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On Offset Free Generalized Predictive Control

Offset Free Reference Tracking of Step and Ramp Reference Signals

Květoslav Belda
Department of Adaptive Systems
Institute of Information Theory and Automation, AS CR
Prague, Czech Republic
belda@utia.cas.cz

***Abstract*—The paper deals with offset free reference tracking problem for reference signals composed of step and ramp functions. The proposed solution arises from usual design of generalized predictive control supplemented with specific modifications suppressing or ideally removing undesirable offset (steady state error) from required behavior. Furthermore, the modifications may improve additionally the behavior of the control process. The solution follows the usual form of the state-space formulation for multi-input multi-output systems and does not change mathematical model describing the controlled system against other solutions. In that context, the dimensionality of the design stays the same as in case of usual predictive control design. The proposed solution is demonstrated by a simulation of simple single-input single-output system of second order, but developed algorithms are general even for multidimensional systems.**

Keywords—generalized predictive control; offset free problem; step and ramp reference signals; single and double integrator

I. Introduction

Offset free reference tracking is a frequent requirement in many industrial control applications. The problem solution is especially important for mechatronic systems, where accuracy, i.e. zero offset from reference signal, is important, required property. Application example is e.g. a speed control of permanent magnet synchronous motor drive [4], [10] (Fig. 1) or control of robotic manipulation system in general [5].

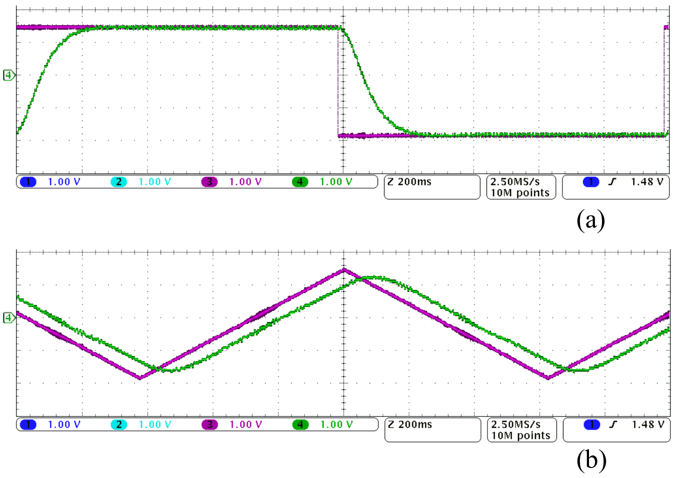

Fig. 1. Example of PMSM drive under single incremental predictive control – record of real exp. response on step (a) and ramp (b) reference signals [4].

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Generalized Predictive Control (GPC) [1] is an efficient due to its flexibility and comprehensibility. However, an undesirable offset of the system output from the reference signal (set points, desired values) may occur in a GPC implementation owing to its positional, proportional character. Furthermore, in any real application, some unmeasured disturbance is present or mathematical model describing a controlled system for control design may be imperfect. These unpleasant circumstances contribute to the occurrence of the increase control error too [3].

There exist several approaches to avoid undesirable offset feature in the literature. The focus is on offset free unknown disturbance and offset free reference tracking problems. The approaches are based on specific state-space estimation techniques [2], where magnitudes of the disturbance or control error are estimated, or simply on specific spreading initial model of controlled system by appropriate number of integrators. Thus, such solutions lead usually to some augmented model and sequentially to augmented state of the system.

The proposed solution embeds integration to the design. However, it does not change initial model of the system, but suitably modifies optimization function of predictive control only. In addition to offset free reference tracking of mentioned reference signals, the modification of penalization of output increments, reducing the ripple of the system output, is included. The modification suppresses oscillations or overshoots caused by inclusion of integrative character into control algorithms.

The paper is organized as follows. In the section II., the used model and its modifications are defined and described. The modifications relate to inclusion of single and subsequently double integrator into the control design to add integral character to Generalized Predictive Control. Section III. briefly discusses features of step and ramp signals considered as reference signals in view of final value theorem for Laplace s-domain. The following, the main, section IV. deals with the design of Generalized Predictive Control for individual reference signals composed of step and ramp functions. There is also description of optimization procedures, result of which is real control actions. The last sections V. and VI. demonstrate the theoretical explanation by comparative example for absolute, and proposed single and double incremental GPC algorithms. They contain brief discussion of the algorithms' features.

II. MODEL DESCRIPTION

In this section, the basal components will be prepared for specific derivation of equations of predictions, which are significant for the design of predictive control. Two components are addressed here. One is a state-space model and its incremental use and the second is an evolution model of control error (offset).

