

# Determinants of Stocks' Choice in Portfolio Competitions

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## Abstract

We study investment competitions in which the players invest a virtual amount of money into financial asset and those with highest returns, measured by the actual prices, are rewarded by fixed prizes. We show that the competition, seen as a game, lacks a pure equilibrium and that the “max-min” solution of the game lies in the extremal point of the feasible set having maximal probability of victory. We show further that if a mixed equilibrium exists then its atoms lie exactly in the extremal points with a non-zero probability of victory and its weights are close to corresponding probabilities of victory.

We analyse empirically a portfolio competition held recently by the Czech portal “lidovky.cz”; we find that the majority of people do not behave according to the game-theoretic conclusions. Consequently, searching for factors influencing a choice of particular stocks, we find that the participants' choice may be explained by several stock traits to a certain extent. We also show that participants tend to choose negatively diversified portfolios.

## Keywords

portfolio competition, game theory, behavioural finance

## 1 Introduction

The subject of our study is a portfolio competition in which their participants divide a virtual amount of money into several (real-life) financial assets; after a specified time, gains of the players are evaluated and several (usually three) best players are rewarded by monetary prizes. If more than one participant achieve the same gain, the prize is divided equally.

As we show below, the strategies in those competitions differ dramatically from a real-life investment: while only the actual return, regardless on the results of the other “players”, matters in real life, so the “player” may afford to reduce her risk by a diversification diversify (see [1]), only the best returns among all the players bring positive gains in the competition which, as shown in Section 2 of the present paper, makes even a risk-averse participant to take the most risky positions. In particular, the only portfolios getting a positive max-min gain are those lying in extremal points of the feasible set. Moreover, we show that if an equilibrium of the game exists then it has to be mixed one with atoms lying in the extremal points.

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An analysis of a particular portfolio competition by Czech internet portal "lidovsky.cz", made in Section 3, however shows that people do not behave according to game-theoretic conclusions; in fact, only 17.6% of participants chose portfolios lying in extremal points.

In Section 4, we propose a method of an explanation of the player's behaviour. In particular, we use multinomial logit - one of the discrete choice models - to determine possible factors driving the participants' choice. It is also shown how the multinomial logit model may emulate the possible game-theoretic behaviour of the participants.

In Section 5, the method is applied to the "lidovsky.cz" competition. The analysis is carried out separately for supposedly rational participants (i.e. those who place their portfolios into the extremal points) and the remaining ones. In both the cases, a hierarchy of models is proposed and subsequently estimated.

The paper is concluded by Section 6.

## 2 Game Theoretic Approach

Let  $R \in \mathbb{R}^n$  be a random vector of asset returns, possibly discounted by a deterministic risk free rate  $r_0$ , with an absolutely continuous joint distribution such that

$$\text{supp}(R) = (-1, \infty)^n.$$

and let the set of feasible actions of the players be defined as

$$S = \{\pi \in \mathbb{R}^n : \gamma \leq 1' \pi \leq 1, 0 \leq \pi_i \leq \alpha, 1 \leq i \leq n\}$$

where  $\alpha$  and  $\gamma$  are some constants; the points  $\pi$  of  $S$  stand for a vector fractions of the initial sum invested into the individual assets.

Let the competitors be risk averse first, the  $i$ -th one having a strictly increasing utility function  $u_i$ . For simplicity, we assume that (the participants act as if) there is only single prize. Then the utility of the  $i$ -th player is

$$v_i = \mathbb{E}(u_i(Z_i))$$

where  $Z_i$  is a gain of the player given by

$$Z_i = Z_i(\pi_1, \dots, \pi_m) = \begin{cases} \frac{1}{k_i} & \text{if } R \in \Gamma_i \\ 0 & \text{otherwise} \end{cases}$$

Here

- $\Gamma_i = \Gamma_i(\pi_1, \dots, \pi_m) := \{r : \pi'_i r > \pi'_j r, j \notin K_i\}$
- $K_i = \{1 \leq j \leq m : \pi'_j R = \pi'_i R\}$ ,
- $k_i = |K_i|$
- $\pi_1, \pi_2, \dots, \pi_m$  are the strategies (portfolios) of individual players.

