that the following set of LMIs is feasible*

$$\begin{bmatrix} -W_2 & 0 & \bar{G}_2^T D_0^T + N_2^T D^T & \bar{G}_2^T D_0^T + N_2^T D^T \\ 0 & -\gamma_1^2 I & D_1^T & D_1^T \\ D_0\bar{G}_2 + DN_2 & D_1 & -I & 0 \\ D_0\bar{G}_2 + DN_2 & D_1 & 0 & W_2 - \bar{G}_2 - \bar{G}_2^T \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} -W_1 & \bar{G}_1^T A^T + N_1^T B^T \\ A\bar{G}_1 + BN_1 & W_1 - \bar{G}_1 - \bar{G}_1^T \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} -Q_1 & 0 & G^T \bar{A}_1^T + \widetilde{N}_1^T \Pi^T & G^T \bar{C}^T \\ 0 & -\gamma_2^2 & \widetilde{B}^T & 0 \\ \bar{A}_1 G + \Pi \widetilde{N}_1 & \widetilde{B} & Q_1 - \bar{E}_1 G + \Pi \widetilde{N}_2 - G^T \bar{E}_1^T + \widetilde{N}_2^T \Pi^T & 0 \\ \bar{C} G & 0 & 0 & -I \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} -Q_2 & 0 & G^T \bar{A}_2^T + \widetilde{N}_2^T \Pi^T & G^T \bar{C}^T \\ 0 & -\gamma_1^2 & \widetilde{B}^T & 0 \\ \bar{A}_2 G + \Pi \widetilde{N}_2 & \widetilde{B} & Q_2 - \bar{E}_2 G + \Pi \widetilde{N}_1 - G^T \bar{E}_2^T + \widetilde{N}_1^T \Pi^T & 0 \\ \bar{C} G & 0 & 0 & -I \end{bmatrix} < 0. \tag{25}$$

If this set of LMIs hold, stabilizing control law matrices are given by

$$K_1 = N_1 \bar{G}_1^{-1}, \qquad K_2 = N_2 \bar{G}_2^{-1}. \tag{26}$$

Proof.
- To show that the LMI (22) guarantees that condition [i] holds, first replace D_0 by $D_0 + DK_2$ in the LMI (9) of Theorem 1. Then introduce new variables $S_2 = \bar{G}_2$ and $N_2 = K_2\bar{G}_2$ to obtain (22).
- The LMI (23) is a known condition for state feedback stabilization of 1D discrete linear systems (see, for example, [6]), and corresponds to condition [j].
- The LMIs (24) and (25) guarantee that conditions [k] and [l], respectively, hold. To prove these claims, apply Theorem 1 with $\bar{A}_1$ replaced by $\bar{A}_{1\text{new}} = \bar{A}_1 + \Pi \begin{bmatrix} K_1 & 0 \end{bmatrix}$, and $\bar{E}_1$ by $\bar{E}_{1\text{new}} = \bar{E}_1 - \Pi \begin{bmatrix} 0 & K_2 \end{bmatrix}$ in the case of (24), and $\bar{A}_2$ by $\bar{A}_{2\text{new}} = \bar{A}_2 + \Pi \begin{bmatrix} 0 & K_2 \end{bmatrix}$ and $\bar{E}_2$ by $\bar{E}_{2\text{new}} = \bar{E}_2 - \Pi \begin{bmatrix} K_1 & 0 \end{bmatrix}$ in the case of (25), respectively. Introducing the additional variables $N_1 = K_1\bar{G}_1$ and $N_2 = K_2\bar{G}_2$ now completes the proof of this part.
- To prove that the matrix G is nonsingular, (22) gives

  $$W_2 - \bar{G}_2 - \bar{G}_2^T < 0$$

  and assume that $\bar{G}_2$ is singular. Then there exists q such that $\bar{G}_2 q = 0$ and hence

  $$q^T(\bar{G}_2 - \bar{G}_2^T)q = q^T W_2 q < 0,$$

  which cannot hold since $W_2 > 0$. Hence $\bar{G}_2$ must be nonsingular. In similar manner, (23) leads to the conclusion that $\bar{G}_1$ must be nonsingular and hence G must be nonsingular. □

Remark 1. To find stabilizing K_1 and K_2 it is necessary to impose a structure on G in Theorem 2, that is, $G = \operatorname{diag}(\bar{G}_1, \bar{G}_2)$. This induces conservativeness but is required to obtain K_1 and K_2 from G, N_1 and N_2. Otherwise if G is a full block matrix (26) does not hold.

