

Optimization of Driving Based on Currently Measured Data*

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Abstract—The paper deals with fuel consumption optimization under condition of keeping the recommended speed. The presented approach is based on data currently measured on a driven vehicle and on external observations. Using adaptive optimal control algorithms under Bayesian methodology, a compromise between fuel consumption minimization and keeping the recommended speed is reached. Research is performed in collaboration with Škoda Auto (www.skoda-auto.com).

I. INTRODUCTION

Reducing of fuel consumption and CO_2 emission is a significant problem concerning both economical and ecological parts of society life. With a gradual emergence of hybrid and electric vehicles in the market a solution might seem to be found, see e.g., [1]. The price of oil is increased and hybrid and electric vehicles promise significant fuel savings in exploitation. From an ecological viewpoint it also seems to be the most appropriate solution: with zero or minimal emissions they are suitable for low-emission zones established in some cities.

Nevertheless, conventional vehicles with combustion motors are still demanded in the market too. Firstly, purchase of hybrid or electric vehicle is still rather expensive (although in recent times reduction of prices is observed) that compensates fuel savings. Moreover, other factors such as (i) natural need of any new technology in refining and improving; (ii) slowly appearing network of charging stations, especially out-of-town; (iii) significant environmental pollution during production and disposal of electric vehicles, etc., realistically predispose to exploitation of conventional vehicles too.

A series of papers can be found in this area, e.g., [2], [3], [4]. Many of the proposed approaches are based on physical model of fuel consumption taking into account surrounding traffic conditions, see, for example, [5], [6].

The presented paper applies a systematic, generally applicable approach of dynamic programming to optimization of driving based on data currently measured on a driven vehicle and on external observations. The key features of the approach are as follows: (i) reaching a compromise between minimizing the fuel consumption and keeping the recommended speed; (ii) using pre-programmed setpoints;

(iii) superordinate deterministic control for ensuring safety speed limits; (iv) involving prior knowledge.

The layout of the papers is organized as follows. Section II formulates a problem and describes generally known solution of dynamic programming of the modified form with pre-programmed setpoints. The main emphasis of the paper is on Section III that applies general solution to the optimization of fuel consumption. Section IV demonstrates results of experiments. Section V provides remarks and future plans.

II. THEORETICAL BACKGROUND

A. Problem Formulation

Let's consider a system that at discrete time instants $t \in t^*$ produces observable outputs y_t influenced by control inputs u_t . The task to design the inputs so that to push the outputs as close as possible to their given set-points s_t .

B. Dynamic Programming

The system is described by the conditional probability density function (pdf)

$$f(y_t | \psi_t, \Theta), \quad (1)$$

taken in the form of multivariate regression model

$$y_t = \theta' \psi_t + e_t, \quad (2)$$

where

- $\psi_t = [u_t, y_{t-1}, u_{t-1}, \dots, y_{t-n}, u_{t-n}, 1]'$ is a regression vector of the n th model order,
- $\theta = [b_0, a_1, b_1, \dots, a_n, b_n, k]'$ is a vector of regression coefficients,
- e_t is a normally-distributed noise with zero mean value and fixed variance r , and $\{\theta, r\} \equiv \Theta$.

Model parameters Θ are unknown and have to be estimated. Bayesian estimation [7] is used for parameter estimation using the following relation:

$$f(\Theta | d(t)) \propto f(y_t | \psi_t, \Theta) f(\Theta | d(t-1)), \quad (3)$$

where the data d_t denote $\{y_t, u_t\}$; $d(t) = \{d_0, d_1, \dots, d_t\}$, where $d_0 \equiv d(0)$ corresponds to prior information about the system; \propto means proportionality (quality up to the normalization constant) and $f(\Theta | d(t-1))$ denotes a prior pdf at the time instant t . In case of the normal regression model (1) the parameter estimation (3) takes the form of a recursive computation of statistics with a conjugated prior Gauss-inverse-Wishart pdf. This technique can be found, e.g., in [7], [8]. Here, it is used both for the model pre-estimation from prior data and for on-line estimation from currently measured data.

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During the control design, the point estimates of parameters are used. This corresponds to a suboptimal solution to the adaptive control [9].