Thus, let us consider the standard linear state-space model of controlled system

$$\begin{aligned} \hat{\mathbf{x}}_{k+1} &= \mathbf{A}\,\mathbf{x}_k + \mathbf{B}\,\mathbf{u}_k \\ \mathbf{y}_{k+1} &= \mathbf{C}\,\mathbf{x}_k \end{aligned} \tag{1}$$

and evolution model of aggregated control error

$$\mathbf{e}_k = \mathbf{e}_{k-1} + \mathbf{w}_k - \mathbf{y}_k \tag{2}$$

Note, that parameters $\mathbf{A}, \mathbf{B}, \mathbf{C}$ of the state-space model may be both time-invariant and also time-variant. The model (1) can be considered also in difference equations as follows

$$\begin{aligned} \hat{\mathbf{x}}_{k+1} - \mathbf{x}_k &= \mathbf{A}\,(\mathbf{x}_k - \mathbf{x}_{k-1}) + \mathbf{B}\,(\mathbf{u}_k - \mathbf{u}_{k-1}) = \Delta\hat{\mathbf{x}}_{k+1} \\ \hat{\mathbf{y}}_{k+1} - \mathbf{y}_k &= \mathbf{C}\,\mathbf{A}\,(\mathbf{x}_k - \mathbf{x}_{k-1}) + \mathbf{C}\,\mathbf{B}\,(\mathbf{u}_k - \mathbf{u}_{k-1}) = \Delta\hat{\mathbf{y}}_{k+1} \end{aligned} \tag{3}$$

Then, after definition of the increments $\Delta\mathbf{x}_k, \Delta\mathbf{y}_k, \Delta\mathbf{u}_k$, the incremental state-space form is

$$\begin{aligned} \Delta\hat{\mathbf{x}}_{k+1} &= \mathbf{A}\,\Delta\mathbf{x}_k + \mathbf{B}\,\Delta\mathbf{u}_k \\ \Delta\hat{\mathbf{y}}_{k+1} &= \mathbf{C}\,\mathbf{A}\,\Delta\mathbf{x}_k + \mathbf{C}\,\mathbf{B}\,\Delta\mathbf{u}_k \end{aligned} \tag{4}$$

This arrangement does not mean any change of the model, but it is base of incremental character of predictive control. However, the whole (absolute) values $\mathbf{y}_k, \mathbf{u}_k$ and appropriate functional predictions are still necessary. Considering equations (3) and (4), the functional formula for estimations of future system outputs can be constructed as follows:

$$\begin{aligned} \hat{\mathbf{y}}_{k+1} &= \mathbf{y}_k + \Delta\hat{\mathbf{y}}_{k+1} \\ \hat{\mathbf{y}}_{k+2} &= \hat{\mathbf{y}}_{k+1} + \Delta\hat{\mathbf{y}}_{k+2} \\ \hat{\mathbf{y}}_{k+2} &= \mathbf{y}_k + \Delta\hat{\mathbf{y}}_{k+1} + \Delta\hat{\mathbf{y}}_{k+2} \\ &\vdots \\ \hat{\mathbf{y}}_{k+N} &= \mathbf{y}_k + \sum_{i=1}^{N} \Delta\hat{\mathbf{y}}_{k+i} \end{aligned} \tag{5}$$

and new control action or input to the system is

$$\mathbf{u}_k = \mathbf{u}_{k-1} + \Delta\mathbf{u}_k \tag{6}$$

The principles of equations (2), (4) - (6) will be used in next explanation for control computation or composition of equations of predictions in main section on predictive control (section IV.).

III. STEP AND RAMP REFERENCE SIGNALS

Step and ramp functions and their combinations belong to a set of basic shapes used for modeling of reference signals. This section sums up their properties important for control design with relation to steady state or offset.

Initially, let a is some arbitrary real constant, which can be steady for considered time interval or deterministically changeable from one constant value to another constant value, where changing period is multiply longer than the sampling period used for control.

Let as consider the description of reference signals or their basal functions with the use of the Laplace transformation and appropriate rules. The step signal is defined in time domain and in Laplace s-domain as follows

$$w(t) = a\,1(t)\,, \quad W(s) = a\,\frac{1}{s} \tag{7}$$

In this case, the constant a determines magnitude of the step function.

In a similar way, ramp signal is defined in time domain and in Laplace s-domain as follows

$$w(t) = a\,t\,1(t)\,, \quad W(s) = a\,\frac{1}{s^2} \tag{8}$$

In this case, constant a determines slope of the ramp function.

From Laplace transformation point of view, the both functions represent single or double integration of the impulse with magnitude a, i.e. $w(t) = a\,\delta(t)$ and $W(s) = a \cdot 1 = a$.

To analyze undesirable offset, the final value theorem [9] applied to the controlled system and reference signal can be used. It should give zero values for zero offsets as is indicated

$$\lim_{t\to\infty} e(t) \;=\; \lim_{s\to 0} s\; \mathrm{G}_{we}(s)\; \mathrm{W}(s) \;\to\; 0 \tag{9}$$

According to this theorem, it is necessary to provide at least one integrator in control loop for step reference signal and at least two integrators for ramp reference signals etc. GPC controller in general does not add any integrator. The inclusion of integration property is described in the next section.

IV. PREDICTIVE CONTROL DESIGN

A. Preliminaries of the Offset Free Control Design

In this subsection, the fundamental terms are listed as follows:

$$\Delta\mathbf{x},\;\; \Delta\mathbf{y},\;\; \mathbf{y},\;\; \mathbf{w},\;\; \mathbf{e},\;\; \Delta\mathbf{u},\;\; \mathbf{u}\,.$$

The subsequent subsections will deal with their mathematical expression, description and utilization. All terms are defined as vectors respecting general multidimensional definition. They represent increments and absolute values of the system state, output, reference, error and control. Defined terms are used in cost functions in a control design.

