13th IFAC Symposium on Large Scale
Complex Systems: Theory and Applications
Shanghai Jiao Tong University
Shanghai, China, July 7-10, 2013

Decentralized stabilization of complex systems with delayed feedback

Lubomír Bakule *, **Manuel de la Sen** ** **Martin Papík** *
Branislav Rehák *

* *Institute of Information Theory and Automation*
Academy of Sciences of the Czech Republic
182 08 Prague 8, Czech Republic (bakule,papik,rehakb@utia.cas.cz)
** *Research Institute for Process Control*
Department of Electricity and Electronics, Faculty of Sciences
University of the Basque Country, Apdo. 644, 48 080 Bilbao, Spain
(wepdepam@lg.ehu.es)

Abstract: The paper studies the problem of decentralized state feedback control design for a class of continuous-time complex systems. These systems are composed of identical nominal subsystems, symmetric nominal interconnections, and nonlinear perturbations. We consider local time-varying delayed feedback at each channel. Single delay as well as multiple delay cases are considered. By exploiting the special structure of the systems, sufficient conditions are derived for the gain matrix selection performed on the design system of reduced dimension under linear matrix inequality approach constraints. It is shown that the robust delay-dependent stability of the global multiple delay closed-loop system is guaranteed when implementing the gain matrix into the global decentralized controller. Moreover, sufficient conditions are derived for the tolerance of local control channel failures in such a global closed-loop system. The fault tolerance can be effectively tested on systems of reduced dimensions.

Keywords: Decentralized control, symmetric interconnected systems, delayed feedback, large-scale systems

1. INTRODUCTION

Large-scale complex systems have been extensively studied since early seventies to deal with complexity as a central problem in system theory and practise. High dimensionality, uncertainty, information structure constraints, and delays are well known major motivating features for development of decentralized control theory as surveyed for instance in Šiljak [1991], Lunze [1992], Bakule [2008], Zečević and Šiljak [2010].

The paper focuses on a class of continuous-time dynamic systems composed of the interconnection of identical subsystems with identical couplings. Such systems are known as symmetric interconnected systems. They appear in very different real world systems as presented for instance in Bakule and Rodellar [1996], Bakule [2005]. In this paper, it is shown that such a structure of subsystems and interconnections enables a special analysis and control procedure. The main feature of this method is the setup of systems with reduced dimension, but keeping the same dynamic properties as shown for instance in Bakule and Lunze [1988], Bakule [2007], Bakule [2005], Bakule and de la Sen [2009]. A more complete survey of theoretic and applied results is presented in Bakule [2008], Bakule and de la Sen [2010], Bakule and de la Sen [2011], Hovd and Skogestad [1994] with the references therein.

The paper is mainly inspired by the previous works on symmetric composite systems with delayed feedback in Bakule and Lunze [1988], Bakule and de la Sen [2010] and Bakule and de la Sen [2011], as well as the results on fault tolerance for this class of systems in Huang et al. [1999] and Lam and Huang [2007]. A common well known feature of these problems is to show how the global synthesis can be simplified through a appropriate transformation to a collection of systems of reduced dimensions.

The first and main contribution of this paper is a procedure for the gain selection of the decentralized delayed feedback which guarantees the robust delay-dependent stability of the global multiple delay closed-loop system. The proper controller design is performed on the generic system of the subsystem's dimension and a single delay in

[1] L.Bakule, B. Rehák, and M. Papík are grateful to the Czech Science Foundation by its support through the Grant 13-02149S. L. Bakule and B. Rehák are grateful to the Ministry of Education, Youth and Sports of the Czech Republic by its support through the Grants LG12014 and LG12008, respectively. M. de la Sen is grateful to the Spanish Ministry of Education and to the Basque Government by their support through the Grants DPI 2012-30651 and IT-269-07, respectively, and to the UPV/EHU through the Grant 11/07.

the feedback. A convex optimization approach is used for the state feedback matrix gain selection. It is shown how to synthesize this gain matrix into the global original system to maintain the required stability property. The result is presented in the form of sufficient conditions.

The second contribution is a sufficient condition for the robust delay dependent stability of the global decentralized multiple delay closed-loop systems in the case when several local feedback controllers fail. The solved problem is to find an integer which corresponds with the smallest number of failures that make the global closed-loop system unstable. We will show that the decomposition approach results in this case to simpler test systems of reduced dimension.