At each time instant t , we define penalization of deviations of outputs from given setpoints and penalization of increments of inputs as follows

$$\mathcal{J}_t = (y_t - s_t)' \omega (y_t - s_t) + (u_t - u_{t-1})' \lambda (u_t - u_{t-1}) \quad (4)$$

where ω and λ are penalization matrices. The task is to design a control minimizing the penalization.

The control strategy can be obtained by minimization of the criterion

$$E \left[\sum_{t=1}^T \mathcal{J}_t | d(0) \right] \quad (5)$$

where T denotes a finite control horizon, $\{u_1, u_2, \dots, u_T\} = u(T)$ and E denotes expectation.

To obtain causal control, the criterion is minimized backward, starting at time $t = T$. Each step of minimization $t = T, T-1, T-2, \dots$, the unknown data (output) at the time must be subdued to expectation on condition of the present output and older data. After expectation the current input is computed so that the criterion is minimized. Thus, the minimization goes recursively against the time.

$$\begin{aligned} & \min_{u(T)} E \left[\sum_{t=1}^T \mathcal{J}_t | d(0) \right] \\ &= \min_{u(T-1)} \min_{u_T} E \left[\mathcal{J}_T + \sum_{t=1}^{T-1} \mathcal{J}_t | d(0) \right] \\ &= \min_{u(T-1)} E \left[\min_{u_T} E [\mathcal{J}_T | u_T, d(T-1)] + \sum_{t=1}^{T-1} \mathcal{J}_t | d(0) \right] \\ &= (\dagger), \end{aligned} \quad (6)$$

and denoting

$$\varphi_T = E [\mathcal{J}_T | u_T, d(T-1)] \quad (7)$$

$$\text{and } \varphi_T^* = \min_{u_T} \varphi_T \rightarrow u_T^* \quad (8)$$

we obtain the following expression, continuing (6) :

$$(\dagger) = \min_{u(T-1)} E \left[\varphi_T^* + \sum_{t=1}^{T-1} \mathcal{J}_t | d(0) \right]. \quad (9)$$

Inserting $\varphi_{T+1}^* = 0$ into (5), we can see that the result has of the same form as the criterion. Thus, the computation is recursive and can be summarized into the following algorithm:

```

Set  $\varphi_{T+1}^* = 0$ 
for  $t = T, T-1, \dots, 1$  do
    expectation
         $\varphi_t = E [\varphi_{t+1}^* + \mathcal{J}_t | u_t, d(t-1)]$ 
    minimization
         $\varphi_t^* = \min_{u_t} \varphi_t \rightarrow u_t^*$ 
end

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where u_t^* is a function of data $d(t-1)$.

C. Setpoint Pre-programming

For model (2) of the second order and penalization (4) we are going to use pre-programmed setpoints s_t , i.e., known at time instant $t = T$ for backward minimization of the criterion. For this aim, from computational reasons it is advantageously to transform the model into the state-space form

$$\underbrace{\begin{bmatrix} y_t \\ u_t \\ y_{t-1} \\ u_{t-1} \\ 1 \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} a_1 & b_1 & a_2 & b_2 & k \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_M \underbrace{\begin{bmatrix} y_{t-1} \\ u_{t-1} \\ y_{t-2} \\ u_{t-2} \\ 1 \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} b_0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_N u_t + \underbrace{\begin{bmatrix} e_t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\epsilon_t}, \quad (10)$$

$$y_t = \underbrace{[1, 0, 0, 0, 0]}_A x_t \quad (11)$$

that allows to easier calculate the subsequent recursion. Rearranging (4) for the state x_t , we denote

$$\mathcal{J}_{x_t} = (x_t - s_t)' \Omega (x_t - s_t) + (u_t - u_{t-1})' \lambda (u_t - u_{t-1}). \quad (12)$$

In (12) the output setpoint s_t must be rearranged as vector $[s_t, 0, 0, 0, 0]'$ in order to correspond to the system state x_t from (10). Similarly, Ω is a diagonal matrix with ω of the appropriate dimension at the beginning of the diagonal (i.e., corresponding to the dimension of y_t) and with zeros instead of the rest of diagonal entries. After substitution of model $x_t = Mx_{t-1} + Nu_t + \epsilon_t$ into (12) and some algebraic rearrangements the obtained result takes the form of the following algorithm

