Double Optimization of Fuel Consumption and Speed Tracking ⋆

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Abstract: This paper presents automatic fuel consumption optimization with simultaneous keeping the recommended vehicle's speed. These tasks are closely related since a simple minimization of fuel consumption leads to stopping a vehicle. The proposed "double" optimization is performed online using combination of two controllers. The first of them is based on fully probabilistic design (FPD) under Bayesian methodology. It optimizes the "driver-vehicle" closed loop with the aim to save fuel and keep the recommended speed, using externally given setpoints. Optimized values serve as setpoints for PID controller, which provides necessary setpoint tracking. Research is performed in collaboration with Škoda auto (www.skoda-auto.com).

Keywords: Control applications; closed-loop control; adaptive systems; stochastic systems; autoregressive models; fuel control; vehicles; constraints.

1. INTRODUCTION

Automotive industry invests a lot in development and support of various approaches to reduce fuel consumption and CO_2 emission. Environment protection and the increasing price of oil are a main motivation for this, see e.g., Barkenbus [2010], Sivak, Schoettle [2012].

Modern conceptual solutions proposed nowadays by automotive industry are mostly found in a form of hybrid and electric vehicles, see Manzie [2010], Wirasingha, Emadi [2011], Moura et al. [2011]. They obviously have a huge potential and probably will become vehicles of future. However, the purchase price both of hybrid and electric vehicles is still rather high, although in recent times reduction of prices is observed. It compensates fuel savings. Other factors such as (i) natural need of any new technology in refining and improving; (ii) slowly appearing network of charging stations, especially out-of-town; (iii) significant environmental pollution during production and disposal of electric vehicles, etc., indicate that conventional vehicles with combustion motor will still be demanded in the marketplace too.

A series of research problems joins both conventional and hybrid and electric vehicles. Modeling an optimal eco-driving strategy is a task desired for all of them since (i) conventional vehicles need it to reduce fuel consumption and emissions; (ii) hybrid vehicles should be driven optimally not to lose a benefit of the use of hybrid powertrain; (iii) electric vehicles need to model a travel range before recharging. This paper focuses on general solution of eco-driving adopted to conventional vehicles' context.

A series of studies confirms relevance of the discussed topic, starting from Ericsson [2001], who investigates which driving pattern factors (speed profile, gear changing, etc.) have main effect on emissions and fuel consumption. Beusen et al. [2009] evaluate the long-term impact of an eco-driving training course by monitoring driving behavior and fuel consumption for several months before and after the course.

Papers found in this area include, for example, the works of Barth and Boriboonsomsin [2009], Raubitschek et al. [2011], Ben Dhaou [2011]. They are mostly devoted to algorithms based on physical description of fuel consumption, taking into account surrounding traffic conditions.

The presented paper proposes a systematic, generally applicable and dynamic, approach to modeling an eco-driving strategy. It is based on data continuously measured on a driven vehicle and on external observations. Extension of general solution up to the hybrid and electric vehicle context is straightforward and is related to available measured data.

A compromise between two contradictory demands – the fuel consumption reducing and the recommended speed tracking – is reached using the presented double optimization. It includes a combination of two controllers. The first one is based on the approach called the fully probabilistic design (FPD) described by Kárný and Guy [2006], Kárný and Kroupa [2012] under Bayesian methodology adopted in Kárný et al. [2005]. The FPD-based controller optimizes the whole "driver-vehicle" closed loop with the aim to save

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fuel and keep the recommended speed, using externally given setpoints (zero consumption and prescribed speed). The resulting FPD-optimized values are used as online generated setpoints for PID controller, which provides more precise setpoint tracking for fuel consumption and speed. The described combination of controllers is used from a reason of high safety requirements and better quality of vehicle's control. A block of logical conditions ensures the post-optimization check of safety constraints.

