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Synthesis of Decentralized Supervisor for Petri Nets Using Decomposition with Overlapping Places

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Abstract

Petri nets are a formal tool for describing processes in a many of important technical and social applications. Their advantage over other related type instruments is very effective modeling of parallelism. This makes it easy to avoid improper behavior process. This article deals with the use of Petri nets for modeling specific type of control of dynamic systems, which reduces improper behavior of these systems. One way to influence the behavior of the system is a suitable feedback that prevents the emergence of adverse conditions. The article presents the partial achievements of our research on loopback control and design supervisors for Petri nets.

Keywords: Perti nets, decentralization, supervisor, loopback control

1 Introduction

The role of each system design is to obtain the required behavior, which doesn't violate constraints, derived from the specification (input) and other than the known conditions can not therefore arise.

This required behavior enforces the implementation of additional implementation so-called supervisor, which can be placed to an existing design as software code or hardware.
Information and communication technologies presently represent one of the main competitive advantages of each organization doing business in almost every sector of the economy. Properly designed, stable and functional information system may be a key competitive advantage (in case of errors can be an disadvantage) the sector.

2 Objectives

The aim of this article is show how you can more accurately manage the ongoing complex process, which is modeled by Petri nets. The Petri net will be seen as a discrete event control system.
Supervisors will consist of places that will be connected to the transition process Petri nets. The purpose of the proposed procedure is to eliminate the illicit network of process conditions. Specifically, the generation of the invariant points and crossings proposed feedback control. Design of control generating constraints. These constraints are expressed using equality, inequalities or logical expressions, which may include a vector marking of Petri nets. Design of control is the numerical solution given by the algebraic equality and inequality or of logical statements. The solution, which the authors of this acrticle concludes, makes it possible to the construction supervisor, when the number of its places (in the description using Petri nets) is a linear function of the number of constraints that must be followed. This property is the proposed solution ensures that growth isn't disproportionate to the complexity of supervisors and the cost of the construction supervisors will be significantly reduced.
To model the processes controlled Petri nets are used in accordance with the following definition.

Definition: **Petri net** is arranged five $N = (P, T, H, w, k)$ where:
1. (P, T, H) is weakly connected bipartite graph in the convention 2 and 3.
Where $(P \cup T, H)$ is graph, sets of nodes P and T are both nonempty and mutually disjoint and the edges in H connecting only nodes from different sets of decomposition $P \cup T$ sets of all verticles of this graph.
2. $w : H \rightarrow N^+$ is mapping the set of all edges in the set of natural numbers, determining the weight of each edge in the graph.
3. $k : P \rightarrow N \cup \{\infty\}$ is mapping, which assigns each place a non-negative whole number or symbol ∞, which determines the capacity of the site.

Acceptable marking Petri nets (for the purposes of this work only for short marking Petri Nets) $N = (P, T, H, w, k)$ is any depiction m: $P \rightarrow N$ of set P *all palces of P* Petri net N into a set of non-negative integers, which satisfies the condition $m(p) \leq k(p)$ for all $p \in P$. For each Petri net select one of its authorized marking m_0, declaring that the initial marking of Petri nets.

Let $N = (P, T, H, w, k)$ is Petri net. For each of the transition $t \in T$ sign $^\bullet t = \{ p \in P$: (p: $(p, t) \in H$) the set of all places, which leads to a transition edge t and $t^\bullet = \{ p \in P$: (p: $(p, t) \in H$) the set of all the places in which a leading edge of a transition t. Transition $t \in T$ is called a feasible transition at marking m, if for all $p \in {}^\bullet t$ is true that $m(p) \le w((p, t))$ and simultaneously for all $p \in t^\bullet$ apply $m(p) \le k(p) - w((t, p))$.