```

set  $A, B, C, D, \mathcal{E}, F = 0$ 
set  $\omega, \lambda$ 
for  $t = T, T-1, \dots, 1$  do
     $R = M' (A + \omega) M,$ 
     $S = M' [(A + \omega) N + C],$ 
     $W = (N' A N + 2N' C + B + \lambda)^{-1},$ 
     $U = N' D + \mathcal{E} - N' \omega s_t,$ 
     $V = M' (D - \omega s_t),$ 
     $A = R - S W S',$ 
     $B = \lambda - \lambda' W \lambda,$ 
     $C = S W \lambda,$ 
     $D = V - S W U,$ 
     $\mathcal{E} = \lambda' W U,$ 
     $F = G - U' W U,$ 
     $u_t = -W (S' x_{t-1} - \lambda u_{t-1} + U),$ 
end

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where the pre-programmed setpoints s_t at time $t = T$ are used for the backward minimization at time $t = T-1$. It

enables to design the control input u_t knowing setpoints in advance.

III. APPLICATION TO FUEL CONSUMPTION OPTIMIZATION

General approach described in the previous section is widely spread and successfully applied in many areas. Using the setpoint pre-programming, we decided to apply it to the specific task of optimization of fuel consumption. Within the current project the control algorithms based on the fully probabilistic design [10] have been already exploited [11] as well its combination with a PID controller [12]. Here we focus on application of dynamic programming with pre-programmed setpoints.

A. Control Aim

Firstly, we have to define the aim of the control task which seems to be obvious: to design a control which would minimize fuel consumption. However, an intuitive solution of this task leads to a reducing speed of a vehicle until full stop. Thus, a compromise between minimizing fuel consumption and keeping reasonable (recommended for the route and not exceeding limits) speed should be reached. A balance of these generally contradicted demands makes the optimum of the solved task.

B. Selection of Variables and Construction of Model

For a considered task, we obtained real data measured on a driven vehicle for a selected route provided by Škoda auto (see www.skoda-auto.com). To ensure necessary dynamics, the data were measured for various economic and not too economic driving styles (8 data samples) with a sampling period 0.2 seconds. The considered route is of a length about 38 kilometers out of Prague with parts of a speed highway, out-of-town roads and roads passing through small towns with corresponding speed limits.

Originally, the available data samples contained significant number of variables. Considering a driver-vehicle system for application of the described approach, we have to select modeled (controlled) outputs y_t , primarily related to a vehicle itself. Among them, we have the outputs to be optimized, i.e., with our demand to push them as close as possible to our desired values, and the outputs not to be optimized, nevertheless bringing the useful information to construct a model of the first ones. Then we have to define input variables u_t that can influence the modeled outputs. Another group is the external variables like altitude, approaching road turn, vehicle position coordinates, etc.

After series of experiments within the project, we selected the following most informative data. For the second order model (2), the output y_t is a five-dimensional vector including the following entries:

The optimized part:

- $y_{1;t}$ – fuel consumption $[\mu l]$,
- $y_{2;t}$ – average rear wheels speed (identified with the vehicle's speed) $[km/h]$.

The non-optimized part:

- $y_{3;t}$ – engine torque $[Nm]$,
- $y_{4;t}$ – engine speed $[rpm]$,
- $y_{5;t}$ – distance traveled from the last measurement $[m]$.

The control input u_t represents pressing the gas pedal. The external variable v_t added to the regression vector is road altitude $[m]$.

Tailored to the context, the control aim is formulated as follows:

- design the control u_t expressing how much the gas pedal should be pressed, so that to
- push the fuel consumption $y_{1;t}$ and the vehicle speed $y_{2;t}$ as close as possible to their setpoints $s_{1;t}$ and $s_{2;t}$ respectively

under natural constraints of the control input: pressing the gas pedal from 0 till 100%.

The required compromise between reducing the fuel consumption and preserving the recommended speed is formulated in the control aim via tracking both the setpoints simultaneously.

C. Choice of Setpoints

The setpoints used in the control criterion are chosen in the following way. The setpoint $s_{1;t}$ for the fuel consumption is taken from the measurements with the lowest average fuel consumption as 85% of the actual fuel consumption at each time instant t .

The setpoint $s_{2;t}$ for the vehicle speed is the recommended speed provided by experts. It represents the speed from the data sample with the lowest average fuel consumption with the applied speed restrictions according to the vehicle position coordinates. It should be noted that currently the task is solved for a known route which means that the recommended speed is prepared beforehand (i.e., all setpoints for the whole route as well). Modelling the recommended speed for unknown route is planned for future work.