A remainder of the paper is organized as follows. Section 2 describes a model of the considered "driver-vehicle" closed loop and formulates the problem. Section 3 presents the fuel consumption optimization via the mentioned combination of controllers. Results are provided in Section 4.

2. "DRIVER-VEHICLE" CLOSED LOOP

Consider a "driver-vehicle" closed loop which in discrete time instants $t \in \{1, \ldots, T\} \equiv t^*$ produces the following observed variables: an output vector y_t, which is influenced by a control input vector u_t and an external variable v_t. The controlled output vector y_t includes $[y_{1;t}, y_{2;t}, y_{3;t}, y_{4;t}, y_{5;t}]'$, where $[y_{1;t}, y_{2;t}]'$ is the optimized output (to be pushed to setpoints as close as possible) and $[y_{3;t}, y_{4;t}, y_{5;t}]'$ is the non-optimized output, about those there are no user's demands. Namely, they are as follows:

- $y_{1;t}$ – the fuel consumption;
- $y_{2;t}$ – average rear wheels speed (identified with the vehicle's speed);
- $y_{3;t}$ – engine torque;
- $y_{4;t}$ – engine speed;
- $y_{5;t}$ – travelled distance;

The control input vector is $u_t \equiv [u_{1;t}, u_{2;t}, u_{3;t}]'$, where

- $u_{1;t}$ is a pressing the gas pedal;
- $u_{2;t}$ is a pressing the brake pedal;
- $u_{3;t}$ is a selected gear of transmission;

The external variable v_t is a road altitude.

The considered closed loop is described by the joint probability density function (pdf)

$$\mathcal{F} = \prod_{t \in t^*} f\left(y_t, u_t | \phi_{t-1}\right) = \prod_{t \in t^*} \underbrace{f\left(y_t | u_t, \phi_{t-1}\right)}_{\text{system model}} \underbrace{f(u_t | \phi_{t-1})}_{\text{controller}}, \quad (1)$$

where $\phi_{t-1} = [y_{t-1}, u_{t-1}, \ldots, v_t]$ and which is factorized according to the chain rule, see Peterka [1981].

2.1 Problem Formulation

The fuel optimization task is formulated as the following servo problem:

- *design the control values $u_{1;t}$ expressing how much the gas pedal should be pressed, $u_{2;t}$ related to pressing the brake pedal and $u_{3;t}$ defining a gear to be selected so that to*
- *push the fuel consumption $y_{1;t}$ towards its setpoint $y^s_{1;t} = 0$ and the vehicle speed $y_{2;t}$ as close as possible to the recommended speed $y^s_{2;t}$*

under the following constraints on the control inputs: pressing the gas pedal from 0 till 100%, pressing the brake pedal from 0.7 till 25bar, gear from 0 (neutral) to 6.

The currently used recommended speed is provided by experts for a known route. It is obtained from available measurements with the lowest fuel consumption under existing speed limits.

3. FUEL CONSUMPTION OPTIMIZATION

It can be seen that the proposed problem formulation calls simultaneously for minimization of the fuel consumption and tracking the recommended speed. This compromise is proposed to be reached via a combination of the following two controllers.

3.1 FPD Controller

The first controller is based on the FPD. This approach brings generality and dynamics to the solution. Here it is presented within the eco-driving context, however, this general approach is not limited by this application. Universality and advantages of the FPD in comparison with other tools are in detail described in Kárný and Kroupa [2012], here they are omitted to save space.

The main idea of the FPD is to select the optimized controller which pushes the "driver-vehicle" closed loop model (1) as close as possible to its ideal model $\mathcal{F}^I$. The ideal model of the closed loop is given by a user, using externally given setpoints (zero fuel consumption and the recommended speed). Having the same form as (1), $\mathcal{F}^I$ is similarly factorized in a product of the ideal system model and the ideal controller.

