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Simple model for urban traffic between two signalized intersections

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Abstract— **This paper introduces a new model of traffic flow between two signalized intersections. The model originated from our effort to improve older, macroscopic model [5] based on the conservation law of vehicles. In order to keep the proposed model simple and numerically tractable, we make several simplifications: we do not take into account acceleration and deceleration of vehicles and different preferences of drivers and we classify the movements of vehicles into two classes: (i) a stopped vehicle, waiting in a queue, and (ii) a vehicle moving with a constant cruising speed. Under these assumptions the flow of vehicles between two intersections can be described by a piece-wise linear function as a combination of several "ramp" functions. We demonstrate the behavior of the proposed model on an simulated scenario of two signalized intersections.**

I. INTRODUCTION

In recent years, many cities have suffered from numerous urban traffic-related problems, including traffic safety, and ecology. As a response to this situation, modern traffic management systems are being deployed [2], [6], [7], which are able to adapt to changing traffic demands and are able to provide coordinated signal settings for large urban areas [4].

While local traffic actuated signal plans can work well at isolated intersections, any control algorithm that supervises a larger urban street network has to optimize signal settings at the whole region at once in order to achieve an optimum traffic flow through the controlled area. This optimization cannot be achieved without some kind of a model describing the current traffic situation in the region. Such a model is periodically updated by measurements from different kinds of traffic detectors [4].

The traffic model provides the user with a set of variables describing the state of the system. The model must connect these variables with control variables that influence the state – signal plans, cycle length, and signal plan offsets. In the approach, described in [5], the vector of queue lengths on the approaches to intersections was chosen as the modeled variable describing the system state. The queues are the main cause of the travel delay and the sum of the queue lengths relates to the travel delay in the system, which is a widely accepted criterion of the level of service.

II. MOTIVATION

In the course of our recent discussions over a discrete-time linear state-space model of urban traffic on a network of signalized intersections [5] we realized that the sub-optimal performance of the model for downstream intersections may be in part attributed to the fact that the model averages the traffic demand over the whole signal plan cycle (by ignoring batches of arriving vehicles due to upstream signals). This fact also complicates our current effort to optimize not only green split (and phase length), but also signal plan offsets.

Prediction of the queue length $\xi_l[k+1]$ on lane l of the controlled system based on the values in the k-th step of the simulation (the modeling step is assumed equal to the cycle length) is described by [5] as

$$\xi_l[k+1] = \begin{cases} \xi_l[k] + D_{\mathrm{in},l}[k] - S_l\, z_l[k] & \text{if } c_l[k] < d_l[k] \\ D_{\mathrm{in},l}[k](1 - z_l[k]) & \text{otherwise} \end{cases} \tag{1}$$

where $D_{\mathrm{in},l}[k]$ is the total demand in the number of vehicles that arrive to the lane the k-th modeling step, S_l is the saturation flow of the lane in vehicles per cycle, $z_l[k]$ is the relative green length with respect to the cycle length, $c_l[k]$ is the capacity of the lane given the length of the green signal, and $d_l[k]$ is the total demand taking into account both arrived and queueing vehicles. As we mentioned above, the dominant disadvantage of the model is the fact that the usage of $D_{\mathrm{in},l}[k]$ implicitly assumes uniform arrivals of vehicles also for interior intersections of the modelled network. This is to some extent plausible assumption for boundary intersections of the system, but it does not hold for traffic filtered by a set of upstream intersections.

Although we see a possibility to improve over the original model (1) by decreasing the sampling period from current 90-120 s to 5-10 s and keeping the model linear, we are facing two major obsacles: first, the amount of collected data rises significantly, and may overload rather slow and outdated communication lines between intersection controllers, and second, our industrial partner reported frequent hardware problems when trying to operate their intersection controllers with such a high sampling frequency.

Provided that we cannot change the original sampling periods, we need a more exact model, describing in more detail the movements of vehicles between two signalized intersections.