If $t \in T$ is feasible transition Petri nets with marking m, then the marking of Petri nets for the switch, also called immediately to the marking m' is defined for all $p \in P$ as follows:

$$m'(p) = \begin{cases} m(p) - w(p,t), & if \quad p \in {}^\bullet t \div t^\bullet, \\ m(p) + w(t,p), & if \quad p \in t^\bullet \div {}^\bullet t, \\ m(p) - w(p,t) + w(t,p), & if \quad p \in {}^\bullet t \cap t^\bullet, \\ m(p) & in\ other\ cases \end{cases} \qquad (3.5)$$

Implementation of transition t from marking m to marking m' we write symbolically $m\left[t\right\rangle m'$.

Let M be the set of all permitted signs of network N. For given marking $m \in$ M mark $\left[m\right\rangle$ intersection of all subsets of $N \subseteq$ M , which is valid for both:

(1) $m \in N$.

(2) If for some transition $t \in T$ and some marking $n_1 \in N$ is $n_1\left[t\right\rangle n_2$, then is also $n_2 \in N$.

This set $\left[m\right\rangle$ is called the set of reachable markings of Petri nets marking $\boldsymbol{m}$. If a fixed initial marking m_0 of Petri net N, then $\left[m_0\right\rangle$ is called a reachable marking set of network $\boldsymbol{N}$, or whether the state space of Petri nets.

Let $N = (P, T, H, w, k)$ is a Petri net. Note np = card(P) as count of places and nt = card(T) as count of transition in this Petri net. Places and transitions to organize freely, and we will continue to be marketed after a row $p_1, p_2, ..., p_{np}$ a $t_1, t_2, ..., t_{nt}$.

For simplicity, we can identify place and transitions in the network, by the help of serial numbers (indexes). If the entries of matrix algebra appears a k-member vectors, we will consider a matrix with k rows and one column (called "column vectors").

Remember too, that the Petri net as a simple graph (no multiple edges) and weakly connected. The set of edges H of Petri net is then subset of set $P \times T \cup T \times P$.

Let $N = (P, T, H, w, k)$ is Petri net. **Input matrix of Petri net** N is a matrix with n_P rows and n_T columns expressing views F^{-}, which display $P \times T$ to N, or $\{1, \dots, np\} \times \{1, \dots, nt\}$ to N, defined as a relationship $F^{-}(p,t) = \underline{w}(t,p)$, where $\underline{w}(p,t) = \{ {}^{w(p,t)\ \text{pro}\ (p,t) \in H}_{0\ \ \text{pro}\ (p,t) \notin H}$. Similarly, the output matrix of Petri net N is a matrix of np rows and nt columns expressing view $P \times T$ do N, or $\{1, \dots, np\} \times \{1, \dots, nt\}$ to N, defined as a relationship $F^{+}(t,p) = \overline{w}(t,p)$, where $\overline{w}(t,p) = \{ {}^{w(t,p)\ \text{pro}\ (t,p) \in H}_{0\ \ \text{pro}\ (t,p) \notin H}$.

Input matrix F^{-} of Petri net N is called *Pre*(N), and the output matrix then called *Post*(N).

Flow matrix F of Petri net N is matrix of type (np, nt), whose elements are ordered tuples $(\underline{w}(p,t), \overline{w}(t,p))$, expressing $P \times T$ view, or $\{1, \dots, np\} \times \{1, \dots, nt\}$ to $N^{+} \times N^{+}$. Then $F(p, t) = (\underline{w}(p,t), \overline{w}(t,p))$.

Matrix of changes D of Petri net N also called incidental matrix or matrix of Petri net N, is defined as matrix of type (np, nt) defined by relationship $D(p, t) = \underline{w}(p,t) - \overline{w}(t,p)$ views expressed $P \times T$ to I . Then $D = Post(N) - Pre(N)$.

For the definition of Petri Nets in this part were used sources [7],[2],[5].