B. Equations of Predictions

The equations of predictions mathematically express, within prediction horizon N, functional estimations of future systems outputs in relation to unknown future control actions. Their composition influences significantly properties of computed control actions.

For both defined tracking problems, it is possible to start from model forms defined in section II. and from a composition based on recursive principle and to continue similarly

$$\begin{aligned} \Delta\hat{\mathbf{x}}_{k+1} &= \mathbf{A}\ \Delta\mathbf{x}_k + \mathbf{B}\ \Delta\mathbf{u}_k \\ \Delta\hat{\mathbf{x}}_{k+2} &= \mathbf{A}^2\Delta\mathbf{x}_k + \mathbf{A}\ \mathbf{B}\ \Delta\mathbf{u}_k + \mathbf{B}\ \Delta\mathbf{u}_{k+1} \\ &\vdots \\ \Delta\hat{\mathbf{x}}_{k+N} &= \mathbf{A}^N\Delta\mathbf{x}_k + \mathbf{A}^{N-1}\mathbf{B}\ \Delta\mathbf{u}_k + \cdots + \mathbf{B}\ \Delta\mathbf{u}_{k+N-1} \end{aligned} \tag{10}$$

From (10) and considering (4), the estimates of future increments of system outputs are defined as follows

$$\begin{aligned} \Delta\hat{\mathbf{y}}_{k+1} &= \mathbf{C}\ \mathbf{A}\ \Delta\mathbf{x}_k + \mathbf{C}\ \mathbf{B}\ \Delta\mathbf{u}_k \\ \Delta\hat{\mathbf{y}}_{k+2} &= \mathbf{C}\ \mathbf{A}^2\ \Delta\mathbf{x}_k + \mathbf{C}\ \mathbf{A}\ \mathbf{B}\ \Delta\mathbf{u}_k + \mathbf{C}\ \mathbf{B}\ \Delta\mathbf{u}_{k+1} \\ &\vdots \\ \Delta\hat{\mathbf{y}}_{k+N} &= \mathbf{C}\ \mathbf{A}^N\Delta\mathbf{x}_k + \mathbf{C}\ \mathbf{A}^{N-1}\ \mathbf{B}\ \Delta\mathbf{u}_k + \cdots + \mathbf{C}\ \mathbf{B}\ \Delta\mathbf{u}_{k+N-1} \end{aligned} \tag{11}$$

They can suitably be written in matrix notation as follows

$$\Delta\hat{\mathbf{Y}} = \mathbf{f}_1\ \Delta\mathbf{x}_k + \mathbf{G}_1\ \Delta\mathbf{U} \tag{12}$$

$$\Delta\hat{\mathbf{Y}} = [\Delta\hat{\mathbf{y}}_{k+1}, \Delta\hat{\mathbf{y}}_{k+2}, \cdots, \Delta\hat{\mathbf{y}}_{k+N}]^T \tag{13}$$

$$\Delta\mathbf{U} = [\Delta\mathbf{u}_k, \Delta\mathbf{u}_{k+1}, \cdots, \Delta\mathbf{u}_{k+N-1}]^T \tag{14}$$

$$\mathbf{f}_1 = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \\ \mathbf{CA}^N \end{bmatrix}, \quad \mathbf{G}_1 = \begin{bmatrix} \mathbf{CB} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{CAB} & \mathbf{CB} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{CA}^{N-1}\mathbf{B} & \mathbf{CA}^{N-2}\mathbf{B} & \cdots & \mathbf{CB} \end{bmatrix} \tag{15}$$

Considering (11) and (5), the estimates of future system outputs are the following

$$\begin{aligned} \hat{\mathbf{y}}_{k+1} &= \mathbf{y}_k + \mathbf{C}\ \mathbf{A}\ \Delta\mathbf{x}_k + \mathbf{C}\ \mathbf{B}\ \Delta\mathbf{u}_k \\ \hat{\mathbf{y}}_{k+2} &= \mathbf{y}_k + \sum_{i=1}^{2}\{\mathbf{C}\ \mathbf{A}^i\}\ \Delta\mathbf{x}_k + \sum_{i=1}^{2}\{\mathbf{C}\ \mathbf{A}^{i-1}\ \mathbf{B}\}\ \Delta\mathbf{u}_k + \mathbf{C}\ \mathbf{B}\ \Delta\mathbf{u}_{k+1} \\ &\vdots \\ \hat{\mathbf{y}}_{k+N} &= \mathbf{y}_k + \sum_{i=1}^{N}\{\mathbf{C}\ \mathbf{A}^i\}\ \Delta\mathbf{x}_k + \sum_{i=1}^{N}\{\mathbf{C}\ \mathbf{A}^{i-1}\ \mathbf{B}\}\ \Delta\mathbf{u}_k \\ &\quad + \sum_{i=1}^{N-1}\{\mathbf{C}\ \mathbf{A}^{i-1}\ \mathbf{B}\}\ \Delta\mathbf{u}_{k+1} + \cdots + \mathbf{C}\ \mathbf{B}\ \Delta\mathbf{u}_{k+N-1} \end{aligned} \tag{16}$$

The estimates (16) can be written also in matrix notation.