The optimization criterion is a minimization of the Kullback-Leibler divergence (KLD), see Kullback and Leibler [1951], between $\mathcal{F}$ and $\mathcal{F}^I$

$$D(\mathcal{F}||\mathcal{F}^I) \equiv \int_{y_t^*, u_t^*} \mathcal{F} \ln\left(\frac{\mathcal{F}}{\mathcal{F}^I}\right) \mathbf{d}[y_t, u_t]. \quad (2)$$

over $\{f(u_t|\phi_{t-1})\}_{t=1}^T$. The used form of the KLD is known to be the optimal tool within the adopted Bayesian methodology, see Bernardo [1979]. The control task with such a criterion is solved using the dynamic programming. General solution for pdfs provides the following form of the optimizing controller:

$$f(u_t|\phi_{t-1}) = \frac{f^I(u_t|\phi_{t-1}) \exp\left[-\omega(\psi_t)\right]}{\underbrace{\int f^I(u_t|\phi_{t-1}) \exp\left[-\omega(\psi_t)\right] \mathbf{d}u_t}_{\gamma(\phi_{t-1})}}, t \in t^*,$$

$$\omega(\psi_t) \equiv \int f(y_t|\psi_t) \ln\left(\frac{f(y_t|\psi_t)}{\gamma(\phi_t) f^I(y_t|\psi_t, y_t^s)}\right) \mathbf{d}y_t, \quad (3)$$

where $\psi_t \equiv [u_t', \phi_{t-1}']$ is a regression vector. Evaluations run against the time course, i.e., for $t = T, \ldots, 1$ and start with $\gamma(\phi_T) = 1$. The factorized form of all pdfs (up to individual vector entries) is used. Proof of this statement is available in Kárný et al. [2005].

3.2 FPD Controller for Normal Models

Throughout this paper, linear normal autoregression models are used for the closed loop description. In this case, the

FPD coincides with a widely spread quadratically optimal control, see Kárný et al. [2005], where penalizations in the squares of variables in the optimality criterion are the main control options. These penalizations are taken as the inversions of the noise variances of the corresponding factors of the closed loop factorized pdf.

In this case, the system model in the closed loop (1) takes a form of a multivariate normal autoregression model

$$f(y_t|\psi_t) = \mathcal{N}_y(\psi_t'\theta, r), \tag{4}$$

where θ are regression coefficients and r are the noise variances of factors. They are estimated at each step of the time cycle with a subsequent substitution of the point estimates during the control synthesis. It means that for the control they are taken as fixed. Parameter estimation is performed using Bayesian approach, see, e.g., Peterka [1981].

The optimized controller (3)

$$f(u_t|\phi_{t-1}) = \mathcal{N}_u(\eta, s) \tag{5}$$

is a part of the closed loop (1) obtained via minimization of the KLD, also in the normal form with expectations η and variances s.

The ideal closed loop model structurally stems from the considered closed loop. However, its individual factors should express the control aims. The ideal system model can be chosen, e.g., as the first order autoregression model

$$f^I(y_t|\psi_t, y_t^s) = \mathcal{N}_y^I(y_t^s, R) \tag{6}$$

with some relatively quick dynamics and constant. Using the factorized form, it can be written as

$$y_{i;t} = a_i y_{i;t-1} + k_i + e_{i;t}, \tag{7}$$

where the parameter a_i provides dynamics, and the constant k_i is set so that the steady-state value of the output entry $y_{i;t}$ is the corresponding value of the setpoint $y_{i;t}^s$. It means that according to the setpoint, the constant is obtained as

$$k_i = y_{i;t}^s(1 - a_i). \tag{8}$$

For non-optimized outputs the results of estimation are used for construction of corresponding factors of the ideal model. The ideal system model noise $e_{i;t}$ in (7) expresses the expected deviations of the ideal values from those produced by the deterministic model. Inversions of their corresponding variances R form penalizations in the quadratic control criterion.

