Portfolio competitions and rationality

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Abstract. We study investment competitions in which the players with highest achieved returns are rewarded by fixed prizes. We show that, under realistic assumptions, a game the participants play lacks a pure equilibrium and that the "max-min" solution of the game lies in one of the extremal points of the feasible set, namely in the one having maximal probability that the portfolio return falls into its normal cone. We analyse empirically a portfolio competition held recently by the Czech portal "lidovky.cz"; we find that the majority of people do not behave according to the game-theoretic conclusions. Consequently, searching for factors influencing a choice of particular stocks, we find that that the only significant determinant of the choice is a size of the stock's issuer.

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1 Introduction

With the public availability of the Internet, various investment competitions started to be held, usually with the following rules: each player obtains a virtual sum of money which he has to divide into several (real-life) financial assets. After a pre-determined time, gains of the players are evaluated (according to the real-life prices) and a selected number of the best players are rewarded by monetary prizes. If several participants achieve the same evaluation, the prize(s) divide(s) equally.

Whether the organizers realize it or not, those games are far from being a simulation of a real-life investment; the main reason for this is the fact that the objectives of "players" in real life differ from those in the game. In particular, while the actual return is simultaneously the gain in real life, which forces a risk averse individual to diversify (see [1]), only the best returns bring positive gains in the competition which, as shown in Section 2 of the present paper, makes even a risk-averse participant to take positions which are the most risky ones from the point of view of portfolio selection theory. In particular, the only portfolios getting a positive max-min gain are those lying in extremal points of the feasible set.

Analysing an actual portfolio game held by Czech internet portal "lidovky.cz", however, we found that people do not behave according to such a conclusion. As shown in Section 3, only 16.8% of participants chose portfolios lying in extremal points.

Section 4 tries to give alternative explanations of the player's behaviour. It is shown that, out of several fundamental and technical-analysis indicators, the only significant factor of a stock's selection is the size of the stock's issuer.

Even if all the game theoretic results and the method of our subsequent analysis are rather straightforward, we regard our work as original because, to our best knowledge, there is no other paper analysing this type of competition.

The paper is concluded by Section 5.

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2 Game Theoretic Approach

Denote $R \in \mathbb{R}^n$ a random vector of asset returns, possibly discounted by a deterministic risk free rate r_0, having an absolutely continuous joint distribution such that

$$\operatorname{supp}(R) = (-1, \infty)^n.$$

We assume that the set of feasible actions of the players is defined as

$$S = \{\pi \in \mathbb{R}^n : \gamma \leq 1'\pi \leq 1, 0 \leq \pi_i \leq \alpha, 1 \leq i \leq n\}$$

where α and γ are some constants. In the definition above, π stands for a vector fractions of the initial sum invested into the individual assets.

We assume the competitors to be risk averse, the i-th one having a strictly increasing utility function u_i. For simplicity, we assume that (the participants act as if) there is only single prize which implies that the utility of the i-th player is

$$v_i = \mathbb{E}(u_i(Z_i))$$

where Z_i is a gain of the player given by

$$Z_i \quad = Z_i(\pi_1, \ldots, \pi_m) = \begin{cases} \frac{1}{k_i} & \text{if } R \in \Gamma_i \\ 0 & \text{otherwise} \end{cases}$$

Here

- $\Gamma_i = \Gamma_i(\pi_1, \ldots, \pi_m) := \{r : \pi_i' r > \pi_j' r, j \notin K_i\}$
- $K_i = \{1 \leq j \leq m : \pi_j' R = \pi_i' R\}$,
- $k_i = |K_i|$
- $\pi_1, \pi_2, \ldots, \pi_m$ are the strategies (portfolios) of individual players.

Remark 1. The vector $(Z_1, \ldots, Z_m)$ is uniquely defined by $\rho = \frac{R}{|R|}$ a.s. where the support of ρ is the unit sphere.

In the present paper, assume all the strategies be the pure ones, i.e. deterministic. Then

Proposition 1. *The set*

$$K_i = \{1 \leq j \leq m : \pi_j' R = \pi_i' R\}$$

is a.s. deterministic.