3 Invariants

For our goal will be essential to find and use in Petri nets so-called invariants. Invariants describe a situation where some objects are invariant for certain events. We can find the properties of Petri nets, which depends only on the topology of the Petri net, not on the initial marking, there are two types of invariant:

Invariant of places in Petri network is a set of places where the sum of all marks is always constant. This invariant can be described by a np row vector X, where np is the count of places in Petri nets, whose nonzero entries correspond to places that are invariant to some places. Each vector X, which satisfies the following condition:

$m^t X = m_0^t X,$ (3.6)

where m_0 is initial marking of Petri net and m represents the following notations defined invariant of places. According to the above equation means that the sum of all brands in the invariant positions remain constant for all markings, and this sum is determined by the initial marking of Petri nets. Invariant of places are defined by all vectors of integers, satisfying the condition:

$X^t D = 0,$ (3.7)

where D is matrix type of ($np \times nt$) describes changes in Petri net as defined in 3.21.

Solved the problem can be formulated as follows:
Given Petri net $N = (P, T, H, w, k)$ and subset $Q \subseteq P$ of selected places of this net. The condition of admissibility $\sum_{p_j \in Q} m_j \leq b$, where b is given integer and m_j is count of makrs in place p_j.

The aim is to:
Deduce method of supervisor construction modeled by Petri net generating invariant of places in the network for following basic structures:
centralized supervisor
decentralized supervisor for complex Petri nets
illustrate the method of solved examples and simulations.

Cyclic Petri nets are common Petri nets which are T-systems, namely cyclic marked graphs as defined in 3.25. Petri nets which don't satisfy this condition, where there are places with multiple inputs or outputs are called non-cyclical. For cyclic Petri nets were designed centralized supervisors by Antsaklis and Moody in [5]. The principle of the proposal is as follows.

4 The description of the calculation

Similarly, all invariants of places that satisfy the condition by $L\, m_p \leq b$, and have incorporated an additional (free) variable m_c, can be expressed in matrix form as follows:
$L\, m_p + m_c \quad = \quad b,$ (4.4)
where m_c is vector $nc \times 1$, that represents the marking of control places.
Invariant of places defined by $m_i + m_j + m_c = 1$, must satisfy the condition of equality invariants $m_i + m_j \leq 1$. The following equality matrix is equality for all invariant of places invariant described in (4.4)

$$X^t D = (L \quad I) \begin{pmatrix} D_p \\ D_c \end{pmatrix} = 0 \quad \Leftrightarrow$$

$$LD_p + D_c = 0 \quad \Leftrightarrow$$

$$D_c = - L\, D_p, \qquad (4.5)$$

where I is the nc dimensional unit matrix. The D_c matrix reduces the edge that connect the control places in transitions in process Petri net. This is due to a procedural model of Petri nets matrix D_p and restriction b, which must comply with the process. The managing member of Petri nets is defined by (4.5).

The initial marking of Petri net supervisor can also be calculated. Initial marking of control places m_{c0} must be invariant of places where equality by (4.4) are satisfied and depends on the initial Petri net marking process, which participates in invariant of places. Equation (4.4) can be written in the form of the initial vector notation:

$L\ m_{p0} + m_{c0} = b \Leftrightarrow$

$$m_{c0} = b - L\ m_{p0.} \qquad (4.6)$$

The following is a summary theorem that describes the design supervisor for the fully controllable transitions.

Theorem: Synthesis supervisor. If the true relationship $b\ \ - L\ m_{p0} \geq 0$, then supervisor

$$D_c \in I^{nc \times nt}, \qquad (4.7)$$

with initial marking m_{c0} is determinated by following matrix

$D_c = -L\ D_p$

$$m_{c\,0} = b - L\ m_{p0,} \qquad (4.8)$$

activates restriction $Lm_p \leq b$ for the system in closed loop with the marking

$$D = \begin{bmatrix} D_p \\ D_c \end{bmatrix}$$

$$m = \begin{bmatrix} m_p \\ m_c \end{bmatrix}$$

$$m_0 = \begin{bmatrix} m_{p0} \\ m_{c0} \end{bmatrix}, \qquad (4.9)$$

Authors of the paper generalize design of supervisor on the basis of invariants included in systems described by cyclic Petri nets and also acyclic systems.