III. PIECEWISE LINEAR MODEL

Vehicle movements in the real world are quite complex and therefore also difficult to model: Different arrival and departure models exist that model the arrivals of vehicles from the outside world into the controlled network and dissipation of the queue on green signal. Also, in a complex urban street network, regularly commuting drivers tend to use their previous experience to select lanes and vehicles

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constantly change their speed. For the purpose of this paper we will simplify the behaviour of vehicles to the maximum possible extent – we will not take into account acceleration and deceleration of vehicles and different preferences of drivers and we will classify the movements of vehicles into two classes: (i) a stopped vehicle, waiting in a queue, and (ii) a vehicle moving with a constant speed of passage v.

The idea to describe the non-linear traffic phenomena in a simplified form using piecewise linear approximation has been first explored by Newell [9] and by Daganzo in his Cell Transmission Model (CTM, [3]). This model simplifies the classical continuous traffic flow models of Lighthill and Whitham [8] and Richards [12] by approximating the fundamental diagram of traffic which relates traffic flow to vehicle density. The CTM is widely used for traffic flow modeling not especially on highways [1], although applications for urban signal controlled networks exist as well [11].

Our model, however, differs from CTM in many aspects. First, CTM works with density-to-flow relationship, while our model computes the vehicle count as a function of time at certain points of the modeled network – this approach makes it to some extent similar to that of Newell [9]. Second, CTM partitions the modeled street segment into several cells of length equal to the cruising speed of vehicles multiplied by CTM time step. In our case we model every street segment as a single element. Third, CTM is able to compute an approximate distribution of the traffic density and flow over the whole segment, while our model is designed to provide us only with the queue length.

Basic building block of our model is a simple ramp function defined by its vector of parameters $\theta = (x_1, x_2, y_1, y_2)$ as

$$\rho(x,\theta) = \rho(x, x_1, x_2, y_1, y_2) = \\ = \begin{cases} y_1 & \text{if } x \leq x_1 \\ y_1 + (x - x_1)\frac{y_2 - y_1}{x_2 - x_1} & \text{if } x_1 < x < x_2 \\ y_2 & \text{if } x \geq x_2 \end{cases} \quad (2)$$

A combination of such ramp functions is used to describe the cumulative flow of vehicles in a single signal plan cycle.

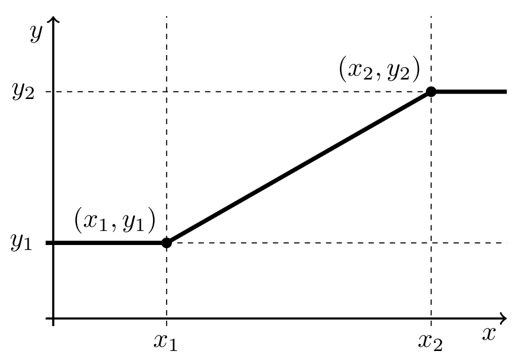

Fig. 1. Piecewise linear function described by (2).

A. Capacity of a lane

Given a signal plan timing, the cumulative capacity $c_{i,l}(\mathrm{t})$ of a lane l at intersection i at time t can be computed using (2) as

$$c_{i,l}(t) = \rho(t, t_{\mathrm{on},i,l}, t_{\mathrm{off},i,l}, 0, \frac{S_{i,l}}{3600}) \quad (3)$$

where $t_{\mathrm{on},i,l}$ and $t_{\mathrm{off},i,l}$ denote beginning and end time of the green signal in the signal plan cycle in seconds, and $S_{i,l}$ stands for the saturation flow of the lane in vehicles per hour.

B. Arrival demand at a boundary intersection

For intersection laying at the boundary of the controlled network, the cumulative demand volume $d_{i,l}(t)$ measured at the stop-bar may be described in a similar manner as

$$d_{i,l}(t) = \rho(t, 0, T_{\mathrm{c}}, 0, D_{i,l}) + \xi_{\mathrm{residual},i,l}. \quad (4)$$

Here, T_{c} is the signal plan cycle length, $D_{i,l}$ denotes the count of vehicles arriving during that cycle from the outside into the controlled network of intersections (and do so through the lane l of the intersection i). Finally, $\xi_{\mathrm{residual},i,l}$ is the residual queue of vehicles that remain in the queue from the previous cycle. Note that Equation (4) simply states that in the absence of further information about arrivals we distribute our $D_{i,l}$ arriving vehicles uniformly in the interval $\langle 0, T_{\mathrm{c}} \rangle$.