Proof. Because $\mathbb{P}[w'S = 0] = 0$ for any absolutely continuous random vector S, any deterministic vector $w \neq 0$ and a constant c, we have

$$e_{i,j} := 1\{\pi_j' R = \pi_i' R\} = 1\{(\pi_j - \pi_i)' R = 0\} \overset{a.s.}{=} 1\{\pi_j = \pi_i\}$$

i.e., is deterministic, so it has to be

$$K_i = \{j : e_{i,j} = 1\}.$$

□

Corollary 1. $v_i = u_i(1/k_i)p_i$ where $p_i = \mathbb{P}(R \in \Gamma_i)$.

The following Proposition shows that some portfolios would never win regardless of the distribution of R

Proposition 2. *If π_i is not an extremal point of $C := \operatorname{conv}(\pi_1, \pi_2, \ldots, \pi_\mathrm{m})$ then $Z_i \equiv 0$ a.s.*

Proof. Assume WLOG that the first k strategies are extremal points of C. From the basic convex analysis (see [2]) we have that $\pi_i = \sum_{j=1}^k \lambda_j \pi_j$ where $\lambda_j \geq 0$. For Z_i to be positive, there should exists at least one possible return value r such that $\lambda_j r'(\pi_i - \pi_j) > 0$ for all $j \leq k$ giving $r'0 > 0$ by summing over all j. □

The following result says that the best max-min strategy is to take the most "advantegeous" corner of S; however, no equilibrium in pure strategies exists whenever there do not exist a group of stocks strongly outperforming the rest.

Theorem 3. *Denote $E = (e_1, \dots, e_r)$ the set of extremal points of S and put*

$$\sigma_i = \mathbb{P}(\rho \in N_S(e_i))$$

where

$$N_S(e) = \{r : r'(\pi - e) \leq 0 \text{ for all } \pi \in S\}$$

is a normal cone.
(i) If $m \geq n + 2$ then

$$\max_{\pi_i} \min_{\pi_j, j \neq i} v_i = 0$$

whenever $\pi_i \notin E$.
(ii)

$$\max_{\pi_i} \min_{\pi_j, j \neq i} v_i \geq u_i(\frac{1}{m})\sigma_i$$

whenever $\pi_i \in E$.
(iii) Denote $I = \lfloor \frac{1}{\alpha} \rfloor$. If there is a player, say the i-th one, such for each $j \geq 1$ there exist $j_1, j_2, \dots, j_{I+1}$, differing from j fulfilling

$$\mathbb{P}(R_{j_k} \geq R_j) > \frac{u_i(\frac{1}{m})}{u_i(1)}, \qquad 1 \leq k \leq I + 1 \tag{1}$$

then there exists no symmetric equilibrium in pure strategies.

Before proving the Theorem note that the RHS of (1) goes to zero with the growing number of participants.

Proof. (i) Assume $\pi_1 \notin E$. Then, by basic convex analysis, there exist $\pi_2, \dots, \pi_{n+2} \in E$ such that π_1 lies in their convex hull so (ii) is implied by Proposition 2.
(ii) If $\pi_1 \in E$, then clearly $N_S(\pi_1) \subseteq \Gamma_1$ for any $\pi_2, \dots, \pi_m$ so

$$v_i = u_i(1/k_i)p_i \geq u_i(1/m)\sigma_i.$$

(iii) Let $\pi \in S$ be an equilibrium. Let j be one of its non-zero components and let j_k fulfil (1) so that $\pi_{j_k} < \alpha$ (such j_k has to exist because at the weight of at most I components may equal to α). Consider portfolio $\tilde{\pi} = (\pi_1, \pi_2, \dots \pi_j - s, \dots \pi_{j_k} + s, \dots, \pi_n)$ where s is small enough for $\tilde{\pi}$ to be feasible. However, if the i-th player holds portfolio $\tilde{\pi}$ and the rest of players hold π then it will be

$$p_i = \mathbb{P}(\tilde{\pi}R > \pi R) = \mathbb{P}(s(R_{j_k} - R_j) > 0) > \frac{u_i(\frac{1}{m})}{u_i(1)}$$

hence the expected utility given $\tilde{\pi}$ (equal to $p_i u_i(1)$) would be greater than that given the equilibrium (being $u_i(1/m)$).

□

Summarizing: if one wants to be sure with a positive expected gain, he has to choose one of the extremal points as his strategy. If, in addition, there are no significant leaders among stocks and/or there is a large number of participants, then no such or another point is a pure equilibrium, i.e., possible common strategy.