To the authors' best knowledge, the problem of decentralized robustly delay-dependent controller design with multiple delay feedback as well as the problem of fault tolerance for this class of complex composite systems have not been solved up to now.

1.1 Outline of the Paper

Section 2 contains the formal state space description of the system in the structured as well as global form and the structure of feedback which is summarized in the problem statement. In Section 3, a single delay controller design as well as its extension to a multiple delay controller design are presented including computation algorithms. This section contains also fault tolerant analysis resulting in an easily calculated test conditions based on certain systems of reduced dimensions. In Section 4, an example for decentralized networked control system illustrates the potential of presented methodology.

2. PROBLEM FORMULATION

2.1 Structured System Description

Consider a nonlinear symmetric system consisting of N subsystems, where the ith subsystem is described as follows

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + h_i(t,x) \qquad x_i(t_o) = x_{io} \qquad i = 1,\dots,N \qquad N > 2 \tag{1}$$

where $x_i(t)$ and $u_i(t)$ are n- and m-dimensional vectors of the subsystem states and control inputs. The interconnections are described in the form

$$h_i(t,x) = \sum_{j=1}^{N} L_{ij}x_j(t) + h_{ij}(t,x_j) \tag{2}$$

where $A, B, L_{ii} = L_d$, and $L_{ij} = L$ denote constant nominal matrices. $h_{ij}(t,\cdot)$ are uncertain arbitrarily time-varying piecewise-continuous functions belonging to a class of piecewise-continuous real functions $\mathbf{H}_{(*)}$ over the domains of continuity $\mathbf{D}_d, \mathbf{D}$ defined as

$$\begin{aligned}\mathbf{H}_{ii} &\stackrel{\text{def}}{=} \{h_{ii}(t,\cdot) : \mathbb{R}^{n+1} \to \mathbf{D}_d | h_{ii}(t,\cdot)^T h_{ii}(t,\cdot) \\ &\leq \alpha^2 x_i^T H_d^T H_d x_i\} \\ \mathbf{H}_{ij} &\stackrel{\text{def}}{=} \{h_{ij}(t,\cdot) : \mathbb{R}^{n+1} \to \mathbf{D} | h_{ij}(t,\cdot)^T h_{ij}(t,\cdot) \\ &\leq \alpha^2 x_j^T H^T H x_j\}\end{aligned} \tag{3}$$

where H_d, H are given constant matrices and $\alpha > 0$ is a given scalar. These functions have the form

$$\begin{aligned}h_{ii}(t,x_i) &= e(t,x_i)H_d x_i(t) \\ h_{ij}(t,x_j) &= e(t,x_j)H x_j(t)\end{aligned} \tag{4}$$

where $e(t,x_j) : \mathbb{R}^{n+1} \to [-1,\ 1]$ represents normalized uncertainty parameter for all i, j.

2.2 Global System

The global system description of (1)–(4) has the form

$$S: \qquad \dot{x}(t) = A^g x(t) + B^g u(t) + h^g(t,x) \qquad x(t_o) = x_o \tag{5}$$

where $x(t) = (x_1(t)^T, ..., x_N(t)^T)^T$ are global states, while $u(t) = (u_1(t)^T, ..., u_N(t)^T)^T$ are global inputs. The nominal matrices are $A^g = (A^g_{ij}), A^g_{ii} = A + L_d, A^g_{ij} = L$ for $i \neq j$, $B^g = \text{diag}(B, ..., B)$. $h^g(t,x)$ are uncertain piecewise-continuous functions satisfying the relation

$$\begin{aligned}\mathbf{H}^g &\stackrel{\text{def}}{=} \{h^g(t,\cdot) : \mathbb{R}^{Nn+1} \to \mathbf{D}^g | h^g(t,\cdot)^T h^g(t,\cdot) \\ &\leq \alpha^2 x^T H^{gT} H^g x\}\end{aligned} \tag{6}$$

where $H^g = (H^g_{ij}), H^g_{ii} = H_d, H^g_{ij} = H$ for $i \neq j$, denote the bounds.

2.3 Delayed feedback

The goal of the paper is twofold as follows

a) Decentralized state stabilization, when the states in the feedback are delayed. The motivation for such an approach appears standardly mainly in networked control systems. Arbitrary time-varying delays acting within a given bounded interval are considered in local loops.

