In 2013 all ECB publications feature a motif taken from the €5 banknote.

NOTE: This Working Paper should not be reported as representing the views of the European Central Bank (ECB). The views expressed are those of the authors and do not necessarily reflect those of the ECB.
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Alexis Derviz
Czech National Bank and Academy of Sciences of the Czech Republic, Institute of Information Theory and Automation; e-mail: alexis.derviz@cnb.cz
ABSTRACT

We explore the ability of a macroprudential policy instrument to dampen the consequences of equity mispricing (a bubble) and the correction thereof (the bubble bursting), as well as the consequences for real activity in a production economy. In our model, producers are financed by both bank debt and equity, and face a mix of systematic and idiosyncratic uncertainty. Positive/negative bubbles arise when prior public beliefs about the aggregate productivity of producers (business sentiment) become biased upwards/downwards. Economic activity in equilibrium is influenced by the bubble size. The presence of macroprudential policy is represented by a convex dependence of bank capital requirements on the quantity of uncollateralized credit. We find that this kind of policy is more successful in suppressing equity price swings than moderating output fluctuations. Economic activity declines with the introduction of a macroprudential instrument in this model, so that the ultimate welfare contribution of the latter would depend on the aggregate default costs.

JEL Classification: G01, G21, G12, E22, D82
Keywords: bank, credit, asset price, bubble, macroprudential policy
NON-TECHNICAL SUMMARY

Policy instruments capable of combating financial instability have occupied a central place on both the macroeconomists’ and financial regulators’ agenda after the recent global financial crisis. The latter demonstrated how a financial crisis can evolve into a severe worldwide recession. Financial sector regulators are usually unable to provide a prompt remedy when a crisis is already in full progress, whereas the monetary policy potential is often quickly exhausted by the zero interest rate bound and limits to non-standard monetary policy. Therefore, a lot of attention is now being dedicated to the group of so-called macroprudential regulatory instruments, such as counter-cyclical capital buffers, loan-to-value (LTV) ratio limits, variable risk weights, and collateral requirements.

The present paper examines macroprudential regulation in a theoretical model of bank lending in a production economy. It can be looked upon as an interface between conventional general equilibrium macro models and partial equilibrium models of financial intermediation driven by information asymmetries. The model introduces equity bubbles as a consequence of incorrect prior public beliefs about aggregate productivity. Share price distortions then persist notwithstanding unbiased signals provided to lenders by borrowers.

It is not our objective here to examine the relevance of various exogenous triggers of bubbly developments in asset markets. Instead, we summarily associate the genesis of an equity bubble with an exogenous informational bias that makes agents believe in an expected productivity growth different from the true one. Compared to the benchmark case of unbiased information exaggerated confidence in productivity growth inflates proceeds from equity sales. The latter mean an increase in the equity-financed component of the physical capital. The firm managers with limited liability, riding the same optimistic wave, take on more debt than what the optimal equity level would imply, causing overinvestment. Leverage grows because debt increases by more than proportionally with equity. There are more defaults with higher losses for lending banks, and the aggregate output is lower. Awareness of this reality justifies the regulator to act against excessive borrowing by macroprudential means, even if the latter cannot remove the source of the original informational distortion. That is, although unsure about the extent and the reason, the authority is sure about adverse consequences of excessive leverage, and acts accordingly.

We model macroprudential policy in the form of convexly growing regulatory capital charges on banks that lend to firms with a high LTV. A restrictive/loose policy stance is in a one-to-one correspondence with the high/low degree of convexity. It is assumed that the authority dislikes losses stemming from high default numbers (e.g. because of the adverse effect on bank capital
and the corresponding risk of public involvement), but preferences of the regulator are not modeled explicitly. Our objective is not to design an omniscient social planner conducting an optimal policy but, more realistically, to assess the real impact of a relatively blunt policy conducted under imperfect knowledge about the true productivity distribution. On the other hand, we operationalize macroprudential policy in an environment with combined debt and equity markets. Such a synthesis is, to the best of our knowledge, novel to the literature. In this paper, a two-period setup is discussed, mainly for reasons of economy of space, although a multi-period generalization would be straightforward.

An immediate consequence of introducing the mentioned macroprudential feedback is the reduction of leverage on the producer side, since the costs of borrowing become more directly driven by the debt-to-equity ratio. However, the most risky borrower segment, that is, the one targeted by the policy, is also the one with the highest ex ante performance. So, the equally intuitive downside of enacting the macroprudential policy is a significant increase in lending rates for all borrowers, leading to a decline in output. Macroprudential capital charges mean that high productivity borrowers experience a disproportionate increase in the price of credit. Since they borrow the most, these firms are the hardest hit from the regulation. In equilibrium, the rise in borrowing costs is compounded by the resulting decline in collateral values. Real interest rates rise by even more than the direct effect of higher capital charges because declining collateral values make corporate credit riskier. The outlined property can be interpreted as a rudimentary collateral cycle effect.

In more detail, we find that

1. macroprudential capital charges on banks are successful in reducing loss given default (LGD) in the model, and marginally alleviate the real economy implications of correcting biased public sentiment. The reason is, obviously, that a policy that penalizes high LTV makes both the worst case- and the mean-losses from defaulting loans go down. When the policy is introduced in a bubbly environment, there is also a second-order effect: the economy loses less to the consequences of suboptimally chosen leverage. In other words, investment mistakes owing to misperception at the heart of the bubble are still made, but smaller ones than they would be without a macroprudential policy.

2. macroprudential tightening fails to reduce default frequency in this model, and leads to an economic contraction of the same order as the aggregate LGD reduction. Two opposite forces are at work here: higher interest rates reduce demand for credit (downward pressure on the probability of default, PD) but also raise debt service (upward pressure on PD). At the same time, high interest rates act against economic activity as a first-order effect.
3. Macropazardential tightening results in a downward pressure on equity (physical capital) prices and a modest reduction of leverage in the model; this result might explain why policymakers find macroprudential restrictions such an attractive strategy in dealing with credit bubbles. However, output losses suggest that the application of macroprudential instruments needs to be conducted also with a view of the possible negative consequences for lending and economic activity. Macropazardential policy is able to alleviate the symptoms of a bubble. Its ability to cure the underlying disease is limited by the fact that it is being conducted only on the lender side of debt contracts, with which it shares imperfect information about the borrower side.

4. By changing the base interest rate (i.e. the rate on deposits and the benchmark linear part of the bank funding cost), one is able to assess the effect of a monetary policy tool aimed at the same equity bubble. The said monetary instrument turns out to be slightly better at reducing default frequency, but slightly worse than the macroprudential tool when it comes to lowering average loss given default. (To be sure, one cannot talk about monetary policy proper in our 2-period environment; the exercise of the paper only deals with a token presence of monetary tightening or easing.) The aggregate implications of monetary and macroprudential policies are very similar and may be difficult to identify empirically. If one policy instrument is used partly to alleviate the adverse side affects of the other (e.g. monetary easing in parallel with macroprudential tightening), the combined effect mostly impacts the bank profit side and has very little effect on the producer/borrower side. Ceteris paribus, macroprudential tightening is more advantageous for banks. This is because monetary tightening uniformly increases financing costs for everyone (banks and firms), whereas macroprudential tightening allows banks to pass increased funding costs on to firms more easily.
I INTRODUCTION

Asset price bubbles, i.e., asset prices deviating from fundamentals due to the prevalence of self-fulfilling incorrect beliefs in the market, are known to have long-lasting implications for the real economy. They cause both misallocation of resources while emerging and severe disruptions of economic activity when the eventual price correction (bubble bursting) takes place, all the more so in the presence of significant amounts of bank credit to the real sector, i.e., when the economy is leveraged. The latest global financial crisis has provided abundant examples of sudden breakdown of credit relationships when insufficiently informed investors revised their previously held views. The aggregate magnitude of the ensuing negative financing shock was big enough to make the financial crisis go over into a worldwide recession. Although initially only a minority of financial institutions was affected by adverse balance sheet developments, businesses seemed to have difficulty finding a replacement for their original lender when the latter became either distressed or overcautious.

Once a crisis has erupted, ex post remedies do not work perfectly: bailouts create incentive problems while monetary policy’s ability to ‘mop up’ after the crash may be limited by the zero lower bound and side effects of non-standard measures. That is why interest in the policy response to asset price developments, i.e., the group of so-called macroprudential instruments, is on the rise. However, the impact of macroprudential policies on the supply side of the economy remains to be analyzed in proper depth, given the lack of both generally accepted theories and sufficient empirical evidence.

The present paper seeks to contribute to the objective of understanding the mutual workings of asset price bubbles and macroprudential policies by proposing a model of imperfect financial intermediaries in a production economy. We define an environment in which firms seek both equity and debt financing under partial opacity: some uncertainties in producer performance are, in principle, resolvable in advance of the financing decision when the appropriate asset management regime is chosen, but incentives in financial institutions may work against the resolution. Other, systemic uncertainties may initially be perceived with a bias by investors, leading to mispricing of both equity and debt and, consequently, to distorted capital formation and output losses in the affected real sector.

Specifically, firms have production functions with two risky components in their total factor productivity. The first is a systemic risk factor with a distribution function known to everyone. In addition, there is a firm-specific component (the firm’s type) which is known to the firm

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1 What is usually meant are regulatory guidelines regarding, inter alia, counter-cyclical capital buffers, loan-to-value ratios, leverage limits, variable risk weights, and collateral requirements.
management but cannot be precisely and credibly communicated to either equity investors or wholesale banks. The firm manager can only send a public signal about the productivity level as a whole, in which systemic uncertainty contaminates the message about the idiosyncratic productivity component value. Only a loan manager with specific expertise (a retail relationship banker) has the necessary non-transferrable skills to learn the borrowing firm’s type. Such a delegated manager can be hired by the wholesale bank for a fee to set the lending rate and collect the proceeds. Altogether, due to the firm type which is unobservable, at least, by the equity market, risk cannot be priced efficiently even in the presence of informed loan managers.

The first distinguishing feature of the model is that return on real investment is affected by a specific input required by the corporate governance mechanism in place in the firm. The input can be thought of as a separate strain of managerial human capital related to production (not to be mixed up with knowledge about technology type, as in the previously mentioned relationship banker case). It is firm-specific and, consequently, no individual is able to distribute its provision among all firms. This circumstance works as an obstacle to full-fledged diversification of equity holdings by retail investors. For simplicity, we concentrate on the extreme case by assuming that each of the retail investors can only observe the human capital level in a single firm. Then, by selecting the right parameters one can generate an economy in which holding shares in other firms is strictly dominated by only holding shares of the firm whose human capital level one knows. Thus, equity financing is possible but share demand only comes from a subset of knowledgeable investors.

Another key element of the model is a specific rationale for the existence of banks. Since, as agreed, the circle of possible equity holders of each firm is limited, firms also seek debt financing, whereas retail investors demand a fixed-rate risk-free (deposit-taking institutions do not default in our model) complement to equity in the form of bank deposits. As opposed to retail investors, the bank can lend to any firm. That is why it can present itself as a diversifying intermediary but, at the same time, extract rents as an exclusive operator of the necessary financial technology.

Due to the mentioned technological exclusivity, the bank, or better said, the financial services sector as a whole, has considerable market power. This is why the well-known agency problem of fund diversion becomes important. According to our view, a financial institution sells claims to the public (here, collects deposits from retail investors) by declaring one investment pattern

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2 For simplicity, we only consider the case in which the loan manager finds out the type precisely, i.e., knows the same thing as the firm manager. Generalizations allowing the relationship banker to learn the type with a noise, although with a higher precision than the public, are possible but do not add much to the qualitative insights of the model.