3 Empirical Evidence

In order to verify whether actual people behave according to the game theoretic conclusions, we analysed a portfolio competition held by Czech news internet portal "lidovky.cz" this year. The competition started in April and is supposed to end in July. According to the rules, its participants could split a virtual million Czech crowns among 27 stocks listed in Table 3, and a (fictitious) bank account yielding 0.4% p.a. The three participants with the highest value of their virtual portfolios, measured on July 9, are promised to obtain 30.000, 20.000, and 10.000 Czech crowns, respectively. If there were more participants with the highest value of their portfolios then the prize would be divided equally.[1] The upper limit α of an investment asset is 40% for stocks, 50% for the bank account, respectively. The rules also say that at least 10% could be invested into a single stock if it is invested into it which, however, was violated by 6 portfolios for unknown reasons.[2]

The data we used come from the internet site of the competition `http://portfolio.lidovky.cz` and a subsequent preprocessing by a special software written in `C++` by us and by a free OCR program `gocr`. As the text recognition appeared to be inaccurate, several consistency checks were performed and, subsequently, manual correction were made; nevertheless, it is still possible that there are minor errors left in data caused by an inaccurate OCR recognition, which may be, however, regarded as noise if the data is analysed statistically .

There was as much as 2699 portfolios competing in the game. Even if it is highly probable that some players created multiple identities to increase their chances, we neglect this suspicion as we have no means to identify those cases.

There is 9828 extremal points of a feasible set in total,[3] 345 of which were occupied by portfolios of 453 (16.8%) participants (the most popular being portfolio CETV 40%, NWR 40%, ORCO 20% which was used 8 times). In other words, only 16.8% of players behaved "rationally" in the sense of Theorem 3. Out of remaining (non-extremal) portfolios, 975 was dominated in the sense of Proposition 2, having no chance for the first prize given the configuration portfolios of the other players. We used Iredundancy problem algorithm to determine which portfolios were dominated (see [3], Chp. 19 for details).

As the participants could optionally publish their gender and age, which was actually done by 2163 of them, 1559 of them, respectively, we tested for a correlation of a type of strategy chosen (possible types being an extremal point, a non-extremal not dominated point and a non-extremal dominated point) with these values. However, no significant results have been found here.

These facts lead us to a conclusion that people did not behave according to game theory given that only pure strategies are assumed.

4 Alternative Explanation.

Opposed to a rational approach, a hypothesis of purely random choice of portfolio, i.e., that the portfolios are chosen from the uniform distribution on the feasible set, suggests itself. This hypothesis, however, is falsified by the fact that SCHHV was never chosen because the probability that some stock has zero weight in all the 2699 portfolios is less than 0.02.

There could be many potential factors possibly influencing the choice of the stocks. In the present introductory paper, we restricted ourselves to considering selected data concerning the individual stocks published by the Prague stock exchange on their website, in particular to

- price-earning ratio (P/E),
- market capitalization (the monetary value of the issued shares), measuring the size of the firm,
- long-time trend (the ratio of the price of the stock at the time of the game's start and the average of the highest and the lowest price form the last year) and
- the short-time trend (the ratio of an OLS trend, computed from observations of the price from the five weeks preceding the competition, and the current price of the stock).

Note that, while the first two factors belong to fundamental analysis, the latter are more technical-analysis ones. In order to discover the dependency of a particular stock's choice on those factors, we run the logistic regression with the relative frequency as a dependent variable and the four mentioned factors

[1] It is, however, not said what would happen in case of equality on the second and/or the third place.

[2] We neglect these lower bounds in our theoretical analysis in Section 2 as they bring non-convexity of the feasible set which consequently complicates the treatment.