Consider the controller for the structured system (1) as

$$u_i(t) = Kx_i(t - \tau_i(t)) \qquad i = 1, ..., N \tag{7}$$

with the bounds

$$0 \leq \tau_i(t) \leq \overline{\tau} \tag{8}$$

where $\overline{\tau}$ is a given positive constant.

The control (7) can be equivalently considered for the global system (5) as

$$u(t) = \sum_{i=1}^{N} D_i K^g C_i x_i(t - \tau_i(t)) \tag{9}$$

where $K^g = \text{diag}(K, ..., K), D_i = \text{diag}(0, ..., 0, I, 0, ..., 0)$, and $C_i = \text{diag}(0, ..., 0, I, 0, ..., 0)$. The matrices D_i and C_i are partitioned into N blocks of identical dimensions, where I denotes the $n \times n$ and $m \times m$ identity matrices located at the ith position of the matrices D_i and C_i, respectively.

b) Fault tolerance analysis means to guarantee the asymptotic stability of the global closed-loop system (5)–(9) under l local feedback channels failures. By exploiting the special structure of symmetric systems, the stability test can be performed easily based on certain system of reduced dimension.

Consider the global closed-loop system as

$$\begin{aligned}S^c: \quad \dot{x}^c(t) &= A^g x^c(t) + \sum_{i=1}^{N} B^g D_i K^g C_i x^c_i(t - \tau_i(t)) \\ &+ h^c(t,x^c) \qquad x^c(t_o) = \Phi^c(t_o) \quad t_o \in [-\overline{\tau}, 0]\end{aligned} \tag{10}$$

where $\Phi^c(t_o)$ denotes the function of initial conditions.

Suppose that any l channels of the system (10) fail, where $l \in \{1, ..., N\}$ during a certain time interval. The dynamics of actuator failures can be modelled, without any loss of generality, by a generic model as follows

$$S^{fc}: \dot{x}^{fc}(t) = A^g x^{fc}(t) + \sum_{i=l+1}^{N} B^g D_i K^g C_i x_i^{fc}(t - \tau_i(t)) + h^c(t, x^{fc}) \quad x^{fc}(t_o) = \Phi^{fc}(t_o)\ t_o \in [-\overline{\tau}, 0] \tag{11}$$

with the first l failed channels. The indices of x^g for l failed channels are dropped in (11) to simplify the notation. The model (11) enables a simplification of the system analysis as presented later.

Remark 1. The system (11) is only the model of actuator failures. An extension of the model to setup active fault tolerant control scheme require a properly designed FDI system to isolate the actuator failures. The model (11) has a potential for such an extension, but we do not deal with it here.

2.4 The Problem

Given the system (5) and the controller (9), the primary goal is to design the gain matrix K so that the controller (9) globally asymptotically stabilizes the closed-loop system (11) for all admissible nonlinearities and a certain valid domain for the delays, i.e. it is robustly delay-dependently stable. The supplementary goal is to derive the conditions for the fault-tolerance of the closed-loop system (11), when a subset of local controller fails. Solve the problem by exploiting the special structure of the symmetric systems which enables an effective system decomposition leading to certain systems of reduced dimensions.

3. MAIN RESULTS

The proposed solution attempts to employ the special structure of a class of symmetrically interconnected identical subsystems for the stabilizing decentralized controller as well as easily calculated fault tolerance of the resulting closed-loop system. First, the gain matrix design procedure for the design model of reduced dimension with a single delay is surveyed. Then, it is shown how to use this result for multiple delay decentralized controller. Finally, a simple test for a fault tolerance of the closed-loop system is derived.

3.1 Single Delay Controller Design

This case means that $\tau_i(t) = \tau(t)$ for all i in (7) and $0 \leq \tau(t) \leq \overline{\tau}$. Then (9) has the reduced form as follows

$$u(t) = K^g x(t - \tau(t)) \tag{12}$$

where $K^g = \text{diag}(K, ..., K)$. The proper control design is performed for the n-dimensional design model

$$\dot{x}_m(t) = A_m x_m(t) + B u_m(t) + h_m(t, x_m) \tag{13}$$

and the controller

$$u_m(t) = K x_m(t - \tau_m(t)) \tag{14}$$

where the model setup including the calculation of components $A_m, h_m(t, x_m) = e(t, x_m) H_m x_m(t)$ including the quadratic bound on a nonlinear perturbation H_m and $0 \leq \tau_m(t) \leq \overline{\tau}$ are presented in detail by Bakule and de la Sen [2009]. Let us summarize the controller gain selection as an algorithm.