The vehicle position coordinates are available at each time instant. It enables to determine the current location and the recommended speed for the current location, as well as for the next one and for the whole route. It means that the setpoints are known in advance and used as pre-programmed, entering the control criterion at time t for its backward minimization. This allows to provide faster reaction of pressing the gas pedal and higher control quality that is especially critical in the case of abrupt changes of the recommended speed. Rather sensitive settings of the penalization matrices ω and λ help to find the balance between tracking both the setpoints simultaneously.

D. Logic Control Block

The presented algorithm optimizes pressing the gas pedal as the control variable, but not the brake pedal. It happens due to the fact that in prior data the brake pedal is not practically used and the model is badly excited. Moreover, when braking is realized by the brake pedal, it is mostly caused by some deterministic traffic event (a sharp turn, a road downhill, etc.).

For such situations, we place a deterministic logic control block in the time cycle after the optimization. It consists of group of logical conditions “if ..., then ...” based on general advices from eco-driving experts, where the main principle is evaluating values of external variables on the route and difference between speed and the permitted speed. However, another important reason to add this deterministic control block is ensuring strict verification of traffic restrictions. The main goal of this block is to provide pressing the brake pedal. Nevertheless, in the case of exceeding the permitted speed it can also correct pressing the gas pedal, even at the cost of losing the optimality. Thus, this block is superordinate to the optimization provided by the dynamic programming. Currently it considers the following driving situations: (i) approaching sharp turns which should be driven with low speed; (ii) exceeding the permitted speed downhill and on the flat road; (iii) driving downhill without pressing the gas pedal; (iv) approaching speed limits; (v) smooth driving after braking or stopping in previous situations; (vi) prohibition of simultaneous pressing the gas and the braking pedal.

Selection of gear is computed as the control variable based on ranges of engine speed and vehicle speed. The logic control block provides a selection of the highest gear always when it is possible as well as the neutral gear choice with a corresponding engine speed. However, optimization of gear selection is a complicated task, which will probably need estimation of a model based on logistic regression. We plan to consider this task in our future research.

Finally all these three variables – pressing the gas and the braking pedals and the selected gear – are used as the control inputs in the time cycle.

IV. RESULTS

The described control scheme is implemented in Matlab. Via serial ports it is connected with a vehicle real-time simulator provided by Škoda auto. The simulator is a separate software based on physical model of a vehicle. It represents a vehicle driven on the route shown in Figure 1 (top).

The data sample about 12000 data items is necessary in order to have the vehicle passed through the whole route with the speed close to the recommended one. For the control purpose, one of the data samples can be selected and then used for comparison with the obtained results. Figure 1 (bottom) demonstrates the route passed by the controlled vehicle corresponding to Figure 1 (top).

We can set the penalization matrix ω in the control criterion according to our actual preferences. With the help of settings of the penalizations we can choose whether we wish to drive faster, but with a higher fuel consumption, or otherwise to drive economically, but slower. It is reached by the choice of bigger penalizations of deviations either of the speed from its setpoint or of the fuel consumption. Results of experiments with various settings of penalizations are provided below. The penalization matrix λ for the input increments remains invariant during the experiments. For better illustration the fragment with 5000 data items are presented.