3 In our model, every retail investor is specialized in a particular economic sector represented by a set of identical firms. This is the sector to which the investor supplies his own human capital (this time in a managerial capacity, so that specialization generates a learning-by-doing effect) and, as a result, has sufficient expertise to pick the right stock within the sector, but not outside it.
for the proceeds, whereas the actual management of borrowed funds follows a different pattern as far as it cannot be fully contracted and verified. The bank accepts deposits with the declared objective to invest them optimally in the whole spectrum of available firms, i.e., to diversify retail investors’ funds for them. Naturally, returns on lending to every individual firm are higher if its type, which is the idiosyncratic component of its technology level, is observed, so that the bank (we use the term *wholesale bank*) is supposed to delegate to a relationship banker. However, the latter, being the exclusive holder of firm-specific knowledge, has considerable bargaining power vis-à-vis the wholesale bank. Therefore, he can drive the required fee up to the level at which the wholesale bank becomes indifferent between employing his services and managing the loan itself based on public information alone. Then, it may happen that negotiations with the retail banker break down, delegation does not take place, and the loan is managed by the wholesale bank “at arm’s length”. In the extreme, these arm’s length loans can be packaged, tranched, and sold to other banks in CDO form. And, at times, this can even be preferable both subjectively (the wholesale bank earns more on its loan portfolio) and socially (lower interest rates on average, more investment of debt-financed capital, and hence higher output). The problem is that the outcome is sensitive, among other things, to the quality of public information.

This brings us to the third key element of the model: public sentiment as a source of equity price bubbles. Namely, we propose the following mechanism of rationalizing the emergence of bubbles. We take public information about firms’ types to be a Bayesian update of a prior belief distribution by an unbiased signal coming from the firm. If the prior distribution is biased, we say that there exists (prejudiced) public sentiment. Since each firm’s public signal is noisy, the prior belief update, although able to reduce the bias, is unable to completely eliminate it. Accordingly, prior prejudice impacts on equilibrium equity prices, lending rates, investment volumes, and output. It is easy to demonstrate that the firm equity price is above/below the unbiased prior sentiment benchmark when the bias sign is positive/negative. In that case, we talk about a positive/negative bubble. Numerical examples in Section 3 illustrate the accompanying loss of output, the price of credit, and default frequencies.

In the extreme, our model allows for accommodating the real consequences of a burst bubble as a manifestation of equilibrium breakdown. For instance, assume that there are just two productivity types, high and low. A priori, both the retail investors and the wholesale bankers may believe that there are more low productivity firms (the mass of the low type is greater than \( \frac{1}{2} \) – pessimistic sentiment), whereas in truth, the high and low types both have mass \( \frac{1}{2} \). (Other possible combinations of truth and sentiment are discussed in section 3.) It is possible that, under a particular sentiment and other exogenous parameter values, there exists an equilibrium
with delegated loan management but no equilibrium with arm’s length management. So, in that case, if wholesale-retail banker bargaining about the compensation of the latter is unsuccessful, there is a big group of firms (in the binary example above, all low-type ones) that cease to operate because they cannot finance production with either equity or debt, and there is a considerable reduction in output.\(^4\) However, more common are situations in which both bubble-free and bubbly equilibria exist, but the latter are associated with a reduction of economic activity.

Although the use of macroprudential policy tools is primarily motivated by financial stability considerations, public debate often associates with them the power to contain the adverse real implications of asset price bubbles as well. We try to give a formal structure to this debate by introducing a macroprudential instrument in the potentially bubbly environment outlined above. Namely, we investigate the impact of additional (and convexly growing) regulatory capital charges on banks that lend to firms with a low relative size of own equity. Although the true advantages and disadvantages of such policy instruments can only become fully visible in a dynamic model (whereas ours is a two-period one), we are nevertheless able to gauge the basic qualitative consequences of the said policy for economic fundamentals within each period.

We consider a joint debt-equity market equilibrium with (partially insider) equity and debt financing of physical capital. For this economy, we investigate the role of lender-borrower informational disparity, economic sentiment, and macroprudential intervention for the probability of default (PD), losses given default (LGD) and the aggregate economic activity, i.e. investment and output. Summarizing our main findings, we establish that

A. macroprudential capital surcharges on banks are successful in reducing LGD, and marginally alleviate the real economy implications of removing an asset price bubble (i.e. incorrect public sentiment)

B. On the downside, macroprudential tightening fails to reduce PD (growing lending rates exert a downward pressure on the distance to default whereas falling lending volumes work in the opposite direction; the two forces almost cancel each other), and leads to a contraction of economic activity of the same order as the LGD-reduction, the effect being visible regardless of the presence of asset price bubbles

C. macroprudential tightening results in a downward pressure on equity (physical capital) prices and a modest reduction of leverage; this feature could explain why immediate effects of macroprudential restrictions might appear attractive to policymakers as a

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\(^4\) This effect generated by the model, as we believe, offers a plausible imitation of at least some instances of the transition from “purely financial” revision of beliefs and the corresponding turbulence in asset markets, to the real adverse impact on investment and GDP, ending up in a recession.
weapon against credit bubbles; however, output losses are a warning that the costs may exceed benefit. In short, macroprudential policy is more successful at dampening the symptoms of a bubble than at curing the underlying disease.

D. aggregate implications of monetary and macroprudential policies aimed at bursting the same asset price bubble, are strongly aligned and may be difficult to separate empirically; but, \textit{ceteris paribus}, macroprudential tightening is more advantageous for banks.

In this paper, we discuss a two-period setup, mainly for reasons of economy of space, although a multi-period generalization would constitute no conceptual problem. Also, our simulations only contain a comparative statics assessment of various business and regulatory models’ implications for economic activity, stopping short of a full-fledged welfare analysis. The main reason is that, for a proper welfare assessment of macroprudential policies in the financial sector, one would need an adequate reflection of social losses associated with default incidence and magnitude (in technical terms, a metric working with, i.a., PD and LGD variables), which is a topic requiring separate research.\textsuperscript{5}

To the best of our knowledge, a model that addresses the informational source of asset price bubbles and operationalizes macroprudential policy in an environment with combined debt and (insider) equity markets, is a novel contribution, although departing points for its individual elements can be found in the literature. The next subsection puts our research in the context of relevant contributions in the area of financial friction modeling.

1.1 RELATED LITERATURE

In an ideal (“Modigliani-Miller”) world of competitive and efficient financial intermediation, there is no place for asset price bubbles. On the contrary, practitioners in the areas of monetary and financial regulation policy are permanently faced by consequences of bubbles and mobilize all sorts of analytical tools to understand them (see Hunter et al., 2005, for a survey; the principle conclusion of this collection of papers is a failure to identify bubbles in real time but a hope to design policies that would induce agents to partly endogenize them). Still, the efficient markets assumption is a popular (while convenient) shortcut in macro models. Although more recent DSGE-with-financial-frictions (hereinafter DSGE-FF) models assign a place to the

\textsuperscript{5} To be sure, general equilibrium models (with financial frictions) that feature welfare-improving effects of a macroprudential intervention, are available, cf. Jeanne and Korinek (2010, 2013). However, their environment is substantially different from ours. First, the role of markets for both equity capital and bank loans is downplayed compared to the present model. Second, the planner’s assumed ability to determine socially optimal quantitative restrictions on borrowing is an important driver of the welfare analysis provided in Jeanne and Korinek (2010, 2013). By contrast, our model does not assume an omniscient social planner, whereas the employed macrorudential tool (based on penalizing high LTV ratios) operates in a market-conform way on the cost side of the lender.
financial sector, they usually rely on a properly functioning financial intermediary as a *propagator of real shocks*. The latest global crisis, and particularly the extent of the credit decline at its peak, has uncovered a certain deficit of attention to improperly functioning financial intermediaries as *a shock source*. However, the bulk of the existing macro literature is preoccupied with orderly market operation, conceding but modest space to shortcomings both on the capital provider and capital consumer sides. Investigations into the interplay of financial and real shocks on the macro level have not been too numerous. The concept of costly state verification (CSV) in contract theory (Townsend, 1979) has been widely used in real business cycle models (first of all by suggesting the appropriate way of modeling default on debt contracts). Inspired by CSV models, the financial accelerator construction of Bernanke et al. (1999) has been an influential example of feeding a financial sector factor into quantitative macroeconomic theory. However, the (fulfilled) objective of Bernanke et al. (1999) was to codify, not necessarily explain, the main realities of financial sector presence in the economy, as they strived to reflect empirically important business cycle phenomena related to financial frictions. In essence, Bernanke et al. (1999) and the succeeding DSGE-FF models (e.g. Christiano et al., 2008) accommodate plausible sources of financial shocks through an ingenious choice of free parameters in otherwise standard optimization problems of agents. As the very term “financial accelerator” suggests, the financial sector shapes the real shock propagation mechanism in the economy, but does not itself originate the events of interest in these models, as capital suppliers do not possess sufficient pre-requisites with regard to either standing in the market or informational endowments. Therefore, they are unable to “misbehave” in a natural way (e.g. in terms of adverse selection, reputation, incomplete contracts, herding behavior, etc.) along the lines drawn by the financial intermediation theory. The latter, on the other hand, relies on toy models which provide only very indirect, if any, empirical guidance. Another insufficiently developed link in the current state of the financial accelerator literature is that with the asset pricing theory. Naturally, the latter, to the degree it is trapped in the efficient market paradigm, does not make synergies any easier. With the outbreak of the global crisis in 2007, a more in-depth synergetic analysis received an unprecedented impulse, but relevant contributions are naturally taking time to materialize. Therefore, most literature to the point is quite recent and many inspiring studies still exist in a preliminary form only.

Logically, in the course of the latest crisis, interest has turned to the propagation of real effects of financial shocks proper, so that empirical evidence of such propagation will no doubt soon

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6 One example is the full competitiveness assumption, which imposes the zero-profit constraint on lenders. What may be a gain in analytic convenience (a reduction of the number of free parameters) is also a loss in flexibility, since the market power of the lender is a feature one would really want to be able to model. Besides, it is often overlooked that zero profit is a two-way “egalitarian” constraint: not just is economic profit prohibited, but also losses are ruled out. But to model a bank without a downside risk would be nearly irrelevant for meaningful applications, for which claim at least the reality of the latest crisis, even if nothing else, provides enough evidence.
abound (cf. Campello et al., 2009). At the same time, the new wave of attention devoted to the role of monetary policy in the run-up to financial crises has rekindled interest in formal modeling of macro-prudential policy tools that augment standard Taylor rule-based interest rate policies. Although the necessity and value added of the macroprudential view of both financial regulation and monetary policy was recognized early enough (see Borio, 2003, for a survey), the latest global financial crisis has delivered a new strong impulse for its development. Some quantitative assessments, based on tentative synthetic techniques, have been conducted under the impression of the financial crisis and global recession (see, for example, Chapter III of the IMF October 2009 World Economic Outlook). The exercise done there uses the approach inspired by, inter alia, Aoki et al. (2004), Iacoviello (2005), and Monacelli (2009). Another possible “shortcut” in approaching macroprudential regulation (Jeanne and Korinek, 2010, 2013) consists in endowing the authority with the ability to figure out welfare-improving taxes on borrowing or directly mandate the socially-optimal level of the latter. Naturally, the less stylized the macroprudential policy analysis one would like to have, the more explicit should be role of financial intermediation compared to earlier macro models. Our paper constitutes a step in this direction, as we propose a fairly general way of introducing macroprudential instruments in a production economy with a financial sector. Unlike some other recent contributions that, although taking both corporate and bank default into consideration, leave systemic driving factors of default outside the model (de Walque et al., 2010), we preserve the main features of the risky lending paradigm of the financial intermediation literature (see, for example, Stiglitz and Weiss, 1981).

Our model output testifies to high sensitivity of economic activity in equilibrium to the incentives within the borrowing firms, delegation within financial intermediaries, and the marginal rate of prudential capital charges. As regards the first two factors, our results are akin to the experience accumulated in the literature that studies implications of the manager incentive scheme choice under separation of ownership and control in DSGE models (cf. Donaldson et al., 2009). Awareness of the costs of the third (non-linear capital charge) factor has so far been widespread among practitioners. The present model complements this awareness with a micro-founded analysis.