The ideal controller takes the following form, using the input reference values $u_t^s = [u_{1;t}^s, u_{2;t}^s, u_{3;t}^s]'$ obtained from measured data:

$$f^I(u_t|\phi_{t-1}, u_t^s) = \mathcal{N}_u(u_t^s, S). \tag{9}$$

It can be chosen as a static model for respective factors

$$u_{i;t} = u_{i;t}^s + \varepsilon_{i;t}, \tag{10}$$

or for the input increments

$$u_{i;t} - u_{i;t-1} = \varepsilon_{i;t}. \tag{11}$$

The chosen ideal controller ((10) or (11)) generates the input values, where inversions of the noise variances S correspond to the inputs penalizations in the control criterion in the case of (10) or to the input increments penalizations with the use of (11).

Under assumption of normality and using the discussed models (4), (6) and (9), the optimized controller $f(u_t|\phi_{t-1})$ (5) minimizes the KLD (2) over all admissible control strategies $\{f(u_t|\phi_{t-1})\}_{t=1}^T$. This formulation leads to the dynamic programming with penalizations of the corresponding factors, resulting in distribution (5), where η are expectations used as the generated inputs.

According to Feldbaum [1961], the dual problem is not feasible. This suggests some suboptimal solution to the adaptive control to be used. For the control implementation, a methodology of receding horizon, see Kárný et al. [2005], can be used, where the newly computed point estimates of parameters are used as fixed for the control design on a given control interval. After realization of one step of control, new data are measured and used for another estimation. The mentioned estimation is performed on-line for the closed loop model including (4) and (5). The ideal system model (6) and the ideal controller (9) are fixed with the exception of the noise variances which are taken from the mentioned closed loop estimation, i.e., in (6) $R = r$ from (4), and in (9) $S = s$ from (5). Thus, the required penalizations in the control criterion become adaptive. Results obtained in Suzdaleva et al. [2012] show that the stabilized values of adaptive penalizations provide better control quality in the considered context. The IST (iterations spread in time) method is recommended, where the repeated solutions to the Riccati equation do not start from initial conditions but from the result achieved in the previous step, see Kárný et al. [1985]. Due to this, a very short control interval can be used.

At each time instant the FPD provides the optimized values of the involved variables obtained with the aim to save fuel and keep the recommended speed. Because of the safety requirements it is extremely important to provide necessary tracking the speed in places with sharp turns and abrupt changes of the speed. To ensure this, it is proposed to use the resulting FPD-optimized values as the setpoints for PID controller.

3.3 PID Controller

A standard PID controller uses the FPD setpoints for speed and fuel consumption at each time instant. Parameters for the PID controller are actually provided by experts.

The PID controller can be also switched between a driver's preference to keep the recommended speed only (that already includes its FPD-optimized setpoint) or both to save the fuel and keep the recommended speed (double optimization).

3.4 Logical Post-Optimization Block

The described automatic control provides driving where braking is realized mostly by engine. To ensure check of strict safety constraints and also for situations, where braking by engine is not sufficient, a block of logical statements "if, then do it" is placed in the time cycle after the optimization. These logical conditions are based on general advices from experts in the eco-driving field. So far, the logical block corrects pressing the gas and the braking pedal and selection of gear for the following situations:

- exceeding the maximal speed downhill and on the flat;
- approaching to the sharp turns with low speed;
- smooth starting after braking or stopping;
- driving downhill without pressing the gas pedal;
- approaching to speed limit points;
- prohibition of simultaneous pressing the gas and the braking pedal.

4. RESULTS

Data and a software vehicle simulator are provided by Škoda Auto (see www.skoda-auto.com). Measurements were conducted on a selected route about 40 kilometers out of Prague. Eight data samples with different types of driving dynamics (slower fuel-saving or faster) are available. A period of sampling is 0.2 seconds. The recommended speed obtained from the data samples with the lowest fuel consumption in the route is provided with the applied speed restrictions (i.e., the speed is cut when it meets them). The proposed approach is implemented in Matlab. Results are provided below.