The following result says that the best max-min strategy is to take the most "advantageous" corner of  $S$ ; however, no equilibrium in pure strategies exists given that there do not exist a group of stocks strongly outperforming the rest.

**Theorem 1.** Denote  $E = (e_1, \dots, e_r)$  the set of extremal points of  $S$  and put

$$\sigma_i = \mathbb{P}(\rho \in N_S(e_i))$$

where

$$N_S(e) = \{r : r'(\pi - e) \leq 0 \text{ for all } \pi \in S\}$$

is a normal cone.

(i) If  $m \geq n + 2$  then

$$\max_{\pi_i} \min_{\pi_j, j \neq i} v_i = 0$$

whenever  $\pi_i \notin E$ .

(ii)

$$\max_{\pi_i} \min_{\pi_j, j \neq i} v_i \geq u_i\left(\frac{1}{m}\right)\sigma_i$$

whenever  $\pi_i \in E$ .

(iii) Denote  $I = \lfloor \frac{1}{\alpha} \rfloor$ . If there is a player, say the  $i$ -th one, such for each  $j \geq 1$  there exist  $j_1, j_2, \dots, j_{I+1}$ , differing from  $j$  fulfilling

$$\mathbb{P}(R_{j_k} \geq R_j) > \frac{u_i\left(\frac{1}{m}\right)}{u_i(1)}, \quad 1 \leq k \leq I + 1 \quad (1)$$

then there exists no symmetric equilibrium in pure strategies.

*Proof.* See [4] □

Note that the RHS of (1) goes to zero with the growing number of participants.

The following result deals with possible mixed equilibria given a risk neutrality of the players. Even though it does not guarantee an existence of a mixed equilibrium, it says that if a symmetric equilibrium exists then it is very close to the mixed strategy with atoms coinciding with the extremal points of  $S$  and with weights equal to the victory probabilities  $\sigma_i$  corresponding to the points.

**Theorem 2.** If  $u_i$  are linear and if  $m \geq m_0$  where

$$m_0 \geq \frac{1}{\sigma_{min}}, \quad \sigma_{min} = \min\{\sigma_i : 1 \leq i \leq |E|\}$$

and

$$\ln(n + 1) + (m_0 - 1)[\ln(1 - \sigma_{min}) + \ln m_0 - \ln m_0 - 1] + \ln m_0 \leq 0$$

then each symmetric equilibrium in mixed strategies  $\Pi = (\theta_i, q_i)_{i \leq r}$  consists exactly from all the extremal points of  $S$  and

$$q_i \geq \sigma_i - \frac{1 - \sigma_i}{m - 1} \geq \sigma_{min} - \frac{1 - \sigma_{min}}{m - 1}$$

Moreover,  $q_i \rightarrow \sigma_i$  as  $m \rightarrow \infty$ .

*Proof.* See [5]. □

Summarizing: if one wants to be sure with a positive expected gain and uses only pure strategies then he has to choose one of the extremal points as his strategy. However, under quite realistic conditions, no symmetric equilibrium in pure strategies exists; hence, if a symmetric equilibrium exists, then it has to be a mixed strategy; however, if such a strategy exists then it has to be a mixture of extremal points with their victory probabilities as weights.

Code	Name	$p$	$a$
AAA	AAA Auto Group N.V.	0.17	3.0
CETV	CE Media Enterprises Ltd.	0.15	3.2
ČEZ	ČEZ, a.s.	0.50	12.2
EFORU	E4U a.s.	0.04	0.7
ENCHE	ENERGOCHEMICA SE	0.06	0.9
ENRGA	Energoaqua, a.s.	0.08	1.3
ERSTE	Erste Group Bank AG	0.42	8.2
FOREG	Fortuna Entertainment Group N.V.	0.37	7.5
JIP	VET ASSETS a.s.	0.04	0.7
KB	Komerční banka, a.s.	0.43	8.3
LAZJA	Jáchymov Property Management, a.s.	0.03	0.4
NWR	New World Resources Plc	0.22	4.7
OCE LH	OCEL HOLDING SE	0.09	1.5
ORCO	Orco Property Group S.A.	0.18	3.7
PEGAS	PEGAS NONWOVENS SA	0.26	5.2
PM ČR	Philip Morris ČR a.s.	0.43	9.2
PRSLU	Pražské služby, a.s.	0.05	0.9
PVT	RMS Mezzanine, a.s.	0.03	0.6
SCHHV	SPOLEK PRO CHEM.A HUT.VÝR.,a.s	0.00	0.0
SMP LY	Severomoravská plynárenská, a.s.	0.12	2.0
TEL. O2	Telefónica Czech Republic, a.s.	0.35	6.9
TMR	Tatry mountain resort, a.s.	0.16	3.3
TOMA	TOMA, a.s.	0.08	1.2
UNI	UNIPETROL, a.s.	0.26	4.7
VCPLY	Východočeská plynárenská,a.s.	0.09	1.6
VGP	VGP NV	0.02	0.4
VIG	VIENNA INSURANCE GROUP	0.23	4.1