Agreement. Denote as $P1$ the gain selection problem presented by Theorem 2 in Yu et al. [2005].

Algorithm 1.

1. Given the system (1) and the constant $\overline{\tau} > 0$. Set up the model (12) by Bakule and de la Sen [2009].

2. Solve the LMI problem $P1$ for the system (13). If the the problem $P1$ is feasible, we get the robustly delay-dependent stabilizing gain K for (14). Then go to step 3. If no feasible result is reached, then go to step 3.

3. End.

Remark 2. To simplify, we omitted the robust stabilization of the plant (13) by the control law (14) with its maximum nonlinear bound in terms of the solvability of LMIs by Yu et al. [2005].

The synthesis leads to the following theorem.

Theorem 1. Given the system (5) and a constant $\overline{\tau} > 0$. Setup the model (13) and solve the problem $P1$ by using Algorithm 1. If the problem $P1$ is feasible, use the resulting gain matrix K to construct the global gain matrix K^g by (12). Then the global closed-loop system (5), (12) is robustly delay-dependently stable.

Proof. It is omitted because it is given in Bakule and de la Sen [2009].

3.2 Multiple Delay Controller Design

We will show that the specific structural properties of the system (5) can be used to derive a special control synthesis procedure for the decentralized control design when local feedback loops have individual time-varying interval bounded delays. An extension of a previous single delay control design can be effectively used also in the case of multiple delay control design.

Consider the system (5) and the controller (9).

Symmetrically interconnected systems represented by a state space realization with circulant matrices of a special structure as presented by (5) are always diagonalizable Bakule and Lunze [1988], Hovd and Skogestad [1994], Massioni and Verhaegen [2009]. Denote the open-loop system (5) as S and the open-loop transformed system as S^d. The states of system S are transformed through a similarity transformation $T_{n,N}$ as follows

$$S \xrightarrow{T_{n,N}} S^d \tag{15}$$

This transformation $T_{n,N}$ has the form

$$T_{n,N} = \frac{1}{N} \begin{pmatrix} (N-1)I & -I & \dots & -I & -I \\ -I & (N-1)I & \dots & -I & -I \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -I & -I & \dots & (N-1)I & -I \\ I & I & \dots & I & I \end{pmatrix} \tag{16}$$

where I denotes the nxn identity matrix.

The system S^d is described as

$$S^d: \quad \dot{x}^d(t) = A^d x^d(t) + B^d u(t) + h^d(t, x^d) \qquad \tilde{x}^d(t_o) = x_o^d \tag{17}$$

where

$$A^d = \mathrm{diag}(A_s, ..., A_s,\ A_o)$$

$$B^d = \frac{1}{N}\begin{pmatrix} (N-1)B & -B & \dots & -B & -B \\ -B & (N-1)B & \dots & -B & -B \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -B & -B & \dots & (N-1)B & -B \\ B & B & \dots & B & B \end{pmatrix} \tag{18}$$

$$h^d(t, \tilde{x}) = \mathrm{diag}(h_s(t, x_1^d), ..., h_s(t, x_{N-1}^d),\ h_o(t, x_N^d))$$

and

$$\begin{aligned} A_s &= A + L_d - L \\ A_o &= A_s + NL \\ h_s(t, x_i^d) &= e(t, x_i^d)(H_d - H)x_i^d(t) \\ h_o(t, x_N)^d &= h_s(t, x_N^d) + e(t, x_N^d)NHx_N^d(t) \end{aligned} \tag{19}$$

There are identical first $N-1$) subsystems in (15). It is evident that such a decomposition considerably simplifies the analysis Bakule and Lunze [1988], Lunze [1992], Bakule and de la Sen [2009].

The synthesis is based on the decomposition approach by Massioni and Verhaegen [2009]. It leads to the following main theorem for the synthesis of the multiple-delay system based on the system of reduced dimension.

Theorem 2. Given the system (5) and a constant $\overline{\tau} > 0$. Setup the model (13) and solve the problem $P1$ by using Algorithm 1. If the problem $P1$ is feasible, use the resulting gain matrix K to construct the global gain matrix K^g by (9). Then the global closed-loop system (5), (9) is robustly delay-dependently stable.