The rest of the paper is organized as follows. Section 2 explains the construction of the model. Section 3 introduces a parametric version of the model, which we solve numerically. In that section, outcomes of various numeric experiments in the presence of equity price bubbles are reproduced. Then, in section 4, we experiment with the interplay between the bubbles and a macro-prudential policy instrument that generates non-linear charges on non-collateralized loans, and confront the outcome with the effect of modifying borrower liability in default. We
also look at similarities and differences between macroprudential and monetary tightening. Section 5 outlines possibilities of further research and concludes.
2 MODEL

2.1 ENVIRONMENT

2.1.1 AGENTS

The economy offers a set \( \mathbf{L} \) of production capacities, or industries, that also serve as opportunities to invest. Each industry is operated by a representative firm differing from other industries by the value \( L \) of its idiosyncratic total factor productivity component. Thus, elements of \( \mathbf{L} \) can be identified with distinct \( L \)-values standing for firm types. Each production capacity has a c.r.s. production function to be described later, which transforms inputs provided in period one into stochastic revenue in period two. All inputs, investment and output are expressed in terms of a single unit of account. There are two periods and three groups of agents: retail investors, firm managers (or simply firms) and banks.

The return on each firm’s equity depends on inputs in the production function. Of those, one is physical capital and the other is a separate managerial human capital. As opposed to physical capital, it is firm-specific and consequently, no individual is able to distribute its provision among several firms. There is one representative retail investor who is specialized in a particular industry, i.e. the representative firm. For this firm, the investor is the exclusive supplier of human capital who also knows the size of inputs in the production function. As a result, this “insider” investor has sufficient expertise to make a qualified choice of stock to own within the industry, but not outside it.\(^7\) We posit an implicit mechanism akin to Townsend’s (1979) CSV one but tied to the knowledge of the firm’s input-output structure, that only allows the insider retail investor to sufficiently discriminate between earnings under different states of nature. The same mechanism precludes banks and all other (“outside”) retail investors who know too little, to hold equity in that firm (consequently, banks do not hold equity at all). Thus, share demand only comes from a subset of knowledgeable investors. Limited participation precludes full diversification of equity holdings by retail investors. Due to the limited participation constraint, non-diversifiable systemic risk can emerge which the regulatory authority may want to counteract, cf. section 4.

There is a representative (wholesale) bank who lends to firms. The bank enjoys market power based on privileged information, so that a borrower cannot shop for cheap credit among several banks (see more in the discussion in Subsection 2.4.1). Accordingly, differently from the long-standing tradition in the DSGE-FF modeling, we do not have to impose the competitive zero profit constraint on banks (cf. footnote 5 in Subsection 1.1).

\(^7\) One can imagine a household of two, with one member supplying equity financing and the other member supplying human capital input to a single familiar industry.
We consider two lending regimes: direct and through a relationship banker. Relationship bankers are loan officers who have better information on borrowers than both the public and the wholesale bank. More precisely, they are able to find out the borrowing firm’s type, i.e. the idiosyncratic component of its technology level; this ability constitutes their non-transferrable human capital. The two basic variants of the model are the one in which relationship banks are active (RB-economy) and the other from which they are absent (arm’s length lending-, or AL-economy). In an RB-economy, the wholesale bank delegates lending to the named superiorly informed loan officers, in an AL-economy, it does not. The reason of introducing this distinction is our intention to confront our model with some established propositions of the financial intermediation theory. In the latter, the concept of banks as delegated loan monitors (Diamond, 1984, Diamond and Rajan, 2000) hinges on the superior ability of banks to collect information on borrowers. This is, roughly, what we have attributed to lenders in the relationship banking regime. On the other hand, recent problems around securitization of bank assets have uncovered the substantial weight of cases in which the said informational advantage is voluntarily given up by loan originators. This is why we wanted to conduct the same formal exercise under the alternative of no superior lender information (our arm’s length regime). For instance, distinguishing between lending regimes is a useful way of assessing the effect of both asset bubbles and macroprudential policies under varying price discrimination in the credit market. On the other hand, since lending technology choice is closely related to the issue of bargaining between the wholesale and the relationship bankers, which we do not model explicitly, we only carry out a comparative statics analysis of its role here. By the same token, one cannot unambiguously identify the socially preferable bank business model (AL or RB) unless details of loan officer negotiation power and remuneration are specified. So, we leave this question out of the analysis of this paper. 

2.1.2 UNCERTAINTY AND INFORMATION

The total factor productivity of the firms, denoted by $A$, is the only source of risk in the model. We assume that productivity is a product of two components: $A=LS$, of which $S$ is the aggregate uncertainty, whose distribution is known to everybody (for simplicity, let it be the same distribution for all firms). On the other hand, $L$ is the firm-specific component, of which the “public”, comprised by retail investors and the wholesale bank, only knows the distribution

8 However, see Derviz and Podpiera (2007) for a model of local manager incentives in a multinational bank, with a possible approach to this issue.

9 One can compare this feature with Bernanke et al. (1999) and successor models: these, too, contain both aggregate and firm-specific uncertainty, but the role of the former is played down, at a fairly high cost for the interpretation of results. Indeed, when systemic uncertainty is present, Bernanke et al. (1999) do not even have a proper debt contract in the model, and the state-contingent hybrid they have to use instead is quite difficult to rationalize. On the contrary, our model faces systemic uncertainty as a key fundamental factor and lets it play due role in both equity and debt pricing.
across the set \( \mathbf{L} \) of industries. There are three groups of agents that have information superior to the public: relationship bankers, firm managers, and retail investors within their own industry. A relationship banker knows the type of every firm that approaches him for a loan. The manager of that firm also knows its type. Managers also know inputs in the production function (they decide upon them), but we assume that they are contractually forbidden to trade in their firm’s stock. Accordingly, note that retail investors do not know the productivity type even of their familiar industries. This type of knowledge is different from their insider knowledge of technology and is restricted to managers and relationship bankers, therewith justifying separate positions of these two agent types in the model. So, information of the inside investor is inferior to that of the manager, qualifying the latter to negotiate a loan with the bank.

Finally, the bank approached by a borrower knows the functional form of its loan demand depending on the lending rate. Under the AL lending regime, this demand function contains a distributed parameter (type \( L \), see more in subsection 2.4).

The above implies a limited knowledge of the overall model by the agents: retail investors buying shares, although they observe the lending rate, are unable to infer the productivity type. Similarly, a bank setting an interest rate knows the equity price of the borrower and anticipates the functional form of its loan demand, but is not sophisticated enough to infer the productivity type from the latter.\(^\text{10}\) We justify this “black-box” nature of perception by the potential presence of unobservable parameters that prevent full identification. (For example, models with continuous supports of random factor distributions do not typically allow for such identification.) This is a usual, even if rarely explicitly articulated, convention of rational expectations macro models.

Next, we describe the objectives and choices of the defined agents one by one.

### 2.2 RETAIL INVESTORS

Each retail investor has a stock of initial wealth \( w_0 \) and a stock \( m_l \) of non-transferrable human capital in exactly one industry \( l \in \mathbf{L} \). This human capital is sold to firm \( l \) in period 1 at price \( z_l \).

For simplicity, we assume that human capital supply is inelastic, i.e. the whole stock \( m_l \) is sold regardless of the value of \( z_l \). This same investor, or the second member of the same household, can use cash \( w_0 + z_l m_l \) available in period 1 to either buy shares in firms of the same industry \( l \) or put it in a bank account offering a fixed interest rate \( i \). One share earns \( y_l(A') \), where \( A' \) is the total factor productivity parameter. Exact expressions will be given in Subsection 2.3. The

\(^{10}\) Accordingly, the possibility of a separating equilibrium is not an issue even in the special case of our model (chosen for ease of exposition only) with only two productivity types, with which we deal in simulations of Sections 3 and 4. See more in Subsection 2.4.1.
important point is that since another member of the same household supplies firm-specific human capital to \( l \), the retail investor household knows the exact levels of inputs in the production function. Therefore, even though productivity realization in period 2 is uncertain, the degree of uncertainty is much lower than it would be if the investor decided to buy shares in another industry \( n \in L \). For an outsider, only return \( y^n \) without a breakdown into factor inputs and productivity would be known, which would combine the uncertainties over \( A^n \), physical capital \( k_n \) (see subsection 2.3) and \( m_n \). Without going into technical detail, we assume that the resulting uncertainty is so high that it is never optimal for any retail investor to reduce share holdings in the inside industry and buy shares in outside ones.

If the investor buys \( x_l \) shares in industry \( l \) at price \( p^l \) (which he takes as given), his wealth in period 2 is equal to

\[
w = x_l y^l + (1 + i)(w_0 + z^l m_l - p^l x_l).
\]

This final wealth, which is uncertain due to the uncertainty in \( A^l \), enters the investor’s utility function, whose conditional expectation in period 1 is maximized with respect to admissible choices of \( x_l \). The interval of admissible choices is \([0,1]\). This means that the number of shares in each industry is normalized to unity and short-selling is not allowed.

Denote the investor utility by \( U \) and his subjective beliefs about the distribution of \( A^l \)-values by \( \phi \). We will only consider continuous non-atomic distributions so that \( \phi \) is a well-defined density. Then the investor solves the problem

\[
\sup_{0 \leq x_l \leq 1} \int - \frac{U(x_l, y^l (A)) + (1 + i)(w_0 + z^l m_l - p^l x_l)}{\phi(A)dA}.
\]

The outcome can be either an internal solution characterized by the first-order condition

\[
\int U'(x_l, y^l (A)) + (1 + i)(w_0 + z^l m_l - p^l x_l)\left[y^l (A) - (1 + i)p^l\right] \phi(A)dA = 0
\]
or a corner solution in situations where the left-hand side of (2) does not change sign for
\(x_l \in (0,1)\). We will exclude from consideration the trivial corner solution \(x_l = 0\) (firms without any outside equity capital) and consider the remaining cases.

The internal solution can be characterized by a formula conventionally appearing in finance theory. Under market clearing (the representative investor holds \(x_l = 1\)) it can be restated as

\[
p' = \frac{1}{1 + i} \int \frac{U'(y'(A) + (1 + i)(w_0 + z'm_l - p'))}{M(z'm_l, p')} y'(A) \varphi(A) dA ,
\]

with \(M(h, p) = \int U'(y(A) + (1 + i)(w_0 + h - p)) \varphi(A) dA\).

Expression (3) can be interpreted as the expected payout on firm \(l\) stock discounted by the subjective stochastic discount factor. The value of the latter under productivity realization \(A\) is equal to \(1/(1+i)\) times the investor’s marginal utility of wealth \(U'\) under \(A\), normalized by expected marginal utility \(M\). But, whereas standard asset pricing theories concentrate on the market pricing of risk that follows from the properties of the stochastic discount factor, we will keep in mind that the right-hand side of (3) also depends on \(p'\), and treat (3) as an equation which determines this price implicitly.\(^{11}\)

Additionally, we are interested in the corner solution \(x_l = 1\) which obtains when objective function (1) of the representative retail investor is increasing in \(x_l\) on the whole interval \((0,1)\). Equivalently, the left-hand side of (2) is everywhere positive in \(x_l\) and the investor actually gets to pay for the whole available stock the price below the expected discounted payout:

\[
p' < \frac{1}{1 + i} \int \frac{U'(y'(A) + (1 + i)(w_0 + z'm_l - p'))}{M(z'm_l, p')} y'(A) \varphi(A) dA .
\]

Naturally, there may be a whole continuum of prices satisfying this inequality. This situation is indeed possible and gives rise to multiple equilibria – an additional source of potential volatility not just in asset prices, but also in interest rates, investment levels and output. Under some parameterizations of the model, a switch from a unique equilibrium implied by the internal price

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\(^{11}\) Note that, being an equation which generalizes the conventional asset-pricing formulae, (3) introduces an equity market-based (co-)determination mechanism for physical capital. Such a mechanism is absent from the existing financial accelerator models.
solution (3) to equilibrium multiplicity corresponding to a continuum of corner price solutions (3C) is possible by a mere shift of sentiment (a formal definition and an extended discussion of the latter can be found in Section 3).