$$\hat{\mathbf{Y}} = \mathbf{f}_1\ \mathbf{y}_k + \mathbf{f}_2\ \Delta\mathbf{x}_k + \mathbf{G}_2\ \Delta\mathbf{U} \tag{17}$$

$$\hat{\mathbf{Y}} = [\hat{\mathbf{y}}_{k+1}, \hat{\mathbf{y}}_{k+2}, \cdots, \hat{\mathbf{y}}_{k+N}]^T \tag{18}$$

$$\mathbf{f}_1 = [\mathbf{I}, \quad \mathbf{I}, \quad \cdots, \quad \mathbf{I}]^T \tag{19}$$

$$\mathbf{f}_2 = \begin{bmatrix} \mathbf{CA} \\ \sum_{i=1}^{2}\mathbf{CA}^i \\ \vdots \\ \sum_{i=1}^{N}\mathbf{CA}^i \end{bmatrix}, \quad \mathbf{G}_2 = \begin{bmatrix} \mathbf{CB} & \mathbf{0} & \cdots & \mathbf{0} \\ \sum_{i=1}^{2}\mathbf{CA}^{i-1}\mathbf{B} & \mathbf{CB} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \sum_{i=1}^{N}\mathbf{CA}^{i-1}\mathbf{B} & \sum_{i=1}^{N-1}\mathbf{CA}^{i-1}\mathbf{B} & \cdots & \mathbf{CB} \end{bmatrix} \tag{20}$$

Considering (16) and (2), the estimates of future control error are like that

$$\begin{aligned} \mathbf{e}_k &= \mathbf{e}_k \quad (= \mathbf{e}_{k-1} + \mathbf{w}_k - \mathbf{y}_k) \\ \hat{\mathbf{e}}_{k+1} &= \mathbf{e}_k + \mathbf{w}_{k+1} - \mathbf{y}_{k+1} \\ &= \mathbf{e}_k + \mathbf{w}_{k+1} - \mathbf{y}_k - \mathbf{C}\ \mathbf{A}\ \Delta\mathbf{x}_k - \mathbf{C}\ \mathbf{B}\ \Delta\mathbf{u}_k \\ \hat{\mathbf{e}}_{k+2} &= \hat{\mathbf{e}}_{k+1} + \mathbf{w}_{k+2} - \mathbf{y}_{k+2} \\ &= \mathbf{e}_k + \mathbf{w}_{k+1} + \mathbf{w}_{k+2} - 2\ \mathbf{I}\ \mathbf{y}_k - (2\mathbf{CA} + \mathbf{C}\ \mathbf{A})\ \Delta\mathbf{x}_k \\ &\quad - (2\mathbf{C}\ \mathbf{B} + \mathbf{CAB})\ \Delta\mathbf{u}_k - \mathbf{C}\ \mathbf{B}\ \Delta\mathbf{u}_{k+1} \\ &\vdots \\ \hat{\mathbf{e}}_{k+N-1} &= \mathbf{e}_k + \sum_{i=1}^{N-1}\{\mathbf{w}_{k+i}\} - (N-1)\ \mathbf{y}_k - \sum_{i=1}^{N-1}\{(N-i)\ \mathbf{C}\ \mathbf{A}^i\}\ \Delta\mathbf{x}_k \\ &\quad + \sum_{i=1}^{N-1}\{(N-i)\ \mathbf{C}\ \mathbf{A}^{i-1}\ \mathbf{B}\}\ \Delta\mathbf{u}_k + \cdots + \mathbf{C}\ \mathbf{B}\ \Delta\mathbf{u}_{k+N-2} \end{aligned} \tag{21}$$

Then, corresponding matrix notation is

$$\hat{\mathbf{E}} = \mathbf{f}_1\ \mathbf{e}_k - \mathbf{f}_0\ \mathbf{y}_k - \mathbf{f}_3\ \Delta\mathbf{x}_k - \mathbf{G}_3\ \Delta\mathbf{U} + \mathbf{W}_s \tag{22}$$

$$\hat{\mathbf{E}} = [\mathbf{e}_k, \hat{\mathbf{e}}_{k+1}, \cdots, \hat{\mathbf{e}}_{k+N-1}]^T \tag{23}$$

$$\mathbf{f}_0 = [\mathbf{0}, \quad \mathbf{I}, \quad 2\mathbf{I}, \quad \cdots, \quad (N-1)\ \mathbf{I}]^T \tag{24}$$

$$\mathbf{f}_3 = [\mathbf{0} \quad \mathbf{CA} \quad \sum_{i=1}^{2}(3-i)\mathbf{CA}^i \quad \cdots \quad \sum_{i=1}^{N-1}(N-i)\mathbf{CA}^i]^T \tag{25}$$

$$\mathbf{G}_3 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{CB} & \mathbf{0} & & \\ \sum_{i=1}^{2}(3-i)\mathbf{CA}^{i-1}\mathbf{B} & \mathbf{CB} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \\ \sum_{i=1}^{N-1}(N-i)\mathbf{CA}^{i-1}\mathbf{B} & \sum_{i=1}^{N-2}(N-1-i)\mathbf{CA}^{i-1}\mathbf{B} & \cdots\ \mathbf{CB} & \mathbf{0} \end{bmatrix} \tag{26}$$

$$\mathbf{W}_s = [\mathbf{0}, \quad \mathbf{w}_{k+1}, \quad \mathbf{w}_{k+1} + \mathbf{w}_{k+2}, \quad \cdots, \quad \sum_{i=1}^{N-1}\{\mathbf{w}_{k+i}\}]^T \tag{27}$$