For acyclic Petri net is a supervisor synthesis is based on the definition of incidence matrix D_p, where transitions are defined by edges between places. The concept of incidence matrix can be generalized by the inclusion of input transitions, ie transitions entering one of the place of Petri nets, but no specific entry. Further generalization concerns the inclusion of output transitions, ie transitions exiting from a given Petri net place, but without entering the transition to one of the places of the network. Suppose that such transitions are controllable. This expands the initial incidence matrix D_p to incidence matrix with input and output transitions labeled D_{pn}. It has the general form

$$D_{pm} = (D_p\ \ D_m), \qquad (4.10)$$

where D_m is incidence matrix of connected array of input and output transitions of the Petri net. This proposal modifies the role of supervisor for Petri nets N_m

$$N_m = (P, T_m, D_m, m_0), \qquad (4.11)$$

where T_m is the set of transitions instead of the original Petri nets and the set of input and output transitions. The problem is to find a supervisor D_{cm} meeting the

specifications
$L\ m_p \leq b$. Let ntm = card(T_m). Synthesis of such a supervisor is similar to [6].
The following definition is true
Theorem: Synthesis supervisor. If the true relationship of $b - L\ m_{p0} \geq 0$, then the supervisor
$D_c \in I^{nc \times ntm}$, (4.12)
with initial marking m_{c0}, is determinated by following matrix
$D_{cm} = -L\ D_{pm}$
$m_{c\,0} = b - L\ m_{p0}$, (4.13)
activates restriction $Lm_p \leq b$ for the system in closed loop with marking

$$D_m = \begin{bmatrix} D_{pm} \\ D_{cm} \end{bmatrix}$$

$$m = \begin{bmatrix} m_p \\ m_c \end{bmatrix}$$

$$m_0 = \begin{bmatrix} m_{p0} \\ m_{c0} \end{bmatrix}, \qquad (4.14)$$

assuming that the transitions with input edges of D_{cm} are controllable.

5 Decentralized supervisors for Petri nets

Another generalization of the authors of the proposal obtained by the invariant supervisor for decentralized control.

Decentralization is one of the well-known methodologies focused on effective solutions to specific problems for complex systems. Decentralization global role means that the original complex problem, divide it into independent or weakly bound sub-problems so solutions to these sub-problems basically solve the global problem. The motivation for the use and development of methods of decentralization is to restrict the information structure in the feedback of the physical nature of the system or reduce the time and memory complexity
of the design procedure and implementation of control algorithms. These general advantages of the decentralized approach also applies to design supervisors for Petri nets. Models based on Petri nets allows to model the current system changes more accurately than models of automata for decentralized control, where is the story of machine based on the space of states. Petri nets model preferably the structural relations and not explicitly state space. The difference between centralized and decentralized supervisor shows schematically on following figure.