### 2.3 FIRMS

#### 2.3.1 OPTIMAL PRODUCTION UNDER LIMITED LIABILITY

Firms transform physical capital $k$ and human capital $m$ into output by means of c.r.s. production functions. Although our main variable of interest is physical capital, the second input allows us to equip the model with both a liquidity constraint on the borrower side and a source of leverage. The latter emerges because (cf. Subsection 2.2) the sum of $zm$ across retail investors acts both as the cash deposited by them in banks (in excess of the initial wealth) and the lower bound of the credit volume granted by banks to firms. Accordingly, our model generates endogenous leverage as a result of varying retail investor and bank balance sheet sizes. This allows one, among other things, to investigate the effects of various policy measures on private sector leverage.

Production functions contain uncertain total factor productivity, split multiplicatively into a systemic risk factor and a firm-specific factor (the firm’s type, recall Subsection 2.1.2). A loan manager with specific expertise (a relationship banker) has the necessary skills to learn the borrowing firm’s type. In RB-economies, such a delegated manager is hired by the wholesale bank to set the lending rate and collect the proceeds.

The internal funds of the firm are insufficient to cover production costs, so it seeks external financing in both equity and debt form. The firm is a price-taker in both those markets. Recall that equity is sold to a subset of retail investors (those who observe the human capital input in the firm), whereas debt financing is reserved to banks. Incorporating the experience of costly state verification modeling (Townsend, 1979), we assume that even delegated loan managers of relationship banks are unable to observe the human capital input with enough precision to support a state-contingent (equity) contract. This allows us to exclude from consideration the case of banks holding equity.

In the sequel, unless this causes ambiguity, we will omit the industry index $l$ when discussing a firm’s actions.

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12 For simplicity, we only consider the case in which the loan manager finds out the type precisely, i.e. knows the same thing as the firm management. Generalizations allowing the relationship banker to learn the type with a noise, although with a higher precision than the public, are possible but do not add much to the qualitative insights of the model.
Human capital input $m$ must be paid for up-front in period 1. For simplicity, we assume that firms have neither sufficient initial cash holdings to do this nor the possibility of raising money for wage payment through an equity sale. The latter possibility is excluded because we think of equity issue as a separate decision taken by the controlling shareholder in advance of the production decisions taken by the managers. We make the latter unconstrained borrowers so that as they can fund the human capital purchase in any desired quantity. This entails a major technical simplification without affecting the essence of the results.

So, a firm using $m$ units of human capital has to borrow from banks at least the amount $zm$. More borrowing may be needed to finance physical capital, for which the identity $k = k_0 + p + b$ holds. Here, $k_0$ is the initial non-traded “foundation” stock, standing for e.g. the stock held by the company founders, $p$ is the “market capitalization”, i.e., the value of shares sold in the equity market (recall that the number of shares is normalized to unity), and, finally, $b$ is the physical capital financed by a bank loan.

In the second period, the firm produces $Af(k,m)$ units of output. We assume that the whole stock of physical capital is then released as a part of firm earnings so that, in total, they are equal to $Af(k,m) + k$. (Since this is a two-period model, it makes little sense to consider capital depreciation explicitly.) Recall that in period 2, the $m$-input has already been paid for out of the bank loan. So, the dividend to stockholders is equal to what remains of the output after the total debt, i.e. $zm + b$, is serviced. Default occurs if output is not enough to repay the debt, in which case the bank seizes all earnings. Let the lending rate be $r$ (taken by the firm as given, see more in the next subsection). Formally, shareholder dividends are

$$y(A) = \max\{Af(k,m) + k - (1 + r)(zm + b), 0\}.$$  

If the firm does not default, this dividend can also be written as

$$Af(q + b, m) + q - (1 + r)zm - rb,$$  (4)

13 In this way, we avoid the need to account for the consequences of possible firm default on wage payment. In principle, we could have defined a contract with $m$-suppliers receiving payment in period 2. Then, under default, these claimholders would have been pooled with the lending bank for the purposes of debt resolution. However, this would have meant unnecessary technical complications without a contribution to the main task of the present analysis, which is to explore the real consequences of interactions between firms and banks.
where $q = k_0 + p$ is total equity capital (traded and non-traded).  

Each firm is run by a risk-neutral manager. For simplicity, we assume that he acts in the best interest of the shareholders (i.e. we abstract from agency effects in the shareholder-firm manager relationship). That is, the manager’s objective is to maximize the expected dividend given the manager’s superior knowledge of productivity, i.e. the knowledge about type.  

The firm manager takes the offered lending rate $r$ and the $m$-price $z$ as given. It is natural to assume that the equity price $p$ and the overall level of equity capital $q$ are also exogenous to him. He decides on optimal levels of $m$ and $b$ knowing that in default, the dividend he strives to maximize is zero. The critical level of systemic production uncertainty above/below which the firms survives/defaults is

$$S^d = \frac{(1 + r)zm + rb - q}{Lf (q + b, m)}.$$  

Therefore, his dividend expectation is calculated over realizations of $S$ exceeding $S^d$. Let us denote the cumulative distribution function of $S$ by $X$ and the corresponding density by $X$. The survival probability is then $X(S^d) = 1 - X(S^d)$, and we will also need the notation

$$\Psi^+(S^d) = \int_{S^d}^{\infty} S X(S), \quad \overline{S}^d = \int_{0}^{\infty} S X(S), \quad \Psi^-(S^d) = \overline{S} - \Psi^+(S^d), \quad \Theta(S^d) = \frac{\Psi^+(S^d)}{\Psi^+(S^d)}.$$  

Note that $\Theta(S^d)$ is the expected systemic productivity component conditioned on survival.

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14 Note the difference between our $q$-variable and the net worth variable of Bernanke et al. (1999) and successors: since the latter models do not have explicit equity markets, their net worth value is monolithic, whereas ours is naturally split into foundation and traded stock.

15 Exact $L$-knowledge by both the firm manager and the delegated loan manager (relationship banker) is a useful technical simplification which, on the other hand, is not central to the qualitative results. What is important is that the degree of knowledge on the firm and the relationship bank side, even if different, is higher than that of the retail investor and the wholesale bank.

16 Although this cutoff value is formally analogous to similar parameters used by Bernanke et al. (1999), Christiano et al. (2008), and related models (the usual notation there is $\Theta$), one should remember that our critical productivity value refers to systemic uncertainty realizations conditional on the given firm-specific uncertainty, whereas the named papers work with the firm-specific component.
Lemma 1 Given the equity capital level \( q \), human capital price \( z \) and lending rate \( r \), the optimal decisions of a firm of productivity type \( L \) on \( m \) and \( b \) are characterized by the first order conditions

\[
\begin{align*}
\theta(S^d) Lf_m(q + b, m) &= (1 + r)z, \quad (6a) \\
\theta(S^d) Lf_e(q + b, m) &= r. \quad (6b)
\end{align*}
\]

(In (6), subscripts denote partial derivatives.) The proof is straightforward given that, when calculating expected dividends, the firm manager integrates only over realizations of \( S \) that exceed \( S^d \). As a consequence, the marginal products enter the first-order condition with the tail expectation multiplier \( \Psi^+(S^d) \), whereas the remaining part of the partial derivative of the dividend expression (5) does so with the survival probability multiplier \( X(S^d) \).

Since we assume a fixed supply of \( m \), (6a) will be interpreted as a market-clearing condition for \( z \), i.e., characterization of the human capital price that equalizes the fixed supply with the demand determined by the marginal product of \( m \). The second optimality condition, (6b), is an implicit characterization of the credit demand \( b = B(r) \) as a – decreasing – function of the lending rate charged. This is the firm manager’s reaction function in the game it plays with the bank (cf. the next subsection). Naturally, \( B \) also depends on \( q, z \), and the parameters of the model, but we omit them in order to simplicity notation.

2.3.2 FEASIBLE DEFAULT THRESHOLDS

Since the production function is c.r.s., by combining (5), (6) and the Euler identity one arrives at the following condition for the survival threshold \( S^d \):

\[
S^d = \theta(S^d) - \frac{(1 + r)q}{Lf(q + \hat{b}(S^d), m)}. \quad (7)
\]

In (7), \( \hat{b}(S^d) \) is the optimal choice of \( b \) implied by (6). The above condition is an equation for \( S^d \) whose solution depends on \( z, r \) and \( q \) as parameters. It turns out that, for typical distributions, production functions and a subset of otherwise realistic parameter values, this equation may have either two solutions or none at all. In the latter case, equilibrium equity+debt financing of
such a firm cannot exist either, regardless of the presence of other firm types in the economy. In the former case, there emerges a possibility of two equilibria corresponding to high/low debt-financed levels of capital and high/low default probability in this firm type. Thus, our model is able to imitate real economic instability as a result of tiny financial shocks.

Another source of instability stemming from the existence of two solutions to equation (7) has to do with possible non-uniqueness of equity price expressed by inequality (3C). The formal reason is a discontinuity in the retail investor’s pricing kernel that comes about if one allows the economy to pick the high default probability (of the high productivity firm) and high debt level solution of (7) whenever an equilibrium based on the low default rate solution is infeasible. This discontinuity creates a whole interval of share prices satisfying (3C). Thus, in economic terms, we have an “overheated” real activity which is, in addition, characterized by instability typical for multiple sunspot equilibria – the effect well in line with popular pictures of a classical overborrowing crisis. It can also be shown that the macroprudential instrument we introduce in Section 4 can sometimes (although not always) remove the overheating outcome by forcing the firms to borrow less and, therewith, disregard the high default rate solution option. Nevertheless, to make possible a consistent comparison of numerical results corresponding to all policy regimes and bank business models, we do not pursue this line of research here and concentrate on the low default rate solution case only.

2.3.3 THE CASE OF DOWNSIDE MANAGER RISK

Limited liability of the manager has far-reaching implications for the risk profile of the firm activity (recall that the manager takes the part of physical capital financed by equity as given, but adds the bank loan-financed part at his discretion). Then, the fact that realizations of $S$ implying default are not taken into account by the manager (he receives zero remuneration under any of them), typically result in overborrowing compared to a hypothetical shareholder-manager liable for all potential loses. The type of instability discussed in the previous subsection is just one of many formal manifestations of the friction linked to limited liability. If downside risk were present, one would have one source of equilibrium multiplicity less to care about. More precisely, assume that the firm manager maximizes the unconditional expectation of after-interest earnings (i.e., including the expectation over those $S$-realizations that would make net earnings negative in the absence of limited liability). Such a manager will borrow the following “unlimited liability” quantity of funds:

$$\bar{S}L_{f_{m}}(q + b, m) = (1 + r)z,$$  \hspace{1cm} (6aUL)
(Recall that $\bar{S}$ is the unconditional mean of systemic productivity component $S$.) That is, although the default consequences for the lender are the same as in the limited liability case, i.e., the bank seizes the output, whose value is insufficient to repay the debt in full, the manager behaves “as if” he bore the full brunt of insolvency. To make managers behave like that, one would need, for example, a compensation scheme that is a function of after-interest earnings, e.g. a fixed fee, plus a percentage of actual – positive or negative – earnings. Similar remuneration schemes of “proportional liability” form, also in a much more general setting than the present one, have been considered by, for instance, Hui (2003).

In any event, firm choices based on (6UL) instead of (6) lead to the following analogue of (7):

$$S^d = \bar{S} - \frac{(1+r)q}{Lf(q + b, m)}. \quad (7UL)$$

Now, the default threshold is uniquely determined by the endogenous variables $b$, $q$, and $r$ and the parameters of the model, i.e. the problem of equilibrium indeterminacy disappears. However, managerial compensation schemes able to induce the said “unlimited liability behavior” are mostly a hypothetical possibility which one rarely encounters in corporate remuneration practice. Therefore, counting on financial intermediation disruptions following from the limited liability case (6), (7) is an empirical necessity.