C. Arrival demand at an interior intersection

For an intersection lane that is located in the interior of the modelled network (i.e. a lane that has an upstream signalised intersection), the demand volume is given as a sum of stop-bar volumes $v_{\mathrm{sb},i,l}(t)$ generated by all relevant lanes of the upstream intersection $\mathcal{U}_{i,l}$, delayed by travel time τ that is needed to travel the distance between both intersections. Similarly to (4), the residual queue from the previous cycle inreases the total demand,

$$d_{i,l}(t) = \sum_{j,n \in \mathcal{U}_{i,l}} v_{\mathrm{sb},j,n}(t - \tau) + \xi_{\mathrm{residual},i,l}. \quad (5)$$

In fact, the travel time τ is not constant – it has to take into account the number of vehicles queuing at lane l on the red signal.

D. Combining demand and capacity

For both the boundary intersections and intersections located in the interior of the modelled network, the actual volume $v_{\mathrm{sb},i,l}(t)$ of vehicles passing over stop-bar of lane l is given by the demand (4) or (5) that is filtered by the capacity of the lane described by Equation (3),

$$v_{\mathrm{sb},i,l}(t) = d_{i,l}(t) \otimes c_{i,l}(t), \quad (6)$$

where the filtration operation $\otimes$ is defined as

$$a(t) \otimes b(t) = \min\left(a(t), b(t), \min(a(t_{\mathrm{off}}), b(t_{\mathrm{off}}))\right).$$

An example of input functions and their output is depicted in Figure 3.

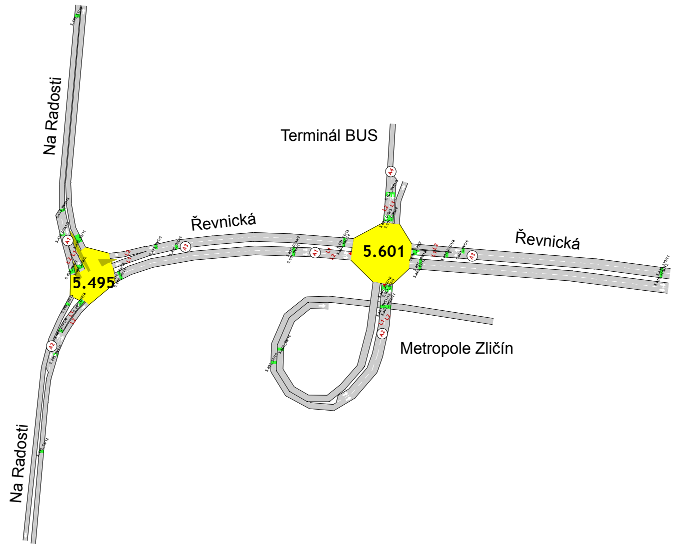

Fig. 2. Test network, consisting of intersections 5.495 and 5.601 at Prague-Zličín. For the demonstration purposes only a subset of the network has been modelled, namely entrances from "Na Radosti" to 5.495 and a segment of "Řevnická" ending with 5.601.

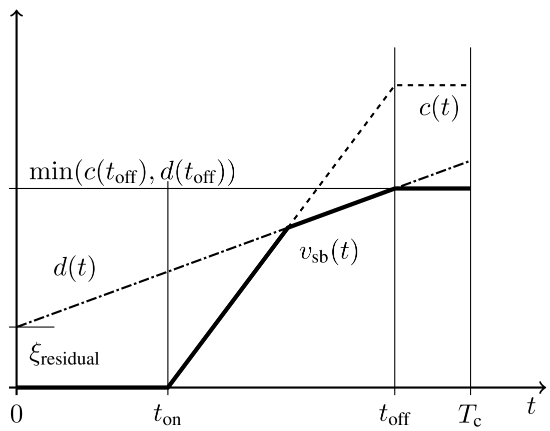

Fig. 3. Graphical demonstration of the operation $v_{\text{sb}}(t) = d(t) \otimes c(t)$ for a boundary intersection. For an interior intersection $d(t)$ would be piecewise linear.