4.1 Results for driver's preference to keep the recommended speed only

Here results are demonstrated for combination of the FPD and the PID controllers with the PID switched to driver's preference to keep the recommended speed only. The obtained average fuel consumption is 5.6 L/100km, which is compared to the original average fuel consumption for the used real data – 6.33 L/100km.

Figure 1 demonstrates the speed tracking for the whole considered route with parts of highway, roads and villages. It can be seen that the FPD setpoint is a bit lower the recommended speed since it is optimized to save the fuel. The PID tracks this setpoint. However, in the places with the very low speed (sharp turns) the FPD does not give the speed low enough to drive through this turn. Here, the combination of the FPD and PID ensures safe driving. In order to show how the course of the speed changes while driving on the whole route, the speed tracking in Figure 1 is shown almost for the whole data sample. However, a rest of data (except for the speed) is not so illustrative, and figures with them for the whole route increase drastically the presented file size to be uploaded. Thus, to save space and for better illustration a fragment of results for 1000 data items is shown for the rest of the data.

Figure 2 shows results for the fuel consumption. It should be noted that despite the driver's preference to keep the speed, the average fuel consumption is lower than the original one. This is obtained due to the FPD optimization. Figure 3 displays pressing the gas pedal (top), braking (middle) and gear (bottom). The braking in Figure 3 (middle) is corrected by the logical block according to the travelled distance. It is performed after the optimization that is why sometimes it does not coincide with the FPD setpoint.

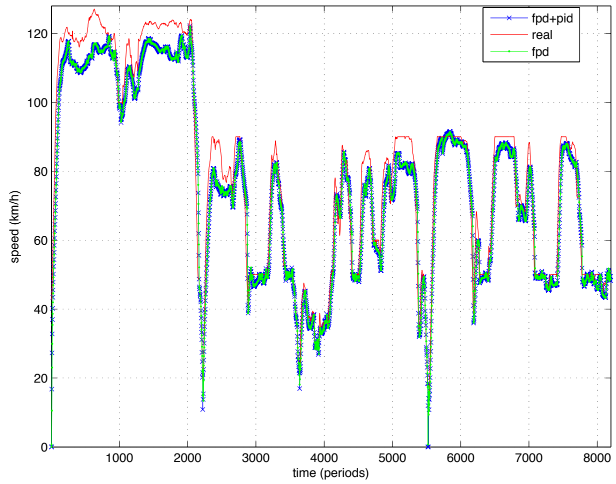

Fig. 1. The speed tracking with driver's preference to keep the recommended speed only

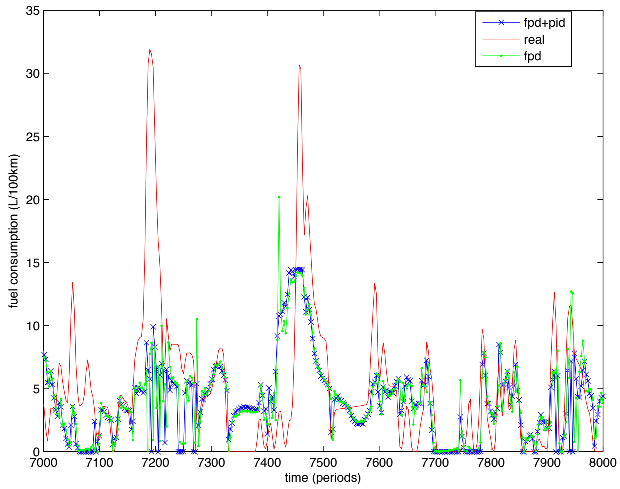

Fig. 2. The fuel consumption optimization with driver's preference to keep the recommended speed only

4.2 Results for driver's preference both to the fuel saving and the speed keeping

The following results are obtained using combination of the FPD and the PID controllers with the PID switched to driver's preference both to save the fuel and to keep the recommended speed. The obtained average fuel consumption is obviously lower than in the previous case, it is 5.04 L/100km. The original average fuel consumption for the used real data is the same – 6.33 L/100km.