C. Quadratic Cost Function for Step Reference Signals

Let us consider the following quadratic cost function

$$
\begin{aligned}
J_k = &\sum_{j=No+1}^{N} \| (\hat{\mathbf{y}}_{k+j} - \mathbf{w}_{k+j})\, \mathbf{Q}_{\mathbf{yw}} \|^2 + \| \Delta\hat{\mathbf{y}}_{k+j}\, \mathbf{Q}_{\Delta\mathbf{y}} \|^2 \\
&+ \| \Delta\mathbf{u}_{k+j-1}\, \mathbf{Q}_{\Delta\mathbf{u}} \|^2
\end{aligned}
\qquad (28)
$$

and its condensed form using vectors and matrices

$$
\begin{aligned}
J_k = &(\hat{\mathbf{Y}} - \mathbf{W})^T \mathbf{Q}_{\mathbf{YW}}^T \mathbf{Q}_{\mathbf{YW}} (\hat{\mathbf{Y}} - \mathbf{W}) + \Delta\hat{\mathbf{Y}}^T \mathbf{Q}_{\Delta\mathbf{Y}}^T \mathbf{Q}_{\Delta\mathbf{Y}} \Delta\hat{\mathbf{Y}} \\
&+ \Delta\mathbf{U}^T \mathbf{Q}_{\Delta\mathbf{U}}^T \mathbf{Q}_{\Delta\mathbf{U}} \Delta\mathbf{U}
\end{aligned}
\qquad (29)
$$

where N is a horizon of prediction, vector $\mathbf{W}$ is an augmented vector of desired values (reference signal) composed as:

$$
\mathbf{W} = [\mathbf{w}_{k+1}, \quad \mathbf{w}_{k+2}, \quad \cdots, \quad \mathbf{w}_{k+N}]^T \qquad (30)
$$

and $\mathbf{Q}_{\mathbf{YW}}, \mathbf{Q}_{\Delta\mathbf{Y}}, \mathbf{Q}_{\Delta\mathbf{U}}$ represent augmented weighting block-diagonal matrices (penalizations) composed from component penalizations $\mathbf{Q}_{\mathbf{yw}}, \mathbf{Q}_{\Delta\mathbf{y}}, \mathbf{Q}_{\Delta\mathbf{u}}$ as indicated in (31):

$$
\mathbf{Q}_{(\mathbf{O})} = \begin{bmatrix} \mathbf{Q}_{(\mathbf{o})} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{Q}_{(\mathbf{o})} \end{bmatrix} \left|\begin{array}{l} \text{where subscripts represent:} \\ (\mathbf{O}) = \{\mathbf{YW}, \Delta\mathbf{Y}, \Delta\mathbf{U}\} \\ (\mathbf{o}) = \{\mathbf{yw}, \Delta\mathbf{y}, \Delta\mathbf{u}\} \end{array}\right. \qquad (31)
$$

The cost function (29) represents function with prepared integration for Δu plus penalization of output increments Δy.

D. Quadratic Cost Function for Ramp Reference Signals

Let us consider another, similar quadratic cost function

$$
\begin{aligned}
J_k = &\sum_{j=No+1}^{N} \| (\hat{\mathbf{y}}_{k+j} - \mathbf{w}_{k+j} - \hat{\mathbf{e}}_{k+j-1})\, \mathbf{Q}_{yw} \|^2 + \| \Delta\hat{\mathbf{y}}_{k+j}\, \mathbf{Q}_{\Delta y} \|^2 \\
&+ \| \Delta\mathbf{u}_{k+j-1}\, \mathbf{Q}_{\Delta u} \|^2
\end{aligned}
\qquad (32)
$$

$$
\begin{aligned}
J_k = &(\hat{\mathbf{Y}} - \mathbf{W} - \hat{\mathbf{E}})^T \mathbf{Q}_{\mathbf{YW}}^T \mathbf{Q}_{\mathbf{YW}} (\hat{\mathbf{Y}} - \mathbf{W} - \hat{\mathbf{E}}) \\
&+ \Delta\hat{\mathbf{Y}}^T \mathbf{Q}_{\Delta\mathbf{Y}}^T \mathbf{Q}_{\Delta\mathbf{Y}} \Delta\hat{\mathbf{Y}} + \Delta\mathbf{U}^T \mathbf{Q}_{\Delta\mathbf{U}}^T \mathbf{Q}_{\Delta\mathbf{U}} \Delta\mathbf{U}
\end{aligned}
\qquad (33)
$$

The equations (32) and (33) are same to the (28) and (29) expect for terms $\hat{\mathbf{e}}_{k+j-1}$ or $\hat{\mathbf{E}}$, which enable GPC to consider second integration in the control, i.e. control error integration.

E. Optimization and Minimization of the Criterion based on Square-Root of Quadratic Const Function

To optimize the equations (29) and (33), let us consider

$$
\min_{\Delta\mathbf{U}} J_k = \min_{\Delta\mathbf{U}} \mathbf{J}_k^T \mathbf{J}_k \rightarrow \min_{\Delta\mathbf{U}} \mathbf{J}_k \qquad (34)
$$

which indicates the use a square-root of the cost function only.