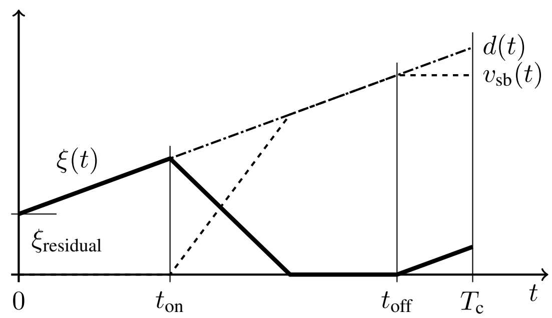

Fig. 4. Queue length $\xi(t)$ for a boundary intersection with demand and capacity corresponding to those in Figure 3.

E. Computing the queue length

Having defined the functions modeling the cumulative demand $d_{i,l}(t)$ and output volume $v_{\text{sb},i,l}(t)$ for a lane of a traffic network, we can conveniently compute also the queue length for the modeled lane. The queue length function $\xi_{i,l}(t)$ is defined as a difference between the demand and output volumes,

$$\xi_{i,l}(t) = d_{i,l}(t) - v_{\text{sb},i,l}(t), \tag{7}$$

where the demand incorporates also the residual queue $\xi_{\text{residual},i,l}$ is the residual queue of vehicles that remain in the queue from the previous cycle. An example of the queue function for a boundary intersection is given in Figure 4.

IV. Experiment

A combination of demand and capacity functions described above makes it now possible to create a model of the traffic flow through a set of intersections. Input parameters of the model are the vehicle counts measured at the boundary of the modelled network and green lengths of particular signal groups governing modelled lanes at all modelled intersections.

As a first step towards the composition of a complete urban traffic network model, the proposed approach has been tested using the TSS Aimsun micro-simulator [13]. The tested network is a subset of a sequence of intersections located in the western suburbs of Prague, around Zličín public transport terminal and shopping centres. It consists of two coordinated intersections and it is depicted in Figure 2.

In our aim to keep the simulation as close to reality as possible, we have used true input data with sampling period 90 s, recorded on 2007/12/12, as shown in Figure 5. In this preliminary phase we also set both intersections to pre-timed control with $T_{\text{c}} = 90\,\text{s}$.

The micro-simulator provided us with the simulated values for maximum queue length at every signal plan cycle and the same output was recorded from the model, where the queue length was computed using the Equation (7). As the model provides queue length as a real number, the modelled value