2.4 BANKS

2.4.1 LENDER-BORROWER NEGOTIATION

The lending bank interaction with the borrower is assumed to take the form of a leader-follower game in which the bank is the leader and the firm is the follower. If a firm approaches a bank with a credit application, the latter makes an interest rate take-it-or-leave-it offer and the former decides on the loan volume based on this offer. That is, the firm formulates an optimal reaction
to every value of the proposed lending rate (reaction function) and the bank sets the lending rate based on the information it has about this reaction function.\textsuperscript{17}

The action of a bank depends on whether it is the original wholesale bank that negotiates the loan or the negotiations are delegated to a relationship banker. In the first case, the bank has a belief distribution over the borrower’s productivity value $A$ as a whole (convolution of beliefs about $S$ and $L$). In the second case, we the delegated loan manager knows type $L$ exactly (just like the firm manager) and only faces aggregate uncertainty regarding $S$. As a result, the wholesale banker sets a common interest rate for all borrowers, whereas relationship bankers under delegation set separate rates for individual types.

The above definitions may evoke a question about the robustness w.r.t. the take-it-or-leave-it option for the firm on the one hand, and, on the other, the possibility of firm type revelation through the loan size choice. To be sure, microeconomics of financial intermediation knows models of borrowers who reveal their quality when they decide about loan volume. However, the hypothetical question whether the AL-regime (firm type unknown) would survive if the bank made a menu offer (rate-volume pair), is of minor importance to us given that in any version of the model with a continuum of types separating equilibria would be irrelevant anyway. Our preference was for a definition that would survive transplantation into a DSGE-conform environment of large random event spaces (i.e. more than one source and continuous distributions of risk, large sets of productivity type realizations, multiple agency problems, variability in time, etc.). A further possible specification, i.e. a simultaneous-move lender-borrower game, would not admit a pure strategy equilibrium for standard concave production functions unless one made exogenous restrictions on maximum and minimum volumes and rates. If such restrictions are introduced, one ends up with a corner solution. That is, the bank-firm bargaining problem becomes moot and the credit market equilibrium is now a function of these ad hoc assumptions for which economic rationale is still to be found, i.e. one is back to field one. Alternatively, equilibria with bank offering a rate-volume menu, like in Besanko and Thakor, 1987, are possible in principle, but would require quite limiting assumptions on the information precision on both sides. Similarly possible is an arrangement with a Nash bargaining solution to the bank-borrower game, as used in an analogous situation by, e.g., Diamond and Rajan, 2000. Also under that option, the determinants of equilibrium in the credit market would not be pinned down but instead hidden in poorly justifiable assumptions, in this case in the selected parameterization of bargaining power of the parties (the outcome would be

\textsuperscript{17} This set-up endows the bank with market power, cf. footnote 1 in subsection 1.2. The fact that, generically, bank-client relationship is not fully competitive on either part was recognized by the literature a long time ago. Santomero (1984) is an example of this early consensus. A more specific (and recent) example of imperfect competition modeling has to do with the concept of client “catch-up” in a specific bank – see e.g. Bonaccorsi di Patti and Dell’Ariccia (2004).
very sensitive to the choice of such a parameterization). Altogether, among the named and other potential alternatives of the microeconomic characterization of lender-borrower interaction, we were looking for the least artificial ones in terms of future embedding in a standard stochastic macroeconomic model, with regard to state space size, technologies and information endowments. Under these criteria, the bank-moves-first option was preferred both for its adequate reflection of the bank market power over established borrowers, cf. Sharpe, 1990, Diamond and Rajan, 2000, or Dell’Ariccia and Marquez, 2004, and a clearly evident empirical relevance (cf. the high share of credit lines among modern corporate loan contracts).

2.4.2 LENDING RATE DETERMINATION

Banks are assumed risk-neutral. The bank faces a cost of funds which, for simplicity, we denote by *i* (the same as the deposit rate for retail investors) and assume a linear funding price regardless of volume. In Section 4, we will look at the consequences of relaxing the last of these three assumptions. Deviations from either of the first two assumptions can be easily accommodated in the model as well but are less important for the subject of the paper.

We formulate the rate-setting problem of the delegated loan manager first. In the notation of the previous subsection, a firm of type \( L \) borrows \( B = zm + \hat{b} \), where the optimal quantities of both components are determined by the optimality conditions (6). Thus, from (6a), with \( \hat{m} \) and \( \hat{b}qk^\hat{m} \) being the optimal levels of, respectively, human and physical capital,

\[
B = \frac{\theta(S^d)(L) f_m(q + \hat{b}, \hat{m})\hat{m} + \hat{b}}{1 + r}.
\]

(8)

Since we have agreed that \( m \) is in fixed supply for each firm (price \( z \) equalizes this supply with optimal demand), one can drop the hat in the notation: \( \hat{m} = m \). Further, \( \hat{b} \) can be expressed through \( L, m, q, r \) and \( \theta=\theta(S^d) \) by using (6b). Often, the expression can be made explicit. For instance, for the Cobb-Douglas production function \( f(k, m) = k^\alpha m^{1-\alpha} \) the named first-order conditions imply that for optimally chosen physical and human capital, \( \hat{k} \) and \( \hat{m} \),

\[
f(\hat{k}, \hat{m}) = (\alpha L r^{-1})^{\frac{\alpha}{\alpha - 1}} \hat{m}.
\]

Then, the preferred loan volume under lending rate \( r \) is equal to
\[
B(\theta, L, m, q, r) = \frac{\alpha + r}{r(1+r)} \theta L f(\hat{k}, m) - q = \frac{\alpha + r}{r(1+r)} \theta^{1/\alpha} L^{1/\alpha} \alpha mr^{-\alpha} - q.
\]

In all cases, we will write \( B = B(r) \) for the firm’s choice of loan volume, by omitting the remaining arguments whenever it does not cause confusion.

**Remark** One can imagine situations in which the optimal level of physical capital is below the already available equity capital \( q \), i.e. the firm does not need to finance physical capital by debt. It only has to borrow \( zm \) to finance first period expenditures, i.e. to pay for the human capital input. However, one can show that limiting lending to \( zm \) is infeasible as an equilibrium outcome for many important special cases. For instance, under the Cobb-Douglas production, banks would be unwilling to lend at a finite rate to such firms. Therefore, we will not consider such cases in this paper. In the numeric examples to be discussed later, the equilibrium debt levels turn out to far exceed the current expenditure needs anyway ♦

The revenue from the loan is \((1+r)B(r)\) if the realization of \( S \) is above \( S' \) (the firms survives) and \( S L f(\hat{k}, m) + \hat{k} \) if \( S < S' \). The cost is \((1+i)B(r)\) in both cases. The expected profit is taken over realizations of \( S \) (\( L \) is known) and can be written as

\[
J^{RB}(L, r) = \Psi^{-1} \left( S' \hat{d} \right) L \hat{f} + X \left( S' \hat{d} \right) \hat{k} + X^+ \left( S' \right) (1+r)B(r) - (1+i)B(r).
\]

In (9), superscript \( RB \) refers to relationship banker and \( \hat{f} \) is a shorthand for the production function value under the optimal choice of the firm. The loan manager chooses \( r \) to maximize the right-hand side of (9) with the knowledge of the loan demand function given by (8). When this maximization problem has a (finite) solution, and, under this solution, the firm equity is priced according to (3) or (3C), we obtain an equilibrium lending rate for the delegated loan management case for the firms belonging to type (industry) \( L \). This rate is type-dependent.

When the wholesale bank sets the rate for all firms itself without delegation, it has the objective function obtained by taking expectation over \( L \) of the right hand side of (9). That is (superscript \( AL \) refers to the arm’s length handling of credit provision),
\[ J^{AL}(r) = \int J^{RB}(L, r) \psi(L) dL, \]  

(10)

where \( \psi \) is the probability density function of the public’s (hence also wholesale banks’) beliefs about \( L \). In both the relationship and the wholesale bank cases, the stock price \( p \) (equivalently, the amount of physical capital financed by equity \( q \)) of the loan applicant is taken as given.

Functions \( J^{RB} \) and \( J^{AL} \) both have at most one internal maximum \( r^*(q) \) in \( r \) for every value of \( q \). It is given by the obvious first-order condition

\[ J_r(r^*) = N_r(r^*) - B_r(r^*) = 0. \]  

(11)

In (11), the superscript is dropped for notational economy and \( N \) denotes the sum of the four first terms on the right-hand side of (9) in the relationship banking case, and their \( L \)-expectation as given by the right-hand side of (10) in the wholesale banking case. Subscripts denote partial derivatives.

For the equilibrium to exist, the curves \( r^*(q) \) and \( q^* = q_0 + p^*(r) \) must intersect in the \((q, r)\)-plane. (Here, \( p^* \) is the stock price of the borrower, determined in subsection 2.2 as a function of lending rate \( r \); this is a function if the price satisfies (3) and a correspondence if it satisfies (3C)). If the curves do not intersect, the equilibrium does not exist. If they intersect at more than one point, there are multiple equilibria.
3 BUBBLY AND BUBBLE-FREE EQUILIBRIA

3.1 BUBBLE DEFINITION

The equilibria of the model differ depending on the lending management regime, so that there are two categories of them. The first is arm’s length (AL) loan management, where there is one lending rate for all borrowers. The second is relationship banking (RB), where there is one lending rate for each borrower type $L$. Recall that the stock price, equal to share capital less the foundation stake ($p = q - k_0$), is in both cases common to all firm types, since retail investors in every stock have the same imperfect information about type as wholesale banks. In the following, we show the results for the simplified situation of just two productivity types, deviating downwards or upwards from the average (so that $L \in \{ L_d, L_u \}$, $L_d < L_u$), in which loan management is either AL or RB for all firms at once. If there were more than two elements in set $L$, one could also consider different wholesale banks choosing different subsets of $L$ in which to try out delegation, but this ramification is left outside the present analysis.

Next, we define bubbles as a by-product of incorrect initial public information.

Information held by retail investors and wholesale banks alike is parameterized by the value $\lambda$ giving the perceived proportion of high-productivity firms in the economy. As a possible justification of the situations in which the public knowledge of $\lambda$ is incorrect, one can think of a biased prior belief distribution common to all agents. Every firm, although unable to communicate its productivity type credibly to anyone but its relationship banker, is nonetheless able to send an unbiased, even if noisy, public signal about its type. Then, the Bayesian belief update procedure results in a reduction (depending on the relative variances of the signal noise and the prior belief distributions), albeit never complete elimination, of the prior error in the public perception. That is, a portion of the prior bias is preserved even though the signal sent by each firm is unbiased and is processed rationally.

When the solution of the equation system (3), (11) is being sought, the relevant value of $\lambda$ is the one characterizing the beliefs and not the actual proportion of high-productivity firms (by the law of large numbers, it should be the same thing when the bias is zero). This is because the perceived $\lambda$ enters both the retail investor and the wholesale banker decision problem (delegated loan managers already know the exact borrower type, so that for them the value of $\lambda$ is irrelevant). The true $\lambda$ is important for determining economy-wide aggregates (e.g. investment, bank credit, and average output) after individual decision problems have been solved and equilibrium established.

18 The assumption of common prior beliefs was made to simplify the analysis of public sentiment implications. It can be easily relaxed if there are reasons to consider belief differentials across important subcategories of economic agents.
In a dynamic extension of the present two-period model, one would probably expect the firms to send signals about productivity type repeatedly, so that the initial bias could become vanishingly small in the absence of shocks to the private productivity component.

Another natural generalization would concern a more rich space of firm types, in which case the definition of prior beliefs affected by sentiment would also have to be modified. This issue is left for future research.