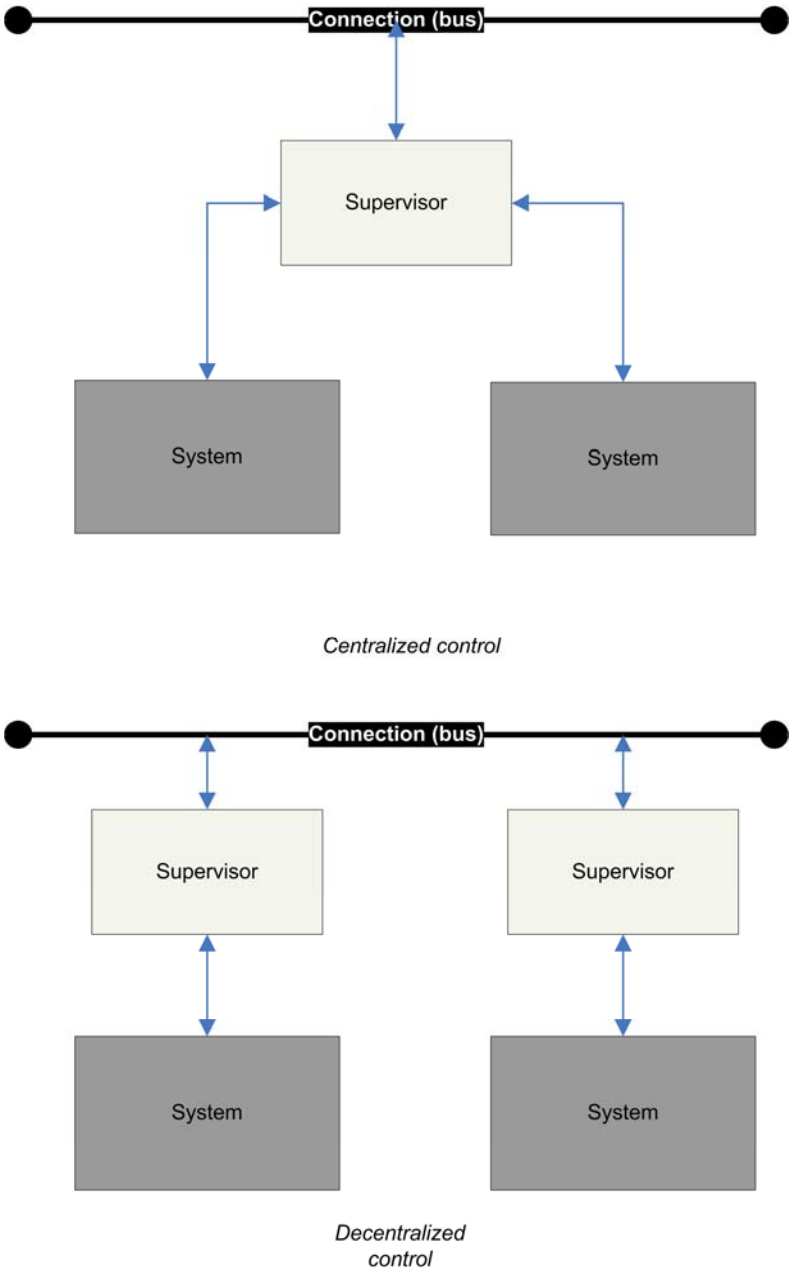

Figure 1: Difference between centralized and decentralized supervisor

The system is specified as a Petri net $N = (P, T, D, m_0)$. A decentralized supervisor is composed of a set of supervisors $\{S1, S2,\dots,Sn\}$ each of that is capable of controlling (observing) the set of transitions driven system. To distinguish the decentralized supervisors $S_1,\dots,S_n$, we will say that everyone S_i is a local supervisor. In addition, mark $T_{o,i}$ ($T_{c,i}$) as subset of the transitions of Petri nets that can be observed (control). Trinity $(N, T_{c,i}, T_{o,i})$ we called subsystem i. Subsystem represents an object of control, which is controlled by and observed by the local supervisor *Si*. The set of unobservable (uncontrolable) transitions *Si* are marked $T_{uo,i} = T \setminus T_{o,i}$ ($T_{uc,i} = T \setminus T_{c,i}$). The system of *N* contains subsystems with uncontrollable and unobservable transitions $T_{uc,i}$ a $T_{uo,i}$ marked as(N, $T_{uc,1}$, ..., $T_{uc,n}$, $T_{uo,1}$, ..., $T_{uo,n}$).

We can formulate a general role of decentralized supervisor as follows:

Given a global specification sets of uncontrollable and unobservable transitions transitions $T_{uc,1}$ $T_{uc,2}$, ..., $T_{uc,n}$ a $T_{uo,1}$ $T_{uo,2}$, ..., $T_{uo,n}$. Find a set of local supervisors $S1$, $S2$, ...,Sn, whose simultaneously behavior ensures that the global specification is satisfied when everyone can control $T \setminus T_{uc,i}$ and observe $T \setminus T_{uo,i}$.