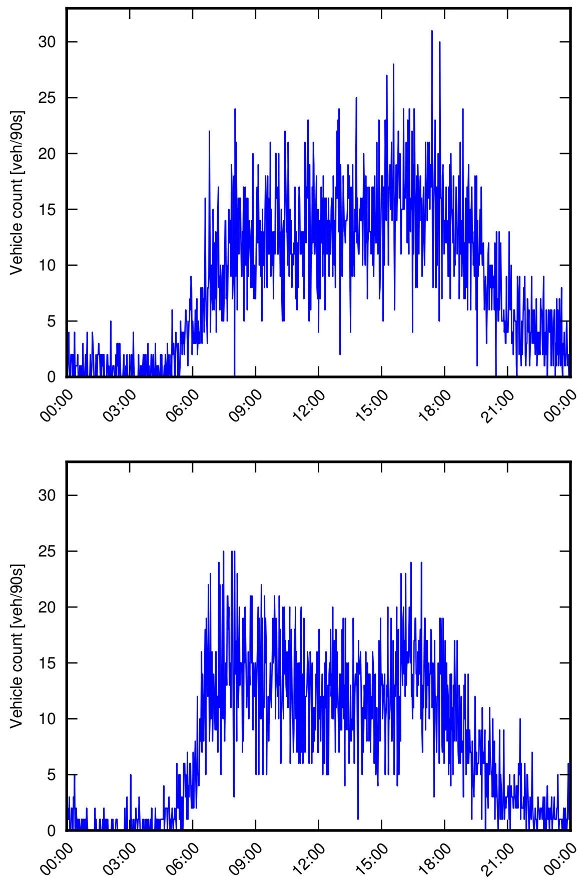

Fig. 5. Vehicle counts for "Na Radosti" – upper figure is the north entrance, lower figure shows the south entrance.

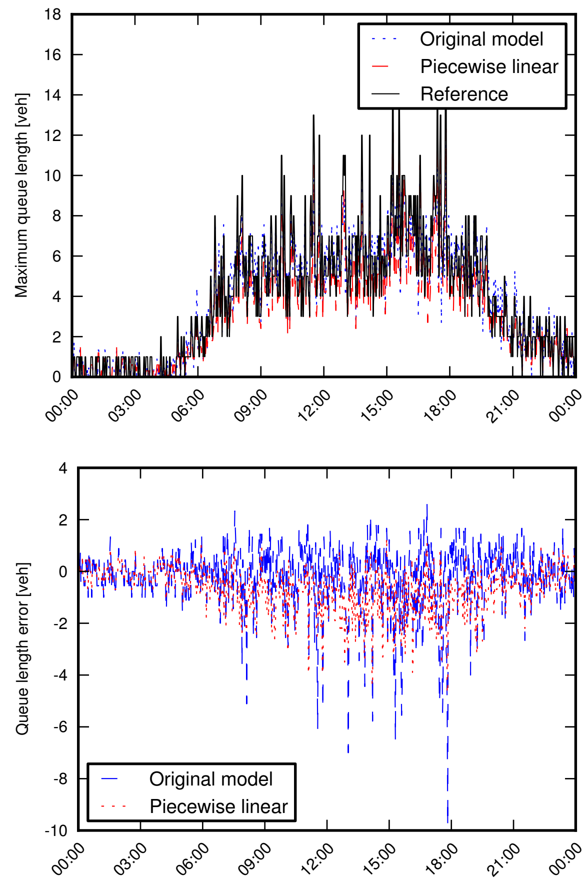

Fig. 6. Comparison of queue lengths predicted by the original model [5], the piecewise linear model presented in this paper, and queue lengths generated by the Aimsun micro-simulator: absolute values (upper) and errors of model predcitions (lower).

for every cycle was rounded to the nearest integer. The output of the comparison is depicted in Figure 6.

We can see that in our example the modelled queue length corresponds relatively well to the simulated queue length. The maximum absolute error in the estimated queue length is 5 vehicles which accounts for approximately 0.5 % of all values. The absolute error in the modelled values is at most one vehicle in 85 % of cases.

In Figure 7 we can see the detail of the modelled situation at the boundary of the test network. Demands at both input lanes at "Na Radosti" are quite low, but as we model the flow as continuum, the output of a particular lane is a real number that is given by the ratio between the arrival demand and the capacity. Both inputs are gated by their respective green signals which have to be distributed in the cycle in such a way that there is no traffic flow conflict. The safety interval between the two signal groups creates a small step when combining the output functions as visible in Figure 7 right.

The next step in the model is to compute traffic flows and queue in the "Řevnická" street. The process is outlined in Figure 8. The demand flow at the downstream intersection is given by the sum of two components: (i) the demand created by vehicles queueing from the previous cycle and (ii) the time-shifted input flow to the lane. This demand is again gated by the green signal governing the lane, resulting in transformation of the demand flow to the output flow (Figure 8 lower right) and into the new queue (Figure 8 upper right). This new queue will form the first component of the demand flow in the next cycle.