Theorem 2 gives a sufficient condition for the solvability of H_∞ control law design problem. A desired control law (26) can be determined by solving the following convex optimization problem:

$$\min \sigma = \alpha_1\mu_1 + \alpha_2\mu_2 \quad \text{subject to (22)–(25)}$$
$$(\text{where } \mu_i = \gamma_i^2,\ i = 1, 2) \tag{27}$$

for selected $0 \le \alpha_1 \le 1$, $0 \le \alpha_2 \le 1$ such that $\alpha_1 + \alpha_2 = 1$. Also it is possible to select $\alpha_1 = 0$ or $\alpha_2 = 0$ and optimize over the other.

As a numerical example, consider the case of (3) when $\alpha = 50$, $k \ge 0$ and

$$\begin{bmatrix} A & B_0 & B & B_1 \\ C & D_0 & D & D_1 \end{bmatrix} = \left[\begin{array}{cc|c|c|c} 2.19 & -1.83 & 0.78 & 0.52 & 1.56 \\ 0.16 & 0.71 & -1.18 & -1.54 & -1.47 \\ \hline 0.2 & 0.99 & -1.3 & -0.75 & -1.25 \end{array}\right]$$

and boundary conditions

$$x_{k+1}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, \quad k \ge 0,$$

$$y_0(p) = 20 + \sin\left(\frac{2p}{\alpha}\pi\right), \quad 0 \le p \le \alpha - 1.$$

This example is not strongly practically stable and solving (27) of Theorem 2 yields the stabilizing control law matrices $K_1 = \begin{bmatrix} -1.755 & 1.770 \end{bmatrix}$, $K_2 = -0.978$ with associated minimum $\mathcal{H}_\infty$ disturbance attenuation $\gamma_1 = 4.29$ and $\gamma_2 = 4.96$ for $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$. The maximum values of the $\mathcal{H}_\infty$ norms of $\hat{G}_a(z_2)$, $\hat{G}_b(z_1)$ and $\hat{G}_c(z_2)$ are 2.8842, 3.3031 and 4.0013, respectively, which confirms that the minimum $\mathcal{H}_\infty$ attenuation is not very conservative in this case, see also Fig. 1.

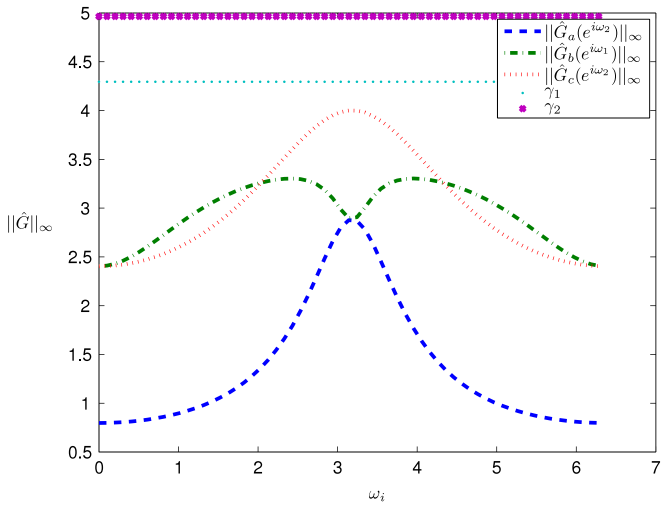

Fig. 1. The $\mathcal{H}_\infty$ norms of $\hat{G}_a(z_2)$, $\hat{G}_b(z_1)$ and $\hat{G}_c(z_2)$ versus the minimum achieved γ_1 and γ_2 for $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$.

5. Conclusions

This paper has developed new results on the stabilization and control of discrete linear repetitive processes with $\mathcal{H}_\infty$ disturbance attenuation that are easily extended to the case where a reference signal is added to the control law. The results given extend in a natural manner to allow design when there is uncertainty associated with the process model. Future research should include the extension of these results to the use of a control law that does not require current pass state information.

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