### 3.2 QUANTITATIVE IMPLICATIONS OF BUBBLES

We proceed by showing the results of the equilibrium calculation in the unbiased sentiment case and then discuss the changes caused by either optimistic or pessimistic prejudice.\(^{19}\)

The following functional forms were used throughout the calculations. Retail investors have a negative exponential utility of final wealth with the absolute risk aversion coefficient 0.3. Firms have a Cobb-Douglas production function with physical capital share \(\alpha = 1/3\) (see the discussion after equation (8) in subsection 2.3). Systemic productivity component \(S\) is log-normally distributed with \(s = \log S\) having mean -0.125 and standard deviation 0.5. Accordingly, the mean value of \(S\) is unity. The chosen values of other relevant parameters are listed in the notes to the individual tables featuring the calculation results.

The results for the unbiased sentiment case are shown in Table 1. As could be expected, more high-productivity firms (i.e. higher value \(\lambda\), both perceived and actual as long as there is no prior bias) in the economy means more equity investment, but also higher lending rates (for everyone in the AL case and on average for the RB case). A less obvious outcome is a fall in bank credit, investment, and output for each individual type at the same time as the aggregate values of these fundamentals grow with \(\lambda\). This is a sort of “aggregate income effect”: under more numerous high-productivity firms, less effort is needed from each individual producer to attain a given level of aggregate expected output.

Further, looking specifically at the equilibria in the relationship banking environment, one sees that lending rates for low-productivity firms fall (moderately) with growing \(\lambda\), whereas they grow with \(\lambda\) for high-productivity firms. At the same time, higher \(\lambda\) corresponds to higher levels of bank credit, investment, and output in the low-productivity segment, but lower levels of the same fundamentals in the high-productivity segment.

Note that this effect obtains in a joint equilibrium in equity and debt markets, whereas it would be absent from parallel partial-equilibrium models of the two markets considered separately. In

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\(^{19}\) All calculations were conducted using Mathematica®.
the latter, investment and output would always fall with the lending rate (like in the IS-equation of the old Keynesian models), same as the equity price. Looking at Table 1, one sees that our approach renders substantially different reduced-form behavior patterns of the basic fundamentals. The loan rate is not just an expression of the price of risk but also reflects the Stiglitz-Weiss (1981) trade-off between solvency and revenue from debt service. So, for instance, in the RB case, high net worth producers pay a high interest rate, a phenomenon unknown to the standard credit risk theory.

Finally, for each fixed $\lambda$, when public sentiment is unbiased, aggregate economic activity is lower in RB economies than in AL ones. The result is due to the relatively low common interest rate that the imperfectly informed wholesale banks charge everybody. Although, in RB economies, credit to high-productivity firms is cheaper than in AL-ones, so that those firms invest and produce more, credit to low-productivity firms is much more expensive and their output is much lower, than in the AL-case. Under unbiased sentiment, the latter effect turns out to dominate the former. That is, in the world we have created it is often welfare improving to know less, i.e. not to delegate lending to loan managers who know the borrower type, rather than more. So, in our model, not unlike the developed economies shortly before the outbreak of the latest crisis, banks may be tempted to refrain from the costly use of intermediary agents with superior information and instead grant loans based on general formal rules (this is the essence of the AL-approach).20

Although formally supported by a different type of model, this outcome should remind situations described by the classical financial intermediation literature dealing with the effect of information quality about borrowers on social optimality of investment choices (see, e.g., Sharpe, 1990, or Rajan, 1992). Consequently, our environment has enough attributes to accommodate the underlying adverse selection and moral hazard effects. A deeper exploration of welfare role of the lending regime can be subsequently undertaken in a richer macro model.

Next, let us turn to the role of bias in public perception, and the resulting equity bubbles. The results are collected in Table 2. Within each borrower type, the determination of the equilibrium equity price and lending rate depends on the perception (not the actual $\lambda$), i.e., it does not depend on prior bias as such. The difference between subjective beliefs and reality matters for the observed economic aggregates. As expected, aggregate bank credit, as well as investment and output, grow along with the actual proportion of high-productivity firms. On the other hand, for every fixed value of actual $\lambda$, economic activity falls with growing perceived $\lambda$. In other

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20 Interestingly, the outcome will be reversed under some realizations of prior prejudice. This should act as a warning that, in a general equilibrium environment, asset bubbles caused by cognitive misperception have potential to overturn conventional intuitions of adverse selection/moral hazard microeconomics.
words, there exists an aggregate cost of incorrect economic sentiment. In this respect, RB economies are slightly less sensitive to prior bias than AL economies, and it may also occasionally happen that the RB output under a particular sentiment value exceeds the AL output (as when perceived $\lambda$ is 0.4 and the actual one is 0.6 in our example). In all cases, inspection of Table 2 suggests that, for a fixed absolute size of sentiment error, it is socially preferable when people are pessimistic. This follows from comparing economic activity for, say, the combination actual $\lambda=0.4$, perceived $\lambda=0.5$ with the combination actual $\lambda=0.5$, perceived $\lambda=0.4$, etc. It remains to be seen to what extent this particular result is influenced by the orthodox efficient market paradigm of equity pricing used. The latter paradigm may also be responsible for the high sensitivity of economic activity values to sentiment changes: whereas the interest rate changes by 0.1 percent, the output values shift by 3 percent or more under a 0.1-size change of sentiment (i.e., the perceived $\lambda$-value).
4 MACROPRUDENTIAL CAPITAL CHARGES AND BUBBLE CONTAINMENT

4.1 POLICY DEFINITION

In this section, we will test the ability of the constructed model to address the real effects of macroprudential regulation of financial intermediaries. The prime objective of such regulation is to maintain financial stability by acting against systemic risk. In our model, the latter is represented by the aggregate random productivity component $S$ of TFP variable $A$. Note that from the viewpoint of a given lender or a given borrower, it plays no role whether, at one extreme, $S$ is common for all producers, on the other, there is an independent identically distributed individual risk factor for each firm, or, intermediately, there is some imperfect correlation between those identically distributed risks. Indeed, a firm manager only cares about its own production taking the physical capital level and the interest rate set by the bank as given. A relationship banker in an RB-economy is a mirror image of the firm manager, only caring about the performance of the latter and not the economy-wide aggregates. Finally, in an AL-economy, the lending rate is set by a pre-specified rule the wholesale bank employs by looking at a demand curve with a distributed parameter – borrower type $L$, possibly distorted by sentiment – but without comparing different individual borrowers.

Consequently, in the present setting, correlations between $S$-realizations of individual producers are solely the policymaker’s concern. Systemic risk is highest when the correlation is perfect, i.e. there is, essentially, a common random disturbance $S$ affecting all firms at once. For simplicity, this is the only case that we will derive numerical results for. Informally (with quantitative policy loss functions relegated to future research), we shall think of the policymaker caring about three factors: economic activity, probability of default and loss given default. For a given equilibrium, the latter two factors become straightforward functions of the distribution of systemic risk factor $S$. These will also be the factors to be acted against by means of a macroprudential policy instrument.

Relying on the evidence that the debt volume grows and its quality deteriorates much faster in the run-up to a financial crisis than in normal times, policymakers have for some time been looking for a way to dampen unusual debt expansions and prevent credit bubbles without tethering “genuine” growth. At the moment, reliable recipes for separating bubbles from sustainable growth are unavailable. So, most probably, in the pursuit of their financial stability

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21 Observe that if, differently from the discussed case of only two representative firms – one for each type – we had a large number of small-size borrowers, perfectly independent firm-level risks would mean no systemic risk whatsoever. In our two-firm setting, the exact size of systemic risk depends, beside correlation, on the relative size of two representative borrowers. This issue is not explored any further in the present paper.
goals, most regulators would resort to simple penalties for suspicious credit expansions by mandating the attribution of increased risk weights to all lending that visibly exceeds the accepted target. That is, the macroprudential policy instruments one is most likely to see are capital requirements for – and hence additional costs of funding of – incompletely collateralized loans, requirements that would grow convexly with loan volume. In the present model, we can accommodate such an instrument by replacing the linear cost-of-funds term in the bank objective function by a linear-quadratic term that contains a surcharge on the loan volume in proportion to growing borrower leverage. That is, we model an instrument formally resembling the much-discussed Financial Stability Levy (FSL) on liabilities of banks, which would force the latter to apply a similar metric to their asset side as well.

Formally, we introduce the following macro-prudential control mechanism into the model. If the target level of physical capital of the borrower is $k$ and the loan size is $B$, the bank is subject to an additional charge (in the form of regulatory capital) that leads to extra funding costs equal to $\frac{a}{2k}B^2$, where $a$ is a positive constant. That is, the funding cost term $(1+i)B$ in (9) (and (10)) is replaced by $(1+i)B + \frac{a}{2k}B^2$. This means that

(a) funding costs are growing as a convex function of the loan volume and not linearly as the original equation (9) stated;

(b) the surcharge is proportional to the product of the loan volume and the borrower’s debt-to-physical capital ratio; every additional unit of credit is penalized unless offset by physical capital collateral financed with equity;

(c) the unit of penalty for uncollateralized credit is $a$, usually a small number in the order of single-digit percentage points.22

In the following subsection, we calculate the equilibria with macroprudential bank capital surcharges in a bubble-free economy. Then, in Subsection 4.3, we analyze the ability of the same instrument to reduce the real effects of a bubble caused by biased sentiment. Finally, in Subsection 4.4 we compare the effect with that of a monetary policy adjustment.

4.2 NO-BUBBLE EQUILIBRIA

The calculation of equilibrium under prudential capital surcharges can proceed in the same way as before, with only term $N$ in equation (11) to be modified in accordance with the new

22 In the calculations, we have used $a=0.01$. 
definition of funding costs. We show the results for the base case of equal borrower type weights and no prior bias in Table 3. For the sake of accurate comparison with the original model without prudential policy instruments, we assume that the extra funding costs carried by the banks are turned back over to the private sector in the form of transfers (e.g. tax relief) and, therefore, are included in the aggregate output measures.

Upon inspecting Table 3 we see that macroprudential policies in the defined form are successful in one immediate objective: to reduce the aggregate loss given default figures. The downside is an increase in the price of credit and the resulting burden on economic activity. The bulk of this burden is carried by high-productivity borrowers, so that their distance from low-productivity ones in terms of investment and output is now smaller. In the present model, high-productivity firms take on more risks and default more frequently than those in the low-productivity segment. So, if the objective of macroprudential policies is to put a check on the expansion of the riskiest segments of the bank loan market, it is being achieved through dampening economic activity in the high-productivity segment. On the other hand, if the stabilization objective of the macroprudential instrument involves the number of defaults (this can be the case if each default carries a negative externality irrespective of size), then its introduction in our environment is clearly counter-productive: the number of defaults is now higher. And since the relationship banking regime in general is more favorable to high-productivity firms, the costs of new policies are higher in RB-economies as well.

Naturally, the actual raison d’être for a macroprudential tool of the above type is its ability to stabilize (asset) prices and output in the medium run, i.e., it can only fully transpire in a dynamic environment. In the latter, if the clean-up after a bursting bubble is a matter of a single-period public expenditure, whereas the output reduction due to a restrictive macroprudential stance is permanent, this type of policy becomes even more costly. This is a possibility to be taken seriously, because the effect in terms of expensive credit and low investment is likely to carry over from the present two-period to a multi-period model. Therefore, it would be always welfare-improving if one had a capital charge mechanism that could minimize the side effects for quality borrowers. To cover this ground, we considered a variant of the present model with proportional liability rules for firm management remuneration, which effectively induces unlimited-liability decisions on capital structure and input purchases. This is how we explore the conjecture that a welfare-improving capital requirement policy may aim at encouraging lending to personally liable borrowers and penalizing excessive exposure to borrowers with conventional limited liability. Supporting evidence in the present setting is provided by a comparison of the outcomes of the benchmark model (limited-liability borrowers) with those of the model under the said imitation of unlimited liability behavior (cf. the remark at the end of
subsection 2.2). The comparison (under equal productivity type weights and unbiased public sentiment) is summarized in Table 4. We see that the induced mimicry of unlimited liability behavior has four major consequences compared to the benchmark:

(a) the lending rates of both productivity types get quite close in the RB case and approach the common lending rate of the LA case,

(b) there is a sharp increase in the equity value of the high-productivity type, and a minor decrease in the equity value of the low-productivity type,

(c) the default probability of the high-productivity type falls substantially, whereas for the low-productivity type, although formally increasing, this probability remains negligible,

(d) economic activity experiences a minor reduction compared to the limited liability borrower behavior.