V. CONCLUSION

We have presented a new model that is capable to predict traffic flow between two signalised intersections with acceptable accuracy. The model describes traffic flow as a piecewise linear function of time. We have demonstrated that for a simple scenario consisting of two intersections the model predicts downstream queue with the tolerance of ± 1 vehicle in 85 % of cases.

This is still just a first step towards a modelling framework that could be used to model the real-world traffic. A more thorough set of simulations has to be accomplished so that we can study the sensitivity of the model especially in saturated conditions. For the purpose of this paper, a simple uniform arrival model has been used to describe arrivals of vehicles at the boundary intersection. While this choice may not be appropriate for larger networks with high traffic

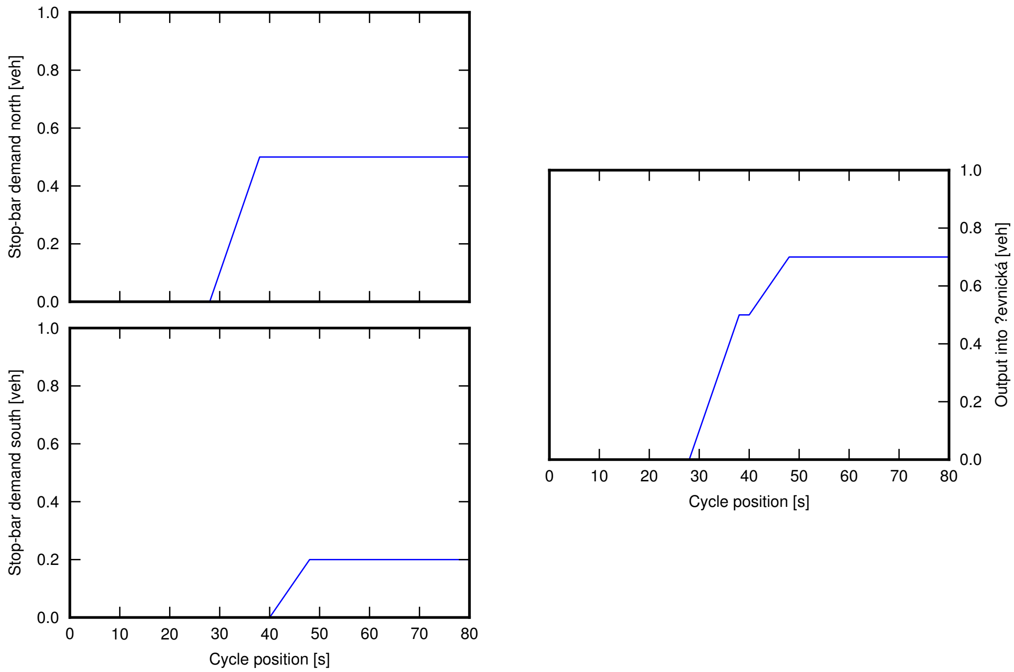

Fig. 7. Example of the input flow composition for a single lane. Demands at street "Na Radosti" are gated by their respectvie green signals so that the input flow into "Řevnická" has the form of a double ramp function.