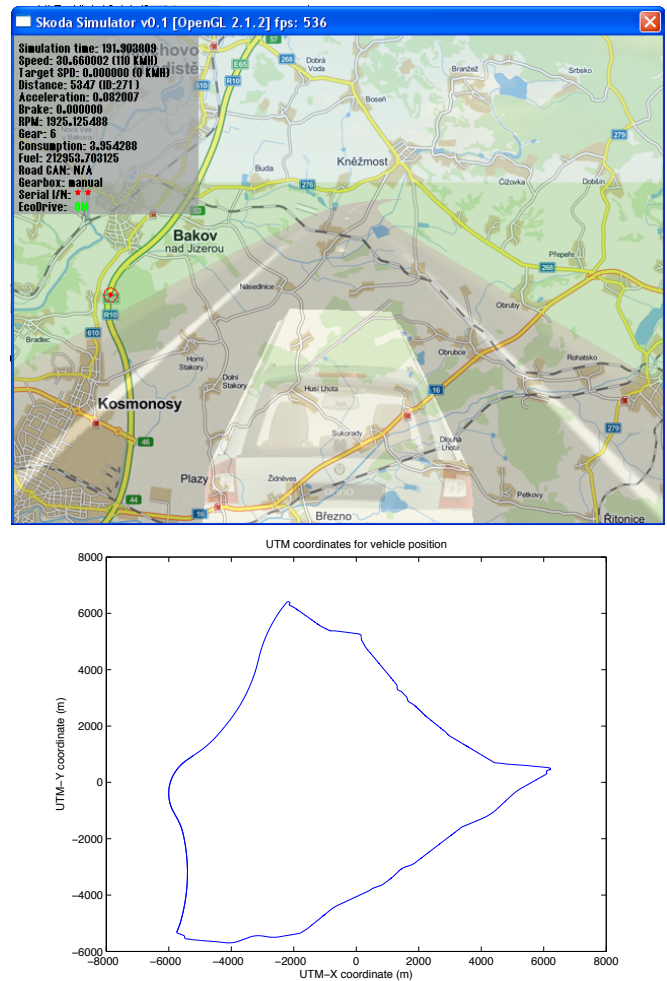


Fig. 1. The top plot provides a screenshot from the simulator with a map of the selected route and a transparent view of a vehicle. The red circle shows a location of the vehicle on the route. The bottom plot shows UTM coordinates for vehicle position after passing the whole route by the simulated vehicle.

A. Results for Control with Faster Speed Preference

Figure 2 shows the speed (top) and the fuel consumption (bottom) with bigger penalization of the speed compared with the recommended and the original real speeds. The controlled speed is really close to the recommended one, however, due to the penalization also of the fuel consumption it is a bit lower. A difference between the controlled speed and the original real speed is explained by the fact that the real speed was faster (than recommended) in the beginning of the route and therefore the time of arrival at some position was accelerated. The fuel consumption in Figure 2 (bottom) does not track its setpoint as close as possible, however, the average fuel consumption obtained for this experiment is 5.2 L/100km that is lower than the real average fuel consumption from the tested data sample: 6.3 L/100km.

Figure 3 (top) demonstrates the pressing the gas pedal that has a smoother course in comparison with the real data. Figure 3 (bottom) provides pressing the brake pedal restricted from 0.8 to 25 bar according to minimal and

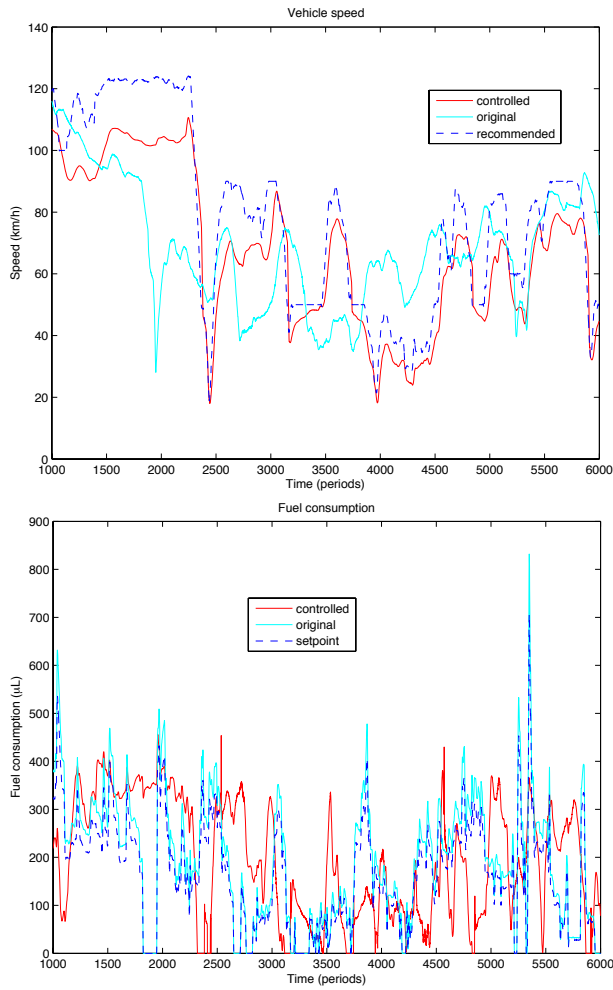


Fig. 2. Speed (top) and fuel consumption (bottom) with bigger penalization of speed deviations.

maximal pressure in the vehicle brake system. Usage of the brake pedal in Figure 3 (bottom) corresponds to the low speed in Figure 2 in places with sharp turns on the route. The obtained selection of gear is not shown here to save space, but it corresponds to general advices of eco-driving experts implemented in the logic control block to choose the highest gear where is possible with the current ranges of the engine speed and the vehicle speed.