If the prime concern of the policymaker is to find a macroprudential policy without a major negative impact on economic activity, unlimited liability mimicry by borrowers, if one could get near it, would have an advantage over the previously considered convex capital surcharge instrument, provided one were at the same time able to encourage delegated loan management in banks. Indeed, suppose that unlimited liability mimicry is impossible to implement in the AL regime, but possible in the RB regime (that is, the relationship banker is able to influence the manager incentive structure in the borrowing firm). In that case, the trade-off for the macroprudential policy is between a convex capital surcharge in wholesale banks against unlimited borrower liability mimicry in the RB regime without capital surcharges. Comparing the upper right panel of Table 3 with the lower right panel of Table 4, one sees that the losses in economic activity caused by abandonment of limited liability are more than compensated by the possibility to give up additional capital requirements. A reduction in default rates comes as a bonus on top of that.

4.3 ECONOMIC ACTIVITY UNDER A BUBBLE, AND CONTAINMENT ALTERNATIVES

Now we go over to the description of the same exercise as in the previous subsection, but in the presence of a bubble. The experiment we conduct uses a positive bubble generated by a prior

23 Recall that the considered behavior of the borrower firm does not mean that the lender gets full repayment in all states of nature, of which in adverse states of nature (output less than debt service, i.e., default) a part of the compensation comes from the borrower’s private wealth. As before, in default the jointly available assets of the firm and its management are insufficient to service the debt. All that is assumed here is that the firm manager compensation is an affine function of firm earnings less debt service. In that case, the manager would select production inputs as if the firm operated under unlimited liability.
belief of 40% low-productivity producers, whereas their true proportion is 60%. The results are collected in Table 5.

In all three lending regimes considered (baseline with limited borrower liability and without macroprudential capital charges, under macroprudential capital charges, and under proportional downside risk of the borrower management), the bubble means higher equity prices compared to the no-bubble benchmark. The extent of equity overvaluation is somewhat higher under delegated loan management. Average output, on the other hand, is lower under the bubble. A little surprisingly, default probabilities are lower compared to the benchmark. Apparently, the mechanism of this reduction has to do with lower interest rates charged to high-productivity firms under optimistic sentiment.

Under the arms’ length bank business model, the most important finding in the context of our objective is that in the presence of the macroprudential instrument, bubble removal (restoration of unbiased sentiment) has a marginally stronger impact on both the equity price and output than in the baseline. In the considered case of a positive bubble, the equity price falls and output rises when the bubble disappears, the same as in the baseline. The difference between bubbly and no-bubble interest rates is also mildly stronger. Unfortunately, this effect is achieved at the cost of a reduced absolute level of economic activity.

Under loan management delegation (relationship banking), the quantitative effects of the macroprudential capital charges are even less convincing, since, compared to the baseline, both the output gain and the bubbly price correction are smaller. The adverse absolute impact on output is even more severe than in the AL business model. Altogether, macroprudential policies in the RB environment can, according to the present analysis, be rather disappointing due to their inability to exploit the informational advantages of delegated loan managers. On the other hand, enforcing firm manager downside risk, if feasible, could both reduce the sensitivity of the real economy to the asset price bubble and (the same as in the no-bubble benchmark) improve the absolute output levels.24

The conducted exercise suggests that the convex macro-prudential capital charge on bank loans has two main advantages. First, it helps investors coordinate on an equilibrium mix of equity and debt financing in situations in which equilibria do not exist in the absence of this

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24 Formally, the improvement can be seen as a consequence of a more efficient risk pricing. The key adverse property of the standard limited liability arrangement is that the borrower is not motivated to distinguish between “dead” and “deader” outcomes under default: the consequences are the same whether insolvency size is large or small. As discussed at the end of subsection 2.3.3, this means that the firm decides input purchases and investment by only taking into account no-default realizations of the uncertainties it faces. Had the managers received some (e.g. delayed) remuneration even under default with an inverse dependence on the loss size, their decisions would have more resembled the socially preferable ones from (6UL). In practice, this means that a part of the manager fee would have to be earmarked and excluded from seizure by the creditor at default. Naturally, such a scheme would require a change in the usual insolvency laws and/or encouragement by an appropriate tax treatment.
instrument. In our model this happens particularly when firm productivity types are distributed very unevenly or when public economic sentiment is highly biased. Second, by discouraging imprudent leverage of highly productive borrowers, it helps to reduce loss given default. Third, it contributes very slightly to suppressing the most immediate consequence of biased sentiment, i.e., equity price bubbles.

On the contrary, the main problems associated with the use of this instrument can be seen in

- a uniform and significant increase in lending rates for all borrowers,
- an increase in default rates (which may be a problem if they are associated with welfare externalities not considered in the model),
- excessive sensitivity of investment and output to small changes of the capital charge rate (this has to do with the additional transmission channel through equity markets).

More generally, there seem to be limits, in terms of economic activity and ex ante welfare costs, to promoting financial stability through policies directed at credit providers. At the same time, policies with the unchanged ultimate objective of credit bubble prevention, but directed at credit consumers, have largely remained unexplored (let alone exploited). Our results indicate that the potential gain from such re-orientation from regulating credit supply to educating credit demand may be worthwhile, notwithstanding numerous implementation difficulties.25

4.4 MACROPRUDENTIAL VS. MONETARY POLICY

Having seen the consequences of bubble containment by means of the macroprudential instrument introduced, we shall now compare the result with the workings of a more traditional monetary policy tool. We model monetary policy as an exogenous simultaneous change in the funding cost and the deposit interest rate value $i$. That is, whilst monetary policy affects the linear component of banks’ financing costs (irrespective of whether this financing comes from deposits or the money market), the macroprudential tool leaves this linear component unchanged but adjusts the curvature of the non-linear cost component.26

For concreteness, calculations were conducted for our basic limited borrower liability case and unbiased sentiment. Variations in the value of $i$ led to equilibria which, with respect to variables characterizing economic activity, were hard to distinguish from the ones obtained from the

25 In a way, one can find in our results on the importance of borrower downside risk for economic activity an echo of findings by Mendoza (2010), on social costs of overborrowing (although in a quite different kind of model, Mendoza, 2010, shows that it is socially preferable to tax fixed income liabilities and subsidize equity issuance, which is a different way to say that downside risk, inherent in equity but absent from debt, supports the socially preferable behavior of producers).

26 Naturally, the sign of a bank’s funding costs dependence on policy rates may vary with its balance sheet structure. Here, we have a stylized textbook case of a bank with just deposits and own capital on the liability side, so that this question does not arise.
baseline case by introducing the macroprudential tool of the preceding subsections. In particular, we found that under the used parameterization, average output, investment, equity prices, and lending rates react to the macroprudential tool in roughly the same way as they would react to a 0.5% increase in the value of interest rate $i$. Conversely, economic activity returns, more or less, to the baseline level when the macroprudential policy is compensated by a 0.5% monetary policy easing (reduction of $i$). The results of the latter experiment are shown in detail in Table 6.

What is the difference, then, between the two policy effects? Inspection of Table 6 suggests that there may be a minor increase in default rates (in the high productivity, hence risky, borrower segment) under active policies compared to the no-policy baseline, but, given the chosen level of precision, this increase barely lies outside the confidence bounds. Much more important is the comparison of the effects on bank earnings: they are significantly higher under the macroprudential tightening-monetary easing policy mix than in the baseline. This fact indicates that, should it come to a counter-cyclical action by the monetary authority, banks would prefer macroprudential activism to monetary tightening. This outcome can be interpreted as follows: monetary policy affects bank funding costs uniformly and jointly with the firm equity and household deposit markets. On the other hand, the considered macroprudential tool immediately affects only the lender-borrower link and allows the bank to pass higher funding costs on to the real sector more easily.

Altogether, both policy instruments, i.e., macroprudential and monetary, contribute superficially to suppressing equity price bubbles, but show a very disappointing record of fighting other credit excess phenomena such as default probabilities. Quite importantly, their performance on the real economy side of bubble containment is marred by the overall dampening impact on investment and output.
5 CONCLUSION

We have constructed a model of a financial sector in a production economy subject to equity price bubbles. Our model accommodates endogenously mixed equity-debt financing of production. Further, it contains a flexible agency-based rationale for the role of banks and a workable quantification of the notion of economic sentiment (often considered a “soft”, non-quantifiable concept) within an integrated macro-financial modeling setup. Additionally, we are able to investigate the real economic implications of macroprudential policies motivated by financial stability considerations. The main building blocks of this model are

- constrained diversification of stock holdings by retail investors,
- exclusive access to financial intermediation technology by wholesale banks,
- choice between delegated and arm’s length loan management by wholesale banks,
- prior beliefs (public sentiment) about the productivity risks of firms, generating equity price bubbles that feed into investment, interest rates, and output.

We formulate an equation system characterizing equilibrium for the cases with and without delegation of lending by wholesale banks. We also introduce a macroprudential regulatory policy resulting in a convex dependence of bank funding costs on the quantity of uncollateralized credit. Then we conduct comparative statics exercises with the help of numeric solutions to this system for a number of important cases. Those include: shifts in the prior economic sentiment of the public which lead to equity price bubbles; variations in the size of non-linear regulatory capital charges linked to unsecured loans; and relaxation of the conventional limited liability constraint on borrower behavior.

Limited liability proves to be one of the main sources of equilibrium fragility: under many combinations of parameter values, equilibria with bank credit do not exist (the only feasible outcome is self-financing by firms) for pure limited liability borrowers but exist when borrowers mimic unlimited liability in their decisions under the pressure of specially designed incentives.

Our exercise has shown that the sensitivity of the economy to biased investor sentiment is not visibly reduced by macroprudential policies of the considered type. Introducing a macroprudential instrument does allow the regulator to reduce loss given default levels (across the bubble sizes considered), even though the default frequency marginally rises instead of falling. The cost is a fall in the ex ante absolute economic activity of the same order of magnitude as the expected LGD-reduction.27 This contrasts with the effect of introducing

27 It then depends on the policymakers’ preferences which of the said opposite effects is deemed more important.
downside risk for borrower management. Such a policy, if one could implement it by an appropriate regulatory design on the lender side or other (e.g. taxation) policies, is able to remove a large portion of the real economic sensitivity to equity price bubbles.

The quantitative characteristics of equilibrium in our model are quite sensitive to several exogenous parameters, such as the initial level of own capital of the borrower. The effect is particularly strong in the textbook Cobb-Douglas production environment, but might be somewhat mitigated in the presence of convex capital installation costs. Still, the main qualitative outcomes of both bubble bursting and changes in macroprudential policy stance survive the introduction of capital installation costs. So, the production side of the model is reasonably robust.