Proof. It is omitted here due to the space limitation.

The synthesis problem is summarized as an algorithm.

Algorithm 2.

1. Given the system (1) and the constant $\overline{\tau} > 0$. Set up the design model (13) by the construction given in Bakule and de la Sen [2009].

2. Compute the gain matrix K by using Algorithm 1. If the problem $P1$ is non-feasible, then go to step 5.

3. Implement the gain matrix K into the controller (9).

4. Check the the robust delay-dependent stability of the global closed-loop system (10).

5. End.

Remark 3. Step 4. of Algorithm 2 can be realized in different ways. For instance, a direct computation using LMIs by Li et al. [2008], indirect verification by simulation or the stability test for a single delay by Yu et al. [2005] can be applied.

3.3 Fault Tolerance

This section deals with the tolerance of the decentralized controller (9) to actuator failures of the closed-loop system (10). Total failures of local controllers within multiple control schemes are considered. That is, entire failed local controllers are completely disconnected from the plant Šiljak [1991]. We suppose that the synthesis has been performed so that closed-loop system (10) is available. Fault tolerance problem means to find the smallest number of failures, i.e. an integer $l = l_o$, that makes the closed-loop system (11) robust delay-dependent unstable. Denote a generic time-varying delay $\tau_g(t), 0 \le \tau_g(t) \le \overline{\tau}$, where $\overline{\tau}$ is given by (8).

Consider the systems with l failures

$$S^{f1}: \quad \dot{x}^{f1}(t) = A_s x^{f1}(t) + BKx^{f1}(t - \tau_g(t)) + h_s(t, x^{f1}) \tag{20}$$

$$S^{f2}: \quad \dot{x}^{f2}(t) = A^{f2}x^{f2}(t) + B^{f3}x^{f2}(t - \tau_g(t)) + h^{f2}(t, x^{f2}) \tag{21}$$

$$S^{f3}: \quad \dot{x}^{f3}(t) = A_s x^{f3}(t) + h_s(t, x^{f3}) \tag{22}$$

where

$$A^{f2} = \begin{pmatrix} A_s + lL & \sqrt{l(N-l)}L \\ \sqrt{l(N-l)}L & A_s + (N-l)L \end{pmatrix}$$

$$B^{f2} = \mathrm{diag}(0, BK)$$

$$h^{f2}(t, x^{f2}) = e(t, x^{f2})\begin{pmatrix} H_s + lH & \sqrt{l(N-l)}H \\ \sqrt{l(N-l)}H & H_s + (N-l)H \end{pmatrix} x^{f2}(t) \tag{23}$$

A_s and $h_s(t, .)$ are defined by (19).

We derive easily computed conditions to test the robust delay-dependent stability in the form of the following theorems.

Theorem 3. Given the system (10) and a constant $\overline{\tau} > 0$. Consider that one local controller fails, i.e. $l = 1$ in the system (11). If the systems S^{f1} and S^{f2} are robustly delay-dependently stable, then the closed-loop system (11) is robustly delay-dependently stable.

Remark 4. The robust delay-dependent stability can be influenced by the selection of the gain K only for $l = 1$.

Theorem 4. Given the system (10) and a constant $\overline{\tau} > 0$. Consider that l local controllers fail in the system (11), where $2 \le l \le N-2$. If the systems S^{f1}, S^{f2}, and S^{f3} are robustly delay-dependently stable, then the closed-loop system (11) is robustly delay-dependently stable.

Remark 5. Both the open-loop system with l subsystems without any control as well as the closed-loop system with $(N-l)$ subsystems with local feedback compose the resulting test system for $2 \le l \le N-2$.

Theorem 5. Given the system (10) and a constant $\overline{\tau} > 0$. Consider that l local controllers fail in the system (11), where $l = N-1$. If the systems S^{f1} and S^{f3} are robustly delay-dependently stable, then the closed-loop system (11) is robustly delay-dependently stable.

Remark 6. The requirement of the robust stability on the system S^{f3} is a necessary condition for the tolerance more than two local controllers.

Proof. Proofs of Theorems 3-5 are omitted here due to the space limitation.

4. EXAMPLE

The solved problem has an important application in decentralized networked control systems, where mainly time-varying delays have a direct interpretation as packet dropouts as considered for instance in Bakule and de la Sen [2009], Bakule and de la Sen [2010], Yu et al. [2005], Bakule and Papík [2012].