The concept of admissibility of the decentralized approach to supervision is here. If we work with uncontrollable and unobservable transitions, it is necessary to ensure that control places will never try to influence uncontrollable transition in the controlled system. At the same time it is requested that no place of supervisor was not influenced or changed by activating unobservable transition in a closed loopcontrolled system. We will call this *d-admissibility* for decentralized system. For differentation to the case of decentralized admissibility will be called *c-admissibility* for centralized system [5].

In the case of decentralized control, we are still interested in the definition of admissibility with regard to the Petri net (N, m_{p0}), including the requirements of the assignment, and a sets of controllable and observable transitions to the different subsystems: $T_{c,1}$ $T_{c,2}$, ..., $T_{c,n}$ a $T_{o,1}$ $T_{o,2}$, ..., $T_{o,n}$. In this case we call decentralized admissibility of d-admissibility. As in the case c-admissibility, we wanted to d-admissibility ensure that the decentralized supervisor can be designed for such a system. Therefore introduce the following definition.

Definition 5.1: Suppose that we have system (N, m_{p0}, $T_{c,1}$, ..., $T_{c,n}$, $T_{o,1}$,..., $T_{o,n}$).
d-admissibility implies that the set of subsystems $C_s \subseteq \{C1, C2, \ldots, Cn\}$ is unempty. Then limitation is c-admissible for system (N, m_{p0}, T_{c}, T_{o}), where

$$T_c = \bigcup_{i \in C} T_{c,i}$$

$$T_o = \bigcap_{i \in C} T_{o,i} \quad (5.1)$$

Set of constraints is d-admissible if every restriction is d-admissible.

Furthermore, there is no need to d-admissible constraints defined by the same set of C_s. Consider the restriction that is c-admissible in relation to a given subsystem 1. Then it means that the restriction is also d-admissible if we have $C_s = \{C1\}$. This can further develop in a more general account, if any restriction in the form of $Lm_p \leq b$ is c-admissible in relation to a specified subsystem, then the $Lm_p \leq b$ is also the d-admissible. Specifically, if each subsystem has full observability and every transition is controllable in relation to a subsystem, then any d-admissible constraints.

Let the given Petri net N with the set of all places P, a set of transitions T, the incidence matrix D_p with initial marking m_{p0}, where $P = P1 \cup P2 \cup P3$, $P1 = \{p_1, \dots, p_{p1}\}$, $P2 = \{p_{p1+1}, \dots, p_{p2}\}$, $P3 = \{p_{p2+1}, \dots, p_{p3}\}$. Then $T = T1 \cup T2 \cup T3$, $T1 = \{t_1, \dots, t_{t1}\}$,
$T2 = \emptyset$, $T3 = \{t_{t1+1}, \dots, t_{t3}\}$.

In this Petri net has the following assumptions:
A1. All transitions are controllable and observable.
A2. Transitions set T1 are only between set of places $PE1 = P1 \cup P2$.

A3. Transitions set T3 are only between set of places $PE2 = P2 \cup P3$.

A4. A set of T2 transitions between points P2 set is empty, ie, $T2 = \emptyset$.

Furthermore, let the restrictions specified in the standard form
$L\, m_p \leq b$, (5.2)

6 Example

For a given complex Petri net N by qualifying A1 - A4 and the constraints (5.2) is goal to design a decentralized supervisor using decomposition with overlapping places P2 using invariant points method.

6.1 Solutions

The first step is to implement the decentralization of the role of Petri nets N and constraints (5.2) on subtasks.

First denote $P2'$ as set of points such that place p_i' of set $P2'$ is a duplicate of places p_i of set $P2$. Furthermore, let $b = b_1 + b_2$.

1. subtask. Design supervisor for Petri net
$N1(PE1, T1, D1_p, m1p0)$, (6.1)
and limitation
$LE1\ m1_p0 \leq b1$, (6.2)
2. subtask. Design supervisor for Petri net
where $D1_p$ is determinated by sets $PE1$, $T1$ and initial marking $m1p0 = (m_{p1}{}^t\ m_{p2}{}^t)$t.
$N2(PE2', T2, D2_p, m2p0)$, (6.3)
and limitation
$LE2\ m2_p0 \leq b2$, (6.4)

where $PE2' = P2' \cup P3$, $D2_p$ is determinated by sets $PE2'$, $T2$ and initial marking
$m2p0 = (m_{p2}{}^{t} \quad m_{p3}{}^{t})t$.