Then, the function (29) for step signal offset free control is

$$
\min_{\Delta\mathbf{U}} \mathbf{J}_k = \min_{\Delta\mathbf{U}} \begin{bmatrix} \mathbf{Q}_{\mathbf{YW}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{\Delta\mathbf{Y}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q}_{\Delta\mathbf{U}} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{Y}} - \mathbf{W} \\ \Delta\hat{\mathbf{Y}} \\ \Delta\mathbf{U} \end{bmatrix} \qquad (35)
$$

which leads to the system of the algebraic equations

$$
\begin{bmatrix} \mathbf{Q}_{\mathbf{YW}}\, \mathbf{G}_2 \\ \mathbf{Q}_{\Delta\mathbf{Y}}\, \mathbf{G}_1 \\ \mathbf{Q}_{\Delta\mathbf{U}} \end{bmatrix} \Delta\mathbf{U} - \begin{bmatrix} \mathbf{Q}_{\mathbf{YW}} \{\mathbf{W} - \mathbf{f}_1\, \mathbf{y}_k - \mathbf{f}_2\, \Delta\mathbf{x}_k\} \\ \mathbf{Q}_{\Delta\mathbf{Y}} (-\mathbf{f}_1)\, \Delta\mathbf{x}_k \\ \mathbf{0} \end{bmatrix} = \mathbf{0} \qquad (36)
$$

or similarly for form suitable for tracking of the ramp function

$$
\min_{\Delta\mathbf{U}} \mathbf{J}_k = \min_{\Delta\mathbf{U}} \begin{bmatrix} \mathbf{Q}_{\mathbf{YW}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{\Delta\mathbf{Y}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q}_{\Delta\mathbf{U}} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{Y}} - \mathbf{W} - \hat{\mathbf{E}} \\ \Delta\hat{\mathbf{Y}} \\ \Delta\mathbf{U} \end{bmatrix} \qquad (37)
$$

which leads to another system of the algebraic equations

$$
\begin{aligned}
&\begin{bmatrix} \mathbf{Q}_{\mathbf{YW}} (\mathbf{G}_2 + \mathbf{G}_3) \\ \mathbf{Q}_{\Delta\mathbf{Y}}\, \mathbf{G}_1 \\ \mathbf{Q}_{\Delta\mathbf{U}} \end{bmatrix} \Delta\mathbf{U} - \\
&\begin{bmatrix} \mathbf{Q}_{\mathbf{YW}} \{\mathbf{W} + \mathbf{W}_s + \mathbf{f}_1\, \mathbf{e}_k - (\mathbf{f}_1 + \mathbf{f}_0)\, \mathbf{y}_k - (\mathbf{f}_2 + \mathbf{f}_3)\, \Delta\mathbf{x}_k\} \\ \mathbf{Q}_{\Delta\mathbf{Y}} (-\mathbf{f}_1)\, \Delta\mathbf{x}_k \\ \mathbf{0} \end{bmatrix} = \mathbf{0}
\end{aligned}
\qquad (38)
$$

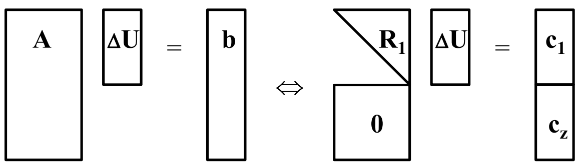
The both systems can be represented by one general system of equations, which can be solved as least squares problem [7] as indicated by (39) - (41)

$$
\mathbf{A}\, \Delta\mathbf{U} \; - \; \mathbf{b} \; = \mathbf{0} \qquad (39)
$$

$$
\mathbf{Q}^T \mathbf{A}\, \Delta\mathbf{U} = \mathbf{Q}^T \mathbf{b} \quad \text{with respect to} \quad \mathbf{A} = \mathbf{Q}\, \mathbf{R}
$$

$$
\mathbf{R}_1 \; \Delta\mathbf{U} = \; \mathbf{c}_1 \qquad (40)
$$

Orthogonal matrix $\mathbf{Q}^T$ transforms the matrix $\mathbf{A}$ to upper triangle $\mathbf{R}_1$ as indicated [8]:



(41)

Vector $\mathbf{c}_z$ is a loss vector. Its Euclidean norm $\|\mathbf{c}_z\|$ equals to the square-root of the optimal cost function minimum: scalar $\sqrt{J}$ (i.e. $J = \mathbf{c}_z^T \mathbf{c}_z$).

Note, the final control action, which should be realized, is

$$\mathbf{u}_k = \mathbf{u}_{k-1} + \Delta\mathbf{u}_k \quad \text{where} \quad \Delta\mathbf{u}_k = \Delta\mathbf{U}_{(j,1)}\Big|_{j=1,2,\cdots,nu} \tag{42}$$

where nu is a number of system inputs (control actions).

The described optimization procedure based on QR decomposition is prepared for time variant model parameters for on-line optimization repeating in every time step. For time invariant models, the results can be formulated in the form of multiplications of constant gains and appropriate real topical values of past system state, output and input [4], [6].