Figure 4 plots the speed tracking. Due to the chosen driver's preference the resulting speed is lower. However, it satisfies to safety requirements. Again, in the places with very low speed (sharp turns) the combination of the FPD and PID proves itself.

Figure 5 demonstrates the resulting fuel consumption, which is lower than in the previous case, Figure 6 – pressing the gas pedal (top), braking (middle) and gear (bottom). Again, the braking in Figure 6 (middle) around 7500 time periods is corrected by the logical block to ensure meeting the speed limits.

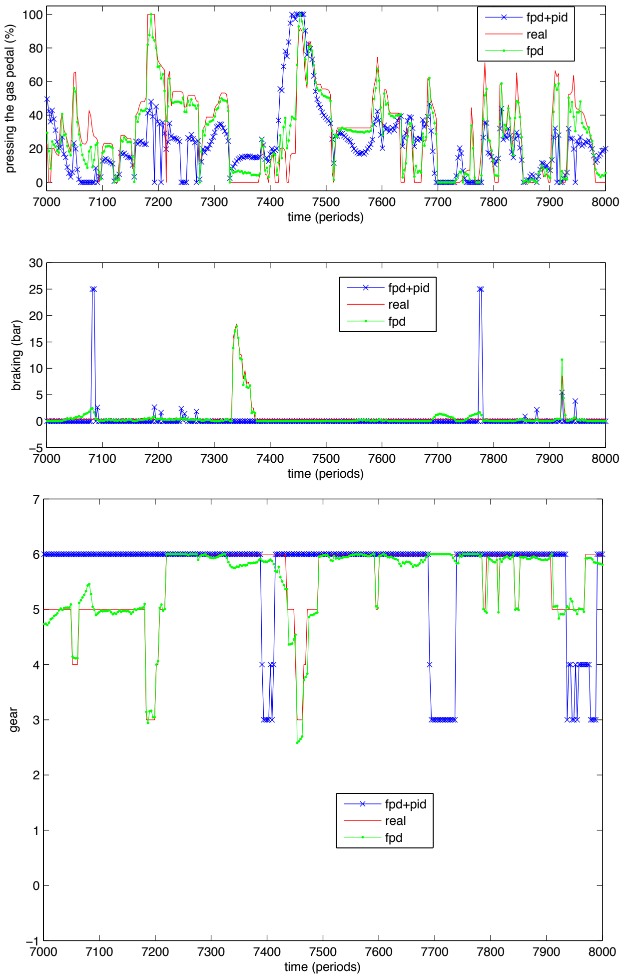

Fig. 3. Pressing the gas pedal (top), braking (middle), gear (bottom) with driver's preference to keep the recommended speed only

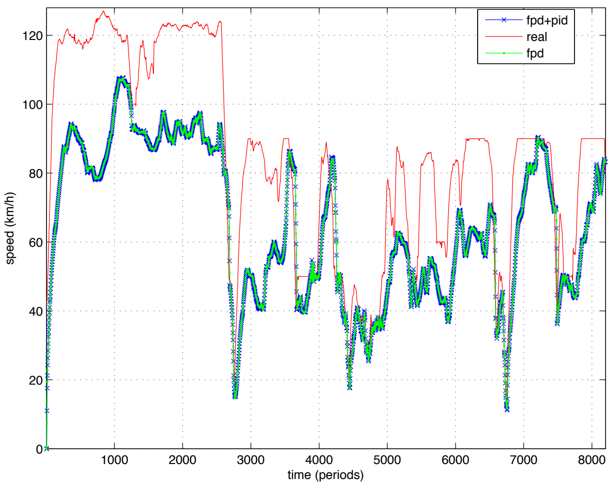

Fig. 4. The speed tracking with driver's preference both to the fuel saving and the speed keeping