Networked Control System

Consider the system (1)–(6) with $N = 3$ as follows

$$\dot{x}(t) = \begin{pmatrix} A&0&0\\0&A&0\\0&0&A\end{pmatrix} x(t) + \begin{pmatrix} B&0&0\\0&B&0\\0&0&B\end{pmatrix} u(t) + \begin{pmatrix}0&L&L\\L&0&L\\L&L&0\end{pmatrix} x(t) + \begin{pmatrix}h(x_1)\\h(x_2)\\h(x_3)\end{pmatrix} + \begin{pmatrix} h'(x_2)+h'(x_3)\\ h'(x_1)+h'(x_3)\\ h'(x_1)+h'(x_2)\end{pmatrix} \qquad (24)$$

where the states are $x(t) = (x_1(t)^T, x_2(t)^T, x_3(t)^T)^T$ with $x_i(t) = (x_{i,1}, x_{i2})^T$. The matrices are defined as

$$A = \begin{pmatrix} -0.3 & 1\\ 0 & -3.4\end{pmatrix} \quad B = \begin{pmatrix}0\\1\end{pmatrix} \quad L_d = \begin{pmatrix}0&0\\0&0\end{pmatrix} \quad L = \begin{pmatrix}-0.1 & 0\\ 0.1 & 0.1\end{pmatrix} \qquad (25)$$

Denote $h_i(t, x_i) = h(s)$ for $s = x_i$ and $h_{ij}(t, x_j) = h'(s)$ for $s = x_j$, $i, j = 1, 2, 3$. The nonlinear perturbations are

$$h(s) = h(s_1, s_2) = \begin{pmatrix} a s_1 \cos s_1 \\ b \sin s_2\end{pmatrix}$$
$$h'(s) = h'(s_1, s_2) = \begin{pmatrix} c s_1 \sin q s_1 \\ d \sin r s_2\end{pmatrix} \qquad (26)$$

The parameters of nonlinear terms are $a = 0.1, b = 0.1, c = 0.05, d = 0.025, q = 1, r = 1$. The quadratic bounds are

$$h^T(s_1, s_2) h(s_1, s_2) = a^2 s_1^2 \cos^2 s_1 + b^2 \sin^2 s_2 \le \begin{pmatrix} s_1 & s_2\end{pmatrix} \begin{pmatrix} a^2 & 0\\ 0 & b^2\end{pmatrix} \begin{pmatrix} s_1\\ s_2\end{pmatrix} \qquad (27)$$

The bound matrix H_p has the form

$$H_d = \begin{pmatrix} a & 0\\ 0 & b\end{pmatrix} \qquad (28)$$

An analogous way of reasoning leads to the relation

$$h'^T(s_1, s_2) h'(s_1, s_2) = c^2 s_1^2 \sin^2(q s_1) + d^2 \sin^2(r s_2) \le c^2 q^2 s_1^2 + d^2 r^2 s_2^2 \qquad (29)$$

The matrix H has the form

$$H = \begin{pmatrix} cq & 0\\ 0 & dr\end{pmatrix} \qquad (30)$$

Consider the initial condition for the system (10) $x(0) = (x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}, x_{3,1}, x_{3,2})^T = (1, -1, 2, -2, 3, -3)^T$.

The goal is to design the gain matrix K in the controller (7) when considering the network with $d_c = 0.1$ and maximal number of the packet dropouts $d_{ik} = 4, i = 1, 2, 3$ and to test local controllers failures.

Results

The control design model is constructed for the matrices A_m and H_m of the system (13) by the method described by Bakule and de la Sen [2009]. The resulting matrices are given as

$$A_m = \begin{pmatrix} -0.35 & 1\\ 0.05 & -3.35\end{pmatrix}$$
$$H_m = \begin{pmatrix} -0.15 & 0 & 0.1 & 0 & 0.025 & 0 & 0.075 & 0\\ 0.15 & 0.15 & 00.1 & 0 & 0.0125 & 0 & 0.0375 & \end{pmatrix} \qquad (31)$$

The gain matrix K with $\gamma = \alpha^{-1} = 0.0001$ has been selected according to Theorem 1 in the form

$$K = \begin{pmatrix} -3.7924 & -1.1525 \end{pmatrix} \qquad (32)$$

State responses, control, and delays in the local feedback loops are shown in Figs.1-3. The initial function is supposed as $x(t_o) = 0$ for $t_o \in [-\overline{\tau}, 0)$.