V. EXAMPLES

The section presents simulation results with single-input single-output system of second order defined as

$$G_s(s) = \frac{1}{s^2 + 2s + 1} \tag{43}$$

$$G_s(z^{-1}) = \frac{0.0047 + 0.0044z^{-1}}{1 - 1.81z^{-1} + 0.82z^{-2}}\Bigg|_{Ts\,=\,0.1s} \tag{44}$$

According to final value theorem (9) (or its Z-transform form), the system (43) reach zero offset $e_{w=step}(t \to \infty) = 0$ in case of step based reference, but nonzero constant offset $e_{w=ramp}(t \to \infty) = 2\ a$ (or $e_{w=ramp}(t \to \infty) = 1.95\ a$ in discrete domain $Ts = 0.1s$, Z-transform) for ramp based reference. The task of GPC design is to speed up the control process and to remove possible offset furthermore with optimizing profile of the system output suppressing ripple (oscillation, overshoot). The GPC should add at least single or double integration. The following set of the four figures Fig. 2 - Fig. 5 demonstrates the results from section IV.

Fig. 2 shows time histories of free open loop excitation of the system (43). Fig. 3 shows time histories of standard GPC with nonzero offsets. Fig. 4 shows time histories of GPC with single integrator/sum. And the last Fig. 5 shows time histories of GPC with double integrator/sum.

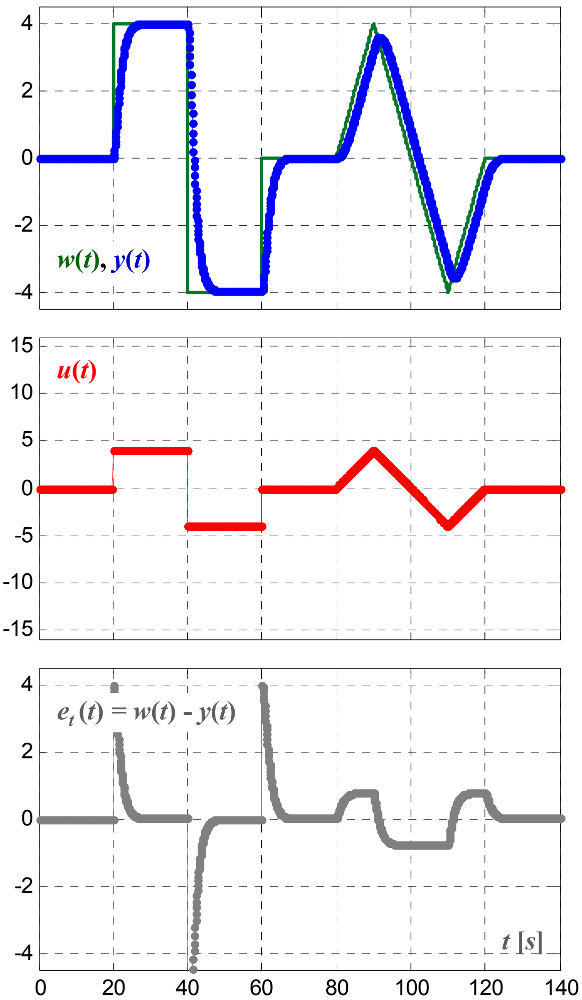


Fig. 2. Open loop excitation, i.e.system (43) without control.

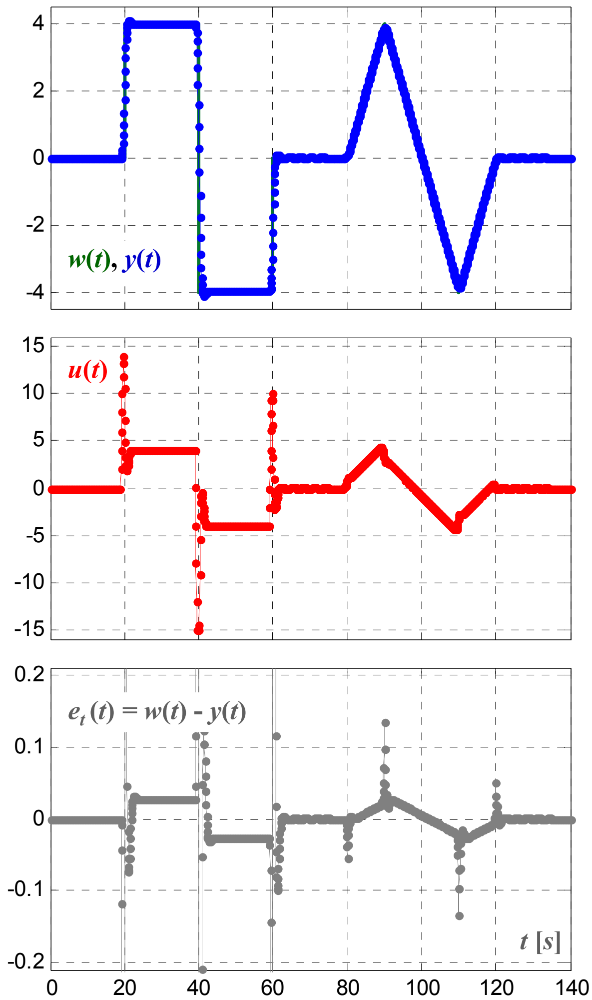


Fig. 3. GPC absolute u: $N = 10$; $qy = 1$; $qu = 0.1$, $Ts = 0.1s$.