For both subtasks (6.1) (6.2) and (6.3) (6.4) now make a design of supervisor by definition 4.1. The result is a matrix $D1_c$ and $D2_c$. Feedback local Petri nets provide limitations and incidence matrices have the following form

$$D_1 = \begin{pmatrix} D1_p \\ D1_c \end{pmatrix} \qquad D_2 = \begin{pmatrix} D2_p \\ D2_c \end{pmatrix} \tag{6.5}$$

with initial marking

$$m1_{c0} = b_1 - LE1m1_{p0} \qquad m2_{c0} = b_2 - LE2m2_{p0}. \tag{6.6}$$

Denote this feedback control Petri nets

$$N1_c(P1_c, T1_c, D1_c, m1_0)$$

$$N2_c(P2_c, T2_c, D2_c, m2_0). \tag{6.7}$$

where $P1_c = PE1 \cup PC1$, $T1_c = T1 \cup TC1$, $m1_0 = (m1_{p0}{}^{t}\; m1_{c0}{}^{t})^{t}$, $P2_c = PE2 \cup PC2$, $T2_c = T3 \cup TC2$, $m2_0 = (m2_{p0}{}^{t}\; m2_{c0}{}^{t})^{t}$. $PC1$ is set supervisor 1 places and $TC1$ isset of transitions including transitions of supervisor $PC1$ and also transitions of supervisor $PC2$ and places $PE1$. Same meaning as in the Petri net $N1_c$ has the terms of Petri nets $N2_c$. Denote further contraction of Petri nets and $N1_c$ and $N2_c$ into one network

$$N_c(P_c, T_c, D_c, m_{c0}), \tag{6.8}$$

make fusion of places $p_i \in P2$ a $p_i' \in P2'$ into one place p_i. This can be interpreted in such a way that the original place p_i in set P2 given Petri net N is after decomposition to the places p_i a p_i' in networks N1 and N2 used for the design of supervisors returned to its original position in the network N_c to all decomposed p_i. For description of global feedback Petri net N_c using a matrix of incidence matrices for D_c determinates by matrices $D1_c$ a $D2_c$. The representation of these matrices is as follows

$$D1_c \qquad\qquad D2_c$$

$$\begin{array}{c} \\ P1 \\ P2 \\ P1_c \end{array}\!\!\begin{array}{c} t_1 \qquad\quad t_{t1} \\ \begin{pmatrix} D1_{p11} & \cdots & D1_{p1t1} \\ D1_{p21} & \cdots & D1_{p2t1} \\ D1_{c1} & \cdots & D1_{ct1} \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ P2' \\ P3 \\ P2_c \end{array}\!\!\begin{array}{c} t_{t1+1} \qquad\quad t_{t3} \\ \begin{pmatrix} D2_{p21} & \cdots & D2_{p2t3} \\ D2_{p31} & \cdots & D2_{p3t3} \\ D2_{c1} & \cdots & D2_{ct3} \end{pmatrix} \end{array} \tag{6.9}$$

Overlap in the original Petri net (1) applies rows $P2$ in $D1_c$ and rows $P2'$ in $D2_c$. Now we make row permutation in matrix $D1_c$ suitable for subsequent presentation

$$\begin{matrix} P1 & & P1 \\ P2 & \rightarrow & P1_c \\ P1_c & & P2 \end{matrix} \qquad (6.10)$$

The purpose of this permutation is to get rows describes overlap in matrix $D1_c$ in last position. To simplify the description still consider this a sign $D1_c$ by a permutation matrix. The resulting incidence matrix of Petri nets formed by network $N1_c$ above permutation andtion, and $N2_c$ is block diagonal matrix where for simplification denoted $r = t1 + 1$.