B. Results for Control with Lower Fuel Consumption Preference

For this experiment we prefer slower economical driving, i.e., with a lower fuel consumption. Thus we set the bigger penalization for the fuel consumption. However, in this case the obtained speed is very low. To reach lower fuel consumption with the reasonable speed, we have to very slightly increase the penalization of the consumption, but remain the previous penalization of the speed.

Figure 4 shows the speed (top) and the fuel consumption (bottom) obtained with the increased penalization of the fuel consumption deviations. The obtained speed is lower than

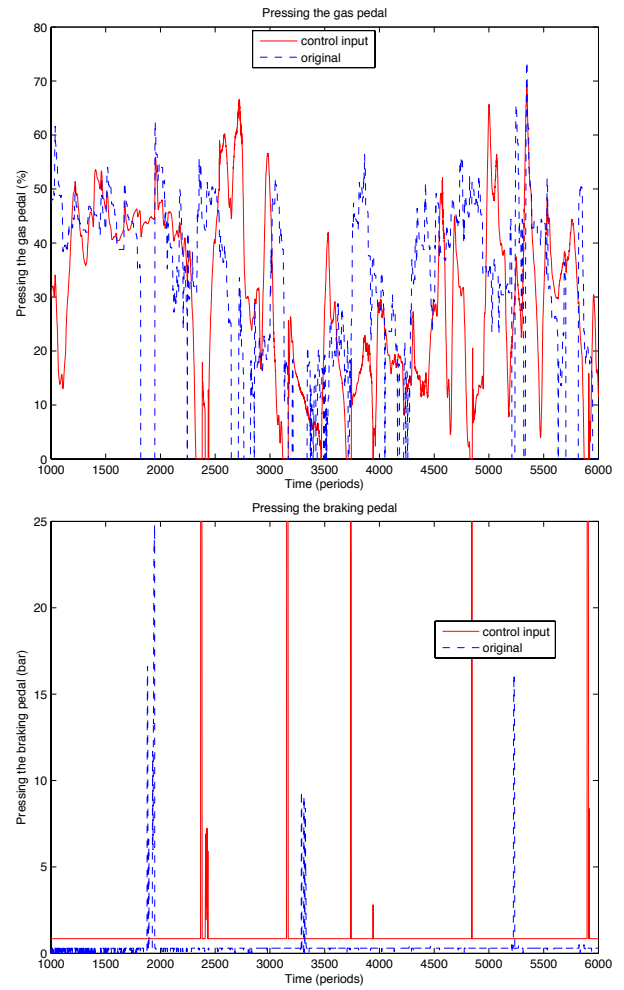


Fig. 3. Pressing the gas (top) and the brake (bottom) pedals with bigger penalization of speed deviations

the recommended one, however it is only slightly lower than the real one. The average fuel consumption obtained in this case is 4.9 L/100km that is lower both than the real fuel consumption and the previous result.

Figure 5 presents pressing the gas (top) and the brake (bottom) pedals obtained with the increased penalization of the fuel consumption deviations. Their plotted courses correspond to general rules of eco-driving and are smoother than in Figure 3. However, the selected gear (not shown here to save space) is mostly low due to the low controlled speed.

C. Discussion

A difference between the obtained results in the presented experiments might seem insignificant. However, it should be noted that even very slight changes in penalizations can bring results in the form of reducing fuel consumption or better preserving the recommended speed. The presented results also confirm the summary obtained using other control approaches within this project [11], [12].