[3] Note that this number depends only on the number of stocks

Code	Name	p	a	MC	P/E	long	short
AAA	AAA Auto Group N.V.	0.17	3.0	1.565	5.98	0.043	0.002
CETV	CE Media Enterprises Ltd.	0.15	3.2	5.797	0	8.753	−0.037
ČEZ	ČEZ, a.s.	0.50	12.2	304.502	7.44	−0.163	−0.011
EFORU	E4U a.s.	0.04	0.7	0.167	10.56	0.022	−0.001
ENCHE	ENERGOCHEMICA SE	0.06	0.9	3.810	0	0.002	0.000
ENRGA	Energoaqua, a.s.	0.08	1.3	1.185	8.92	0.056	0.000
ERSTE	Erste Group Bank AG	0.42	8.5	223.957	0	0.108	−0.021
FOREG	Fortuna Entertainment Group N.V.	0.37	7.5	5.096	15.21	0.105	0.020
JIP	VET ASSETS a.s.	0.04	0.7	0.010	0	−0.052	0.000
KB	Komerční banka, a.s.	0.43	8.3	144.095	15.53	0.038	0.008
LAZJA	Jáchymov Property Management, a.s.	0.03	0.4	0.477	89.91	−0.025	0.000
NWR	New World Resources Plc	0.22	4.7	17.877	0	−0.318	−0.019
OCELH	OCEL HOLDING SE	0.09	1.5	3.798	0	0.000	0.000
ORCO	Orco Property Group S.A.	0.18	3.7	5.899	0.12	−0.148	−0.014
PEGAS	PEGAS NONWOVENS SA	0.26	5.2	4.661	12.92	0.064	−0.009
PM ČR	Philip Morris ČR a.s.	0.43	9.2	22.302	12.74	0.051	0.001
PRSLU	Pražské služby, a.s.	0.05	0.9	0.795	18.09	−0.066	0.000
PVT	RMS Mezzanine, a.s.	0.03	0.6	1.225	0	0.036	0.032
SCHHV	SPOLEK PRO CHEM.A HUT.VÝR.,a.s	0.00	0.0	775.763	0	0.000	0.000
SMPLY	Severomoravská plynárenská, a.s.	0.12	2.0	13.251	17.05	0.007	0.000
TEL. O2	Telefónica Czech Republic, a.s.	0.35	6.9	93.245	10.67	−0.181	−0.019
TMR	Tatry mountain resort, a.s.	0.16	3.3	7.814	0	0.029	0.004
TOMA	TOMA, a.s.	0.08	1.2	1.006	7.8	0.010	−0.001
UNI	UNIPETROL, a.s.	0.26	4.7	31.190	0	0.003	0.000
VCPLY	Východočeská plynárenská,a.s.	0.09	1.6	6.657	13.94	−0.037	−0.018
VGP	VGP NV	0.02	0.4	6.504	19.35	0.000	0.000
VIG	VIENNA INSURANCE GROUP	0.23	4.1	123.110	12.93	0.104	0.000

Table 1 Menu of stocks: p - frequency of choice, a - average weight (in %), *MC* - market capitalization, *P/E* - price to earning ration, *long* - long term trend, *short* - short term trend.

	Coefficient	Std. Error	t-ratio	p-value
const	-2.02345	0.238733	-8.4758	0.0000***
MC	8.09090e–06	2.55727e–06	3.1639	0.0047***
P/E	-0.0145792	0.0103617	-1.4070	0.1740
$long$	-0.0203306	0.125300	-0.1623	0.8727
$short$	-12.5787	16.7717	-0.7500	0.4616

Sum squared resid	16.92489	S.E. of regression	0.897746
R^2	0.462091	Adjusted R^2	0.359632
$F(4,21)$	4.510016	P-value(F)	0.008704
Log-likelihood	-31.31136	Akaike criterion	72.62272
Schwarz criterion	78.91320	Hannan–Quinn	74.43415

Table 2 Result of logistic regression $p = 1/(1 + e^{-(1,MC,P/E,long,short)'\beta})$

as independent ones.[4] The results, shown by table 3, clearly show that only the market capitalization comes out as significant.[5]

5 Conclusion

We analysed a rather general case of a portfolio competition. As the behaviour of players in an actual game of this type appeared to be inconsistent with the from the game-theoretical point of view, we tried to give a simple behavioural explanation: in particular, we found that - out of four factors - players take only the size of the stock's issuer into account when constructing their portfolios.

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[3] Goodman, J.E., and Rourke J.O. eds. *Handbook of Discrete & Computational Geometry.* Chapman and Hall/CRC, 2010.

[4] Even if there is a dependency between choices of the particular stocks by the individuals, we omit it relying on the law of large numbers promising a closeness of the relative frequencies with the probabilities.

[5] We excluded SCHHV from the regression as it is clear from its data that the stocks is not traded.