In contrast, robustness is less likely on the equity market side. In the present version of the model, we have employed the standard (“Walrasian”) asset pricing paradigm. The latter is used to price company equity and hence also determine – without frictions, in a one-to-one relation – the level of equity-financed physical capital. In this environment, cases in which equilibrium either becomes indeterminate or falls apart are common. It is possible that the relaxation of this unrealistic frictionless link between the stock price and investment will be able to reduce some counter-intuitive effects, such as the behavior of default frequency in the high-productivity borrower segment. However, the main conclusion about the economic activity implications of macroprudential tightening is unlikely to be reversed. The quantitative treatment of this question is left to future research.
REFERENCES


Table 1: Economic fundamentals in equilibrium with unbiased sentiment

<table>
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<tr>
<th>Indicator</th>
<th>$\lambda=0.4$</th>
<th>$\lambda=0.5$</th>
<th>$\lambda=0.6$</th>
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<tr>
<td>$q$</td>
<td>3.244</td>
<td>3.328</td>
<td>3.398</td>
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<tr>
<td>$r$</td>
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<td>0.075</td>
<td>0.076</td>
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<td>$k_d$</td>
<td>17.256</td>
<td>20.072</td>
<td>17.017</td>
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</table>

Notes: The foundation stake $q_0$ in firm equity is at level 0.2. The cost of lendable funds (deposit rate) is 0.03. For firms of type #, $q_#$ is total equity capital, $r_#$ is the borrowing rate, $B_#$ is the volume of credit taken, $k_#$ is the total investment in physical capital, $y_#$ is expected gross output (when the systemic productivity factor takes its expected value of 1), AL is arm’s length loan management, and RB is relationship banking (delegated loan management).
### Table 2 Main fundamentals under changing sentiment

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<th>True value of $\lambda$ →</th>
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<th>0.5</th>
<th>0.6</th>
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<th>0.5</th>
<th>0.6</th>
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</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>0.074</td>
<td>0.074</td>
<td>0.074</td>
<td>0.075</td>
<td>0.075</td>
<td>0.075</td>
<td>0.076</td>
<td>0.076</td>
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</tr>
<tr>
<td>$r_d$</td>
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<td>0.081</td>
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<td>0.080</td>
<td>0.080</td>
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<td>0.079</td>
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</tr>
<tr>
<td>$r_u$</td>
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<td>0.070</td>
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</table>

**Notes:** The foundation stake $q_0$ in firm equity is at level 0.2. The cost of lendable funds (deposit rate) is 0.03. For firms of type $\#, q_\#$ is total equity capital and $r_\#$ is the borrowing rate; variables without subscripts denote economy-wide aggregates; $B$ is the volume of credit taken, $k$ is the total investment in physical capital, $y$ is expected gross output (when the systemic productivity factor takes its expected value of 1), AL is arm’s length loan management, and RB is relationship banking (delegated loan management).
### Table 3: Economic fundamentals in equilibrium with and without prudential capital surcharges

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<td>r</td>
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<td>24.809</td>
</tr>
<tr>
<td>$y_u$</td>
<td>28.786</td>
<td>24.709</td>
</tr>
</tbody>
</table>

### Notes:
- Results are shown for the perceived share $\lambda=0.5$ of high-productivity borrowers and no prior bias. The base capital surcharge $a$ is 1 percent per 1st unit of credit uncollateralized by physical capital. The foundation stake $q_0$ in firm equity is at level 0.2. The cost of lendable funds (deposit rate) is 0.03. For firms of type $\#$, $q_\#$ is total equity capital, $r_\#$ is the borrowing rate, $B_\#$ is the volume of credit taken, $k_\#$ is the total investment in physical capital, $y_\#$ is expected gross output (when the systemic productivity factor takes its expected value of 1), AL is arm’s length loan management, and RB is relationship banking (delegated loan management). Loss given default is the expectation with respect to the productivity distribution conditioned on default.
Table 4 Economic fundamentals in equilibrium when borrower incentives replicate either limited or unlimited liability conditions

<table>
<thead>
<tr>
<th>Borrower incentives →</th>
<th>LL-consistent</th>
<th>UL-consistent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aggregate</td>
<td>Aggregate</td>
</tr>
<tr>
<td>Indicator ↓</td>
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<tr>
<td>$q$</td>
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<tr>
<td>$r$</td>
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<tr>
<td>Default probability</td>
<td>$L_d$</td>
<td>+0</td>
</tr>
<tr>
<td></td>
<td>$L_u$</td>
<td>+0</td>
</tr>
<tr>
<td>$L_u$</td>
<td>0.025</td>
<td>0.0086</td>
</tr>
<tr>
<td>$B_d$</td>
<td>16.055</td>
<td>15.505</td>
</tr>
<tr>
<td></td>
<td>19.823</td>
<td>18.916</td>
</tr>
<tr>
<td>$B_u$</td>
<td>23.590</td>
<td>22.326</td>
</tr>
<tr>
<td>$k_d$</td>
<td>17.017</td>
<td>17.053</td>
</tr>
<tr>
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<td>20.324</td>
<td>20.047</td>
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<td>$k_u$</td>
<td>23.632</td>
<td>23.042</td>
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<tr>
<td>$y_d$</td>
<td>20.832</td>
<td>20.870</td>
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<tr>
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<td>24.809</td>
<td>24.535</td>
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<tr>
<td>$y_u$</td>
<td>28.786</td>
<td>28.200</td>
</tr>
<tr>
<td>$q_d$</td>
<td>3.138</td>
<td>3.072</td>
</tr>
<tr>
<td>$q_u$</td>
<td>3.413</td>
<td>3.881</td>
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<tr>
<td>$r_d$</td>
<td>0.080</td>
<td>0.076</td>
</tr>
<tr>
<td>$r_u$</td>
<td>0.072</td>
<td>0.077</td>
</tr>
<tr>
<td>Default probability</td>
<td>$L_d$</td>
<td>+0</td>
</tr>
<tr>
<td></td>
<td>$L_u$</td>
<td>0.000051</td>
</tr>
<tr>
<td>$L_u$</td>
<td>0.022</td>
<td>0.0062</td>
</tr>
<tr>
<td>$B_d$</td>
<td>14.433</td>
<td>15.879</td>
</tr>
<tr>
<td></td>
<td>19.516</td>
<td>18.522</td>
</tr>
<tr>
<td>$B_u$</td>
<td>24.599</td>
<td>21.164</td>
</tr>
<tr>
<td>$k_d$</td>
<td>15.299</td>
<td>16.606</td>
</tr>
<tr>
<td></td>
<td>19.988</td>
<td>21.907</td>
</tr>
<tr>
<td>$k_u$</td>
<td>24.677</td>
<td>21.907</td>
</tr>
<tr>
<td>$y_d$</td>
<td>18.981</td>
<td>20.390</td>
</tr>
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<td>24.446</td>
<td>23.684</td>
</tr>
<tr>
<td>$y_u$</td>
<td>29.912</td>
<td>26.979</td>
</tr>
</tbody>
</table>

Notes: Results are shown for the perceived share $\lambda=0.5$ of high-productivity borrowers and no prior bias. The foundation stake $q_0$ in firm equity is at level 0.2. The cost of lendable funds (deposit rate) is 0.03. For firms of type $#$, $q,#$ is total equity capital, $r,#$ is the borrowing rate, $B,#$ is the volume of credit taken, $k,#$ is the total investment in physical capital, $y,#$ is expected gross output (when the systemic productivity factor takes its expected value of 1), AL is arm’s length loan management, and RB is relationship banking (delegated loan management). Borrower management incentives are either consistent with limited liability (LL) or imitate unlimited liability (UL).
Table 5 Bubble, bank business model, and regulatory stance

<table>
<thead>
<tr>
<th>Bubble: Y/N</th>
<th>Equity price</th>
<th>Output</th>
<th>Interest rate</th>
<th>Default probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>3.198</td>
<td>23.534</td>
<td>7.57%</td>
<td>+0.0163</td>
</tr>
<tr>
<td>No</td>
<td>3.044</td>
<td>24.443</td>
<td>7.40%</td>
<td>+0.0384</td>
</tr>
<tr>
<td>Productivity type</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Baseline</td>
<td>2.871</td>
<td>20.170</td>
<td>8.54%</td>
<td>+0.0263</td>
</tr>
<tr>
<td>Macroprudential instrument</td>
<td>2.731</td>
<td>20.961</td>
<td>8.37%</td>
<td>+0.0580</td>
</tr>
<tr>
<td>Relative difference between bubble and no-bubble outcomes</td>
<td>5.07%</td>
<td>-3.72%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Downside borrower risk</td>
<td>3.724</td>
<td>23.606</td>
<td>7.51%</td>
<td>+0.0007</td>
</tr>
<tr>
<td>Relative difference between bubble and no-bubble outcomes</td>
<td>0.04%</td>
<td>-1.80%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AL</td>
<td>3.136</td>
<td>23.009</td>
<td>7.69%</td>
<td>+0.0153</td>
</tr>
<tr>
<td>Relative difference between bubble and no-bubble outcomes</td>
<td>5.92%</td>
<td>-3.41%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Macroprudential instrument</td>
<td>2.776</td>
<td>19.430</td>
<td>8.77%</td>
<td>+0.0241</td>
</tr>
<tr>
<td>Relative difference between bubble and no-bubble outcomes</td>
<td>5.53%</td>
<td>-3.97%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Downside borrower risk</td>
<td>3.337</td>
<td>22.524</td>
<td>7.58%</td>
<td>+0.0005</td>
</tr>
<tr>
<td>Relative difference between bubble and no-bubble outcomes</td>
<td>9.36%</td>
<td>-0.84%</td>
<td></td>
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</tr>
<tr>
<td>RB</td>
<td>3.052</td>
<td>22.714</td>
<td>7.73%</td>
<td>+0.0008</td>
</tr>
<tr>
<td>Relative difference between bubble and no-bubble outcomes</td>
<td>0.0007</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: AL – arm’s length loan management, RB – relationship banking (delegated loan management)
Table 6 Macroprudential capital surcharges compensated by a monetary policy easing: change of economic fundamentals against the baseline

<table>
<thead>
<tr>
<th>Policy stance</th>
<th>Baseline</th>
<th>Macroprudential instrument + low key interest rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indicator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td>3.328</td>
<td>3.310</td>
</tr>
<tr>
<td>$r$</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td>Default</td>
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<td></td>
</tr>
<tr>
<td>probability</td>
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<td></td>
</tr>
<tr>
<td>$L_d$</td>
<td>+0</td>
<td>+0</td>
</tr>
<tr>
<td>$L_u$</td>
<td>0.025</td>
<td>0.028</td>
</tr>
<tr>
<td>Loss given default</td>
<td>20.593</td>
<td>20.814</td>
</tr>
<tr>
<td>$B_d$</td>
<td>16.055</td>
<td>16.165</td>
</tr>
<tr>
<td>$B_u$</td>
<td>23.590</td>
<td>23.814</td>
</tr>
<tr>
<td>$k_d$</td>
<td>17.017</td>
<td>17.104</td>
</tr>
<tr>
<td>$k_u$</td>
<td>23.632</td>
<td>23.822</td>
</tr>
<tr>
<td>$y_d$</td>
<td>20.832</td>
<td>20.925</td>
</tr>
<tr>
<td>$y_u$</td>
<td>28.786</td>
<td>28.985</td>
</tr>
<tr>
<td>Bank earnings</td>
<td>0.883175</td>
<td>0.9801465</td>
</tr>
</tbody>
</table>

| $q_d$          | 3.138    | 3.106                                              |
| $q_u$          | 3.413    | 3.400                                              |
| $r_d$          | 0.080    | 0.080                                              |
| $r_u$          | 0.072    | 0.072                                              |
| Default probability | $L_d$ | +0       | +0                                                 |
| $L_u$         | 0.022    | 0.025                                              |
| Loss given default | 22.937 | 21.845                                           |
| $B_d$         | 14.433   | 14.395                                             |
| $B_u$         | 24.599   | 24.906                                             |
| $k_d$         | 15.299   | 15.233                                             |
| $k_u$         | 24.677   | 24.951                                             |
| $y_d$         | 18.981   | 18.914                                             |
| $y_u$         | 29.912   | 30.206                                             |
| Bank earnings | 0.8817475 | 0.976822                                          |

Notes: Results are shown for the perceived share $\lambda=0.5$ of high-productivity borrowers and no prior bias. The base capital surcharge $\alpha$ is 1 percent per 1 unit of credit collateralized by physical capital. The foundation stake $q_0$ in firm equity is at level 0.2. The cost of lendable funds (and the deposit rate) is 0.03 in the baseline and 0.025 under macroprudential measures. For firms of type $\pi$, $q_\pi$ is total equity capital, $r_\pi$ is the borrowing rate, $B_\pi$ is the volume of credit taken, $k_\pi$ is the total investment in physical capital, $y_\pi$ is expected gross output (when the systemic productivity factor takes its expected value of 1), AL is arm’s length loan management, and RB is relationship banking (delegated loan management). Borrowers have limited liability.