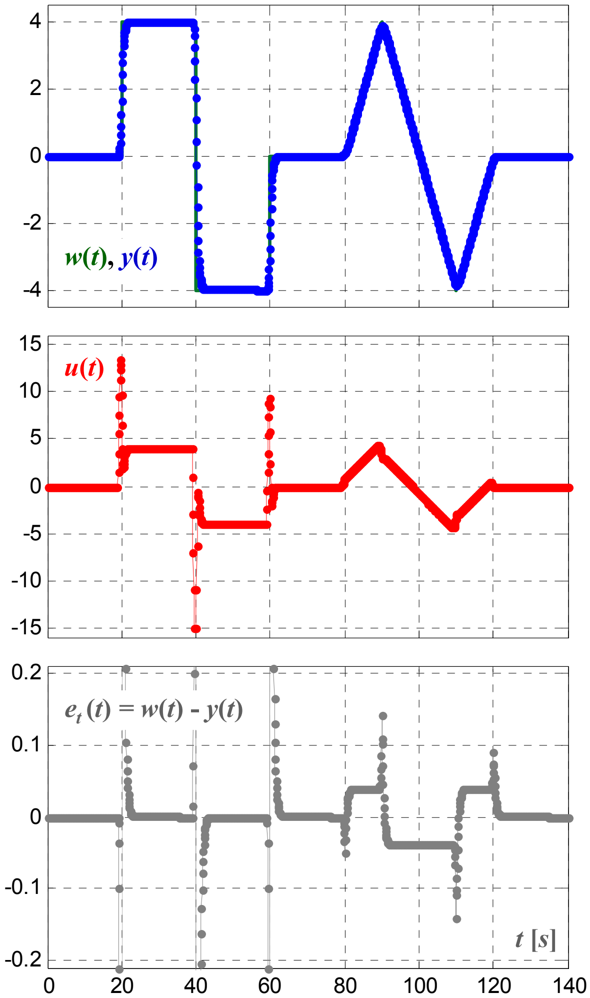


Fig. 4. GPC incr. Δu: $N = 10$; $qy = 1$; $q_{\Delta}y = 4$; $q_{\Delta}u = 0.1$, $Ts = 0.1s$.

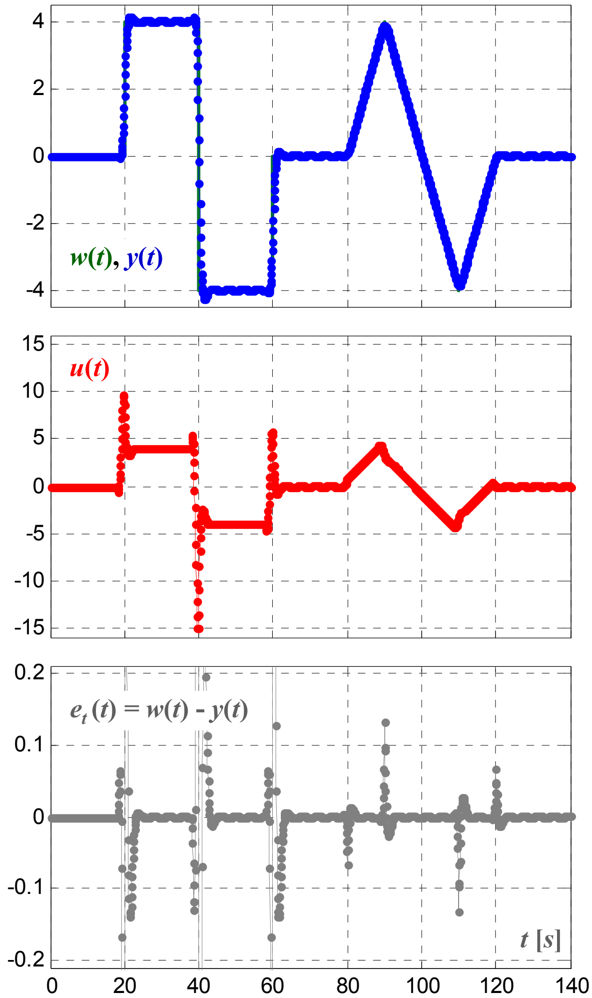


Fig. 5. GPC incr. Δe Δu: $N = 20$; $qy = 1$; $q_{\Delta}y = 20$; $q_{\Delta}u = 0.1$, $Ts = 0.1s$.

The control parameters written in figure captions represent used prediction horizon N, diagonal elements $q_{\Delta}y$ and $q_{\Delta}u$ of appropriate penalization matrices and Ts is sampling period.

In Fig. 5, the removing of all undesirable offsets is obvious for both step and ramp functions including suppression of output oscillations. The horizon N is longer so that the control actions would be comparable with previous algorithms.

VI. Conclusion

The paper deals with offset free Generalized Predictive Control. It addresses the solution especially for reference signals composed of step and ramp functions. However, the proposed solution influences offset behavior of control for any reference signals. This is caused by changing simple proportional character (P) of Generalized Predictive Control to proportional-integral character (PI, PI^2) with keeping features of the initial predictive control algorithm.

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