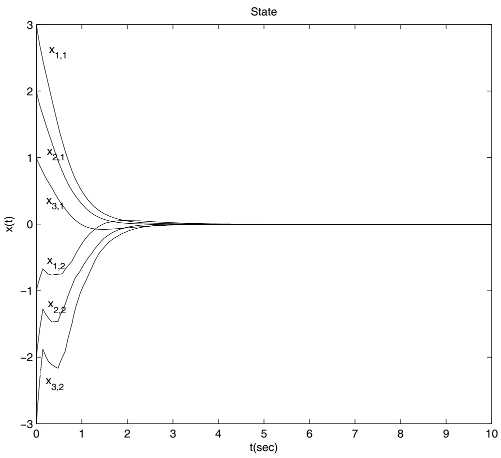

Fig.1. State responses

Figure 1 plots the state responses of the overall closed–loop network system with individual delays in the local feedback loops. It illustrates the robust-delay dependent stability of the global system.

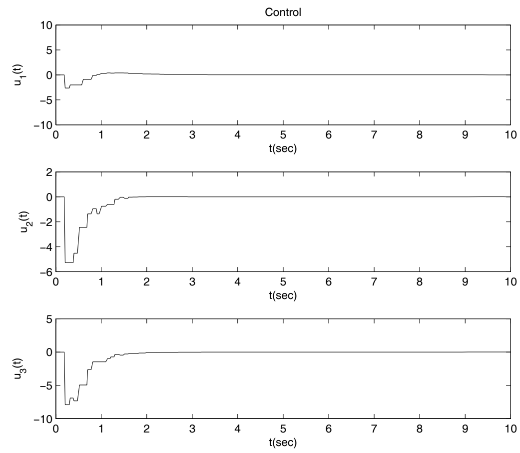

Fig.2. Local control

Figure 2 plots local control of the global closed–loop system with ZOH in the controller-to-actuator part of the feedback loop.

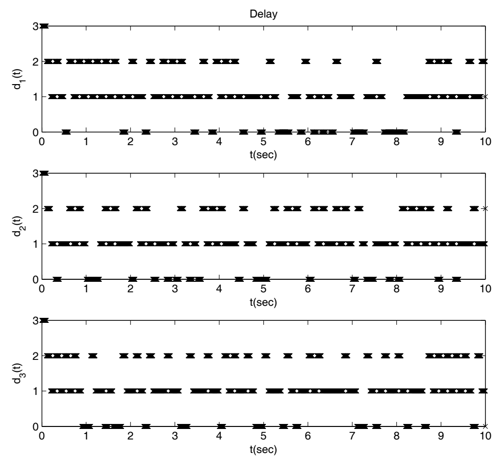

Fig.3. Delays in the local loops

Figure 3 plots individual delays $\tau_i(t)$ caused by the packet dropouts with the upper bound $\overline{\tau} = 0.5$ and the sampling period $\Delta = 0.1$. The minimal value of the delays $\tau_i(t)$ is 0.1. The delays of the packet dropouts were generated randomly by using the uniform distribution.

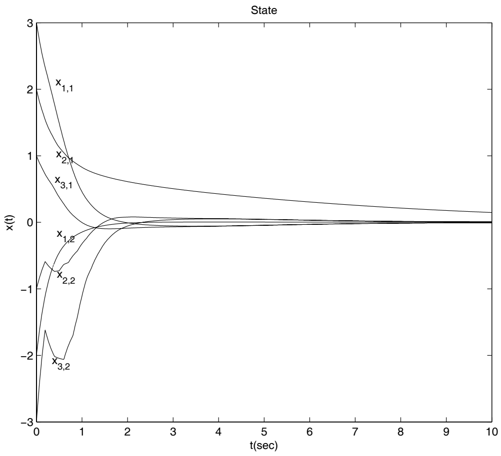

Fig.4. State responses with $l = 1$

Figure 4 plots the states with an actuator failure in the 2nd controller. The global closed-loop system maintains its stability under one channel failure but with a worse performance as shown in Figs. 1 and 4. It corresponds with a-priori expectations. More than one failure results in instability of the global system. The gain selection by Algorithm 1 was solved using the Sedumi 1.1 package in Matlab R2012a. Predefined values of options were not changed. To facilitate the LMI problem definition, the Sedumi Interface 1.04 was used. The delays in the control loop are uniformly distributed, their values are generated by the function randn.