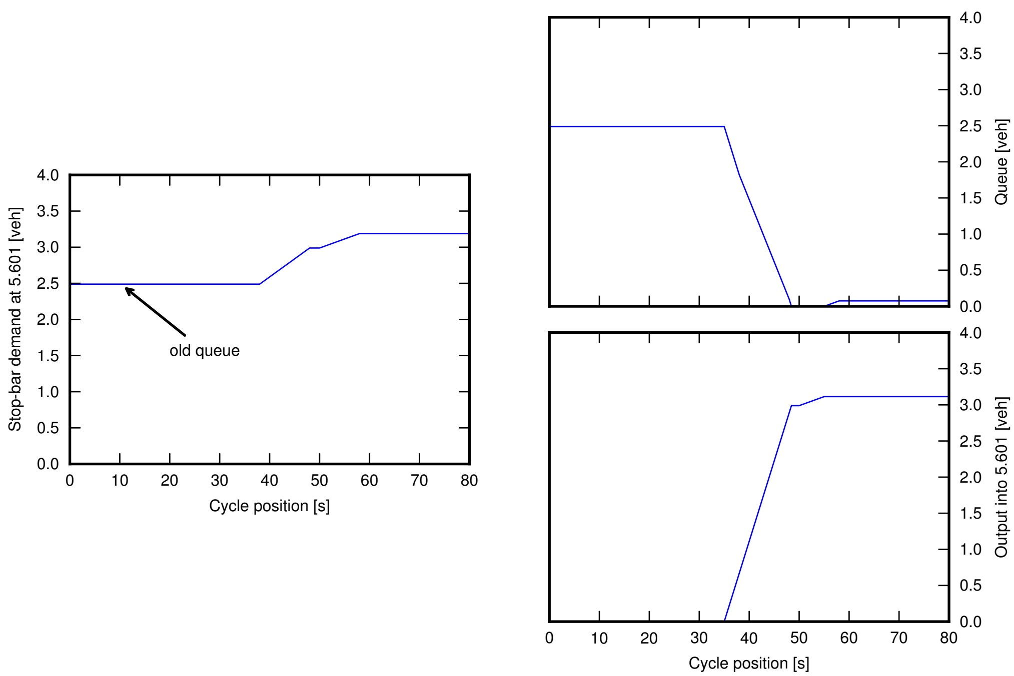

Fig. 8. Example of the flow composition at the stop-bar of intersection 5.601. The stop-bar demand flow is equal to the time-shifted input flow arriving from 5.495, depicted in Figure 7.

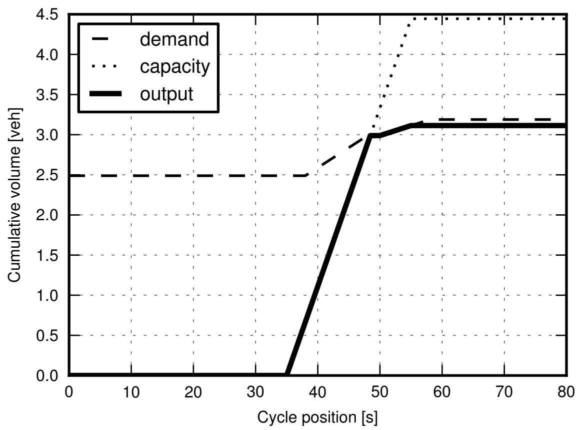

Fig. 9. Graphical detail of the output flow computation. The output demand flow is determined by the combination of the existing queue and by the arival flow from the upstream intersection. The demand is limited by the capacity of the intersection, given by the length of the green signal.

volumes, on the other hand, the effect of "filtering" the input demand by other signalised intersection may well wash out the difference for a set of intersections.

The model considers only constant speed of vehicles. Extension to several classes of vehicles with different cruising speeds is straightforward: the resulting model (which will ignore possible overtaking or lane blocking by slower vehicles) can be computed as a superposition of the partial models for different vehicle classes. Incorporating also the acceleration and deceleration modes into the model will destroy its piecewise linearity: even if we limited ourselves to constant acceleration and deceleration, the model will become piecewise quadratic.

The demonstrated model allows only single-lane approaches to an intersection, and shall be extended to allow multiple lanes, as depicted in Figure 2. The model also does not take into account the fact that in some case a signal group reaches over the actual end of the cycle T_c, i.e. $t_1 > t_2$ (the green signal begins close to T_c and ends in the next cycle). This is a typical situation that occurs with traffic-actuated signal controllers.

As with any other model, due to the necessary simplifications in the model structure and due to uncertainty in the behaviour of drivers, the values of vehicle count and queue length provided by the model do not entirely correspond to the measurements provided by traffic detectors. Hence, the model has to be extended with some kind of error estimation and correction mechanism, probably based on a non-linear versions of Kalman filter, as is the DD1 filter [10] used in our earlier works.

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