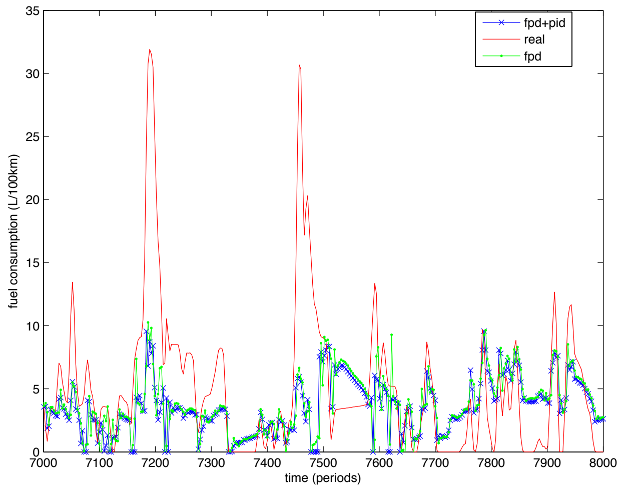

Fig. 5. The fuel consumption optimization with driver's preference both to the fuel saving and the speed keeping

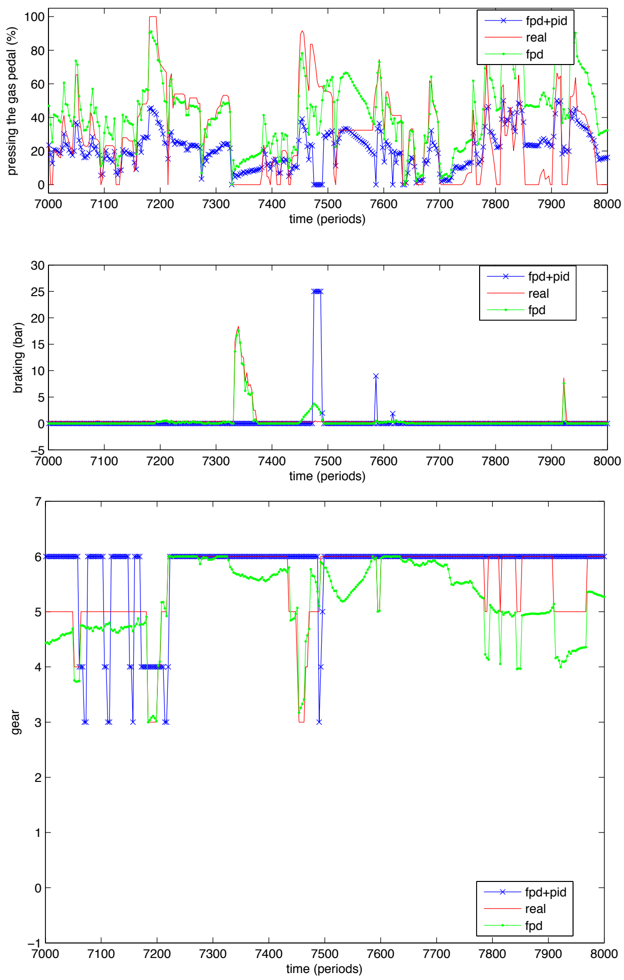

Fig. 6. Pressing the gas pedal (top), braking (middle), gear (bottom) with driver's preference both to the fuel saving and the speed keeping

4.3 Discussion

It can be said that, even with the driver's preference to keep the recommended speed switched in the PID controller, the FPD optimization still gives desired fuel savings in comparison with the real data. Use of the double optimization (FPD and PID) with driver's preference both to reduce the fuel consumption and to keep the recommended speed provides more economic and safe driving.

5. CONCLUSION

The paper describes the current state of the presented research project and continues a line starting in previous works, see Suzdaleva et al. [2012], Suzdaleva et al. [2012]. The obtained results are promising from the viewpoint of the fuel consumption reducing. However, the software simulator is not a real vehicle and in reality the results can be different. Tests in a real driven vehicle are planned.

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