5. CONCLUSION

In this paper, we have presented new procedures for designing decentralized state feedback controllers for a special class of continuous-time interconnected systems composed of identical interconnection and identical subsystems. We consider local time-varying delayed feedback at each channel. Single delay as well as multiple delay cases are considered. By exploiting the special structure of the systems, it is possible to decompose synthesis problems into a set of smaller ones. The problem of robust delay-dependent stability of the global system is solved by the gain matrix selection for a reduced-order design system. Moreover, the problem of the fault tolerance of the stabilized global system is solved by the decomposing the problem into a set of three lower order problems. The method has been applied for the gain matrix design within the networked control system example. The presented methodology has a potential to setup an active fault tolerant system with the robust delay-dependent stability issues for interconnected systems.

REFERENCES

L. Bakule. Complexity-reduced guaranteed cost control design for delayed uncertain symmetrically connected systems. In *Proceedings of the 2005 American Control Conference*, pages 2590–2595, Portland, OR, 2005.

L. Bakule. Stabilization of uncertain switched symmetric composite systems. *Nonlinear Analysis: Hybrid Systems*, 1(2):188–197, 2007.

L. Bakule. Decentralized control: An overview. *Annual Reviews in Control*, 32:87–98, 2008.

L. Bakule and M. de la Sen. Decentralized stabilization of networked complex composite systems with nonlinear perturbations. In *Proceedings of the 7th IEEE International Conference on Control and Automation*, pages 2272–2277, Christchurch, New Zealand, 2009.

L. Bakule and M. de la Sen. Decentralized resilient H_∞ observer-based control for a class of uncertain interconnected networked systems. In *Proceedings of the 2010 American Control Conference*, pages 1338–1343, Baltimore, MD, USA, 2010.

L. Bakule and M. de la Sen. Decentralized H_∞ quantizers design for networked complex composite systems. In *Proceedings of the 18th World Congress of the IFAC*, pages 2674–2679, Milano, Italy, 2011.

L. Bakule and J. Lunze. Decentralized design of feedback control for large-scale systems. *Kybernetika*, 24(3–6): 1–100, 1988.

L. Bakule and M. Papík. Decentralized control and communication. *Annual Reviews in Control*, 36(1):1–10, 2012.

L. Bakule and J. Rodellar. Decentralized control design of uncertain nominally linear symmetric composite systems. *IEE Proceedings – Control Theory and Applications*, 143:630–536, 1996.

M. Hovd and S. Skogestad. Control of symmetrically interconnected plants. *IEEE Transactions on Automatic Control*, 30:957–973, 1994.

S. Huang, J. Lam, G.H. Yang, and S. Zhang. Fault tolerant decentralized H_∞ control for symmetric composite systems. *IEEE Transactions on Automatic Control*, 44: 2108–2114, 1999.

J. Lam and S. Huang. Decentralized H_∞ control and reliability analysis for symmetric composite systems with dynamic output feedback case. *Dynamics of Continuous, Discrete and Impulsive Systems. Series B: Applications and Algorithms*, 14(3):445–462, 2007.

B. Li, B. Xu, and X. Shi. Delay-dependent stability criteria for a class of linear systems with multiple time-varying delays and nonlinear perturbations. In *Proceedings of the 7th World Congress on Intelligent Control and Automation*, pages 7246–7249, Chongquing, China, 2008.

J. Lunze. *Feedback Control of Large-Scale Systems.* Prentice Hall, London, 1992.

P. Massioni and M. Verhaegen. Distributed control for identical dynamically coupled systems: A decomposition approach. *IEEE Transactions on Automatic Control*, 54 (1):124–135, 2009.

D.D. Šiljak. *Decentralized Control of Complex Systems.* Academic Press, New York, 1991.

M. Yu, L. Wang, and T. Chu. Sampled-data stabilisation of networked control systems with nonlinearity. *IEE Proceedings-Control Theory and Applications*, 152(6): 609–614, 2005.

A. Zečević and D.D. Šiljak. *Control of Complex Systems. Structural Constraints and Uncertainty.* Springer Verlag, Berlin–Heidelberg, 2010.