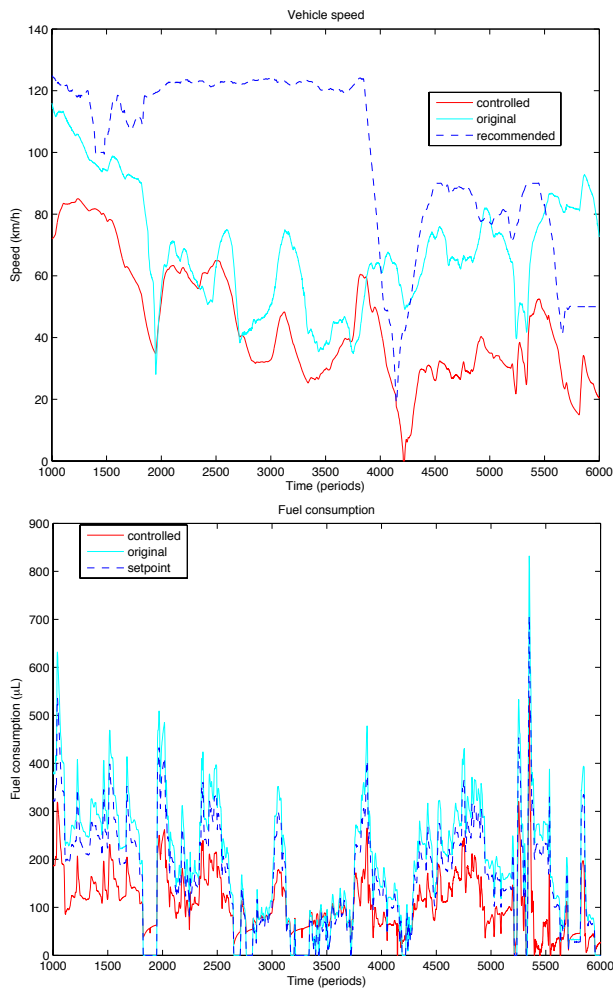


Fig. 4. Speed (top) and fuel consumption (bottom) with the increased penalization of the fuel consumption deviations

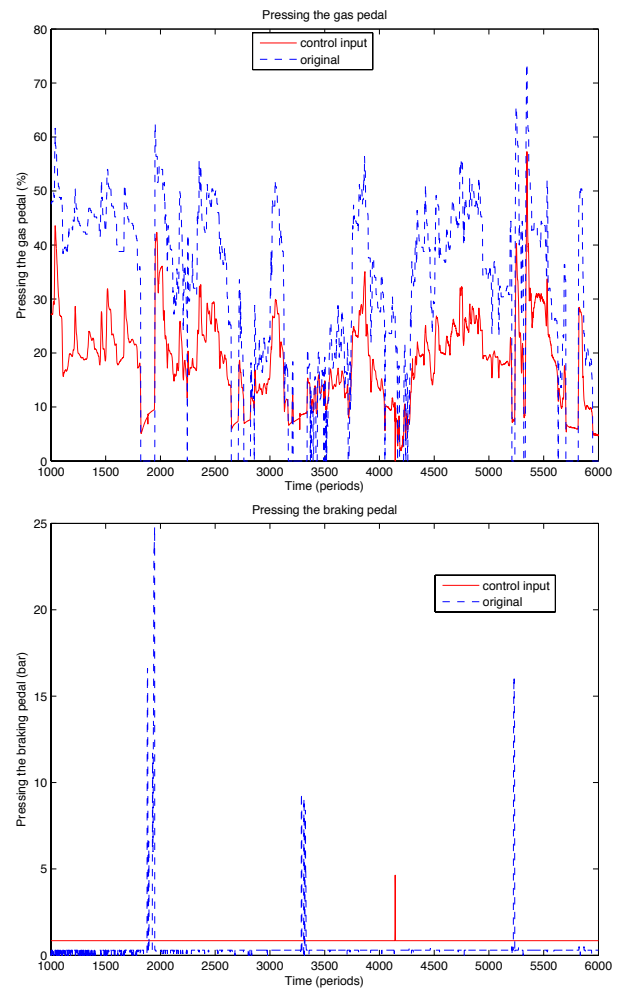


Fig. 5. Pressing the gas (top) and the brake (bottom) pedals with the increased penalization of the fuel consumption deviations

V. CONCLUSIONS

In this paper we presented the intermediate results of our project aimed at development of algorithms for optimization of driving based on currently measured data. The future work we plan is to test the algorithms on the hardware simulator of a vehicle provided by Škoda auto.

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