$$D_{ce}$$

$$\begin{matrix} & t_1 & & t_{t1} & t_r & & t_{t3} \\ P1 & D1_{p11} & \cdots & D1_{p1t1} & 0 & \cdots & 0 \\ P1_c & D1_{c1} & \cdots & D1_{ct1} & 0 & \cdots & 0 \\ P2 & D1_{p21} & \cdots & D1_{p2t1} & 0 & \cdots & 0 \\ P2' & 0 & \cdots & 0 & D2_{p2r} & \cdots & D2_{p2t3} \\ P3 & 0 & \cdots & 0 & D2_{p3r} & \cdots & D2_{p3t3} \\ P2_c & 0 & \cdots & 0 & D2_{cr} & \cdots & D2_{ct3} \end{matrix}$$

(6.11)

$$D_c$$

$$\begin{matrix} & t_1 & & t_{t1} & t_r & & t_{t3} \\ P1 & D1_{p11} & \cdots & D1_{p1t1} & 0 & \cdots & 0 \\ P1_c & D1_{c1} & \cdots & D1_{ct1} & 0 & \cdots & 0 \\ P2 & D1_{p21} & \cdots & D1_{p2t1} & D2_{p2r} & \cdots & D2_{p2t3} \\ P3 & 0 & \cdots & 0 & D2_{p3r} & \cdots & D2_{p3t3} \\ P2_c & 0 & \cdots & 0 & D2_{cr} & \cdots & D2_{ct3} \end{matrix}$$

(6.12)

Design of local supervisors by solving subtasks 1 and 2 leads to the desired places constraints on networks $N1_c$ and $N2_c$. This raises a logical question; how becomes these invariants contractions networks $N1_c$ and $N2_c$ into network N_c. The answer is the following theorem described in bibliography [1]. Let denote a set of points $P1I \subseteq P1 \cup PC1, PI \subseteq P2, P2I \subseteq P3 \cup PC2, PI' \subseteq P2'$ in the networks $N1_c$ and $N2_c$. Suppose that $p_i \in PI$ has for all i the correspondence in $p_i' \in PI'$. Then $\text{card}(PI) = \text{card}(PI')$.

The results can be summarized in the following theorem.

Theorem: Let given Petri nets $N1_c$, $N2_c$ a N_c. If $P1I \cup PI$ are the invariant places of network $N1_c$ and similarly $P2I \cup PI'$ are invariant places in metwork $N2_c$, then $P1I \cup PI \cup P2I$ are the invariant of a network N_c. Furthermore, the restrictions (6.2) (6.4) and in $N1_c$ and $N2_c$ is satisfied by constraints (5.2) in the network N_c.

The following procedure leads to the following algorithm design method of decentralized supervisor subsystems overlap.

The following procedure leads to the algorithm of design decentralized supervisor for subsystems with overlap.

6.2 Algorithm

We have a fully controllable Petri net N with a requirement for restriction (5.2) and satisfying the assumptions A1 - A4.

Determination of the distribution structure of the network N to subsystems with overlapping.

Expansion of Petri net N and formulation of subtasks.

Design local supervisors for proposal subtasks.

Contraction of local loopback control Petri nets N1c and N2c into global Petri net Nc.

7 Results

The original work proposed method largely enhances design supervisor for complex Petri nets and simplifies design by allowing you to decentralize supervision. This gives the possibility of algorithmic design and feedback systems for these procedures has not been possible to use. Generalization of methods for acyclic networks is important for ensuring interoperability of systems operating in diverse environments and interfaces for interconnection between different systems.

Limits of the proposed method consist of two requirements:

The method presupposes the full controllability and full observability of the system modeled in Petri nets. This requirement may be modified transformation procedure for changing the constraints that are known in the literature.

The method assumes that the restriction on the behavior of the controlled system can always be expressed using matrix inequalities valid for the vector giving the number of brands in different parts of the network.

The new generalization was tested on selected examples and simulations.

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