Table 1: Menu of stocks:  $p$  - frequency of choice,  $a$  - average weight (in %)

### 3 Data

In the present Section we analyse a particular portfolio competition, namely the one held by Czech news internet portal "lidovky.cz" this year. The competition started in April and ended in July. According to the rules, its participants could split a virtual million Czech crowns among 27 stocks listed in Table 1, and a (fictitious) bank account yielding 0.4% p.a. The three participants with the highest value of their virtual portfolios, measured on July 9, were promised to obtain 30.000, 20.000, and 10.000 Czech crowns, respectively. If there were more participants with the highest value of their portfolios then the prize would be divided equally.<sup>4</sup> The upper limit  $\alpha$  of an investment asset is 40% for stocks, 50% for the bank account, respectively. The rules also said that at least 10% could be invested into a single stock if it is invested into it which, however, was violated by 6 portfolios for unknown reasons.<sup>5</sup>

The data we used come from the internet site of the competition <http://portfolio.lidovky.cz>

<sup>4</sup>It is, however, not said in the rules what would happen in case of equality on the second and/or the third place.

<sup>5</sup>We neglect these lower bounds in our theoretical analysis in Section 2 as they bring non-convexity of the feasible set which consequently complicates the treatment.

and a subsequent preprocessing by a software written by us in C++ and by a free OCR program `gocr`. As the text recognition appeared to be inaccurate, several consistency checks were performed and, subsequently, manual corrections were made; nevertheless, it is still possible that there are minor errors left in data caused by an inaccurate OCR recognition, which may be, however, regarded as noise if the data is analysed statistically.

There was as much as 2699 portfolios competing in the game. Even if it is highly probable that some players created multiple identities to increase their chances, we neglect this suspicion as we have no means to identify those cases.

There is 9828 extremal points of a feasible set in total,<sup>6</sup> 365 of which were occupied by portfolios of 477 (17.68%) participants (the most popular being portfolio CETV 40%, NWR 40%, ORCO 20% which was used 8 times). In other words, no more than 17.68% of players behaved "rationally" in the sense of Theorem 1. Out of remaining (non-extremal) portfolios, 975 (36.1 %) was dominated (i.e. there were enclosed into a convex hull established by other portfolios), having no chance for the first prize given the configuration of the other portfolios. We used Iredundancy problem algorithm to determine which portfolios were dominated (see [3], Chp. 19 for details).

Figure 1 shows average weights of individual stocks in the participants' portfolios; differences among the stocks are visible at the first look, and, even if the differences between participants who chose extreme portfolios (we call them "extremists" in the rest of the paper) and the others could not be proved solely from the numbers displayed in the graph (the standard deviation of the difference is up to 0.015), a more detailed statistical analysis (the goodness-of-fit test of distributions of two most weighted stocks in portfolios of extremists and the others) shows this difference to be significant, too. Therefore we decided to analyse the two groups separately.

## 4 Methodology

In econometrics, situations when  $K$  subjects choose between  $J$  alternatives is usually treated by means of discrete choice models, the multinomial logit model especially. We use this approach, too.

The multinomial logit model assumes the  $k$ -th subject to choose the alternative  $j_0$  if and only if

$$j_0 = \operatorname{argmax}_j u_{k,j}, \quad u_{k,j} = \beta'_{k,j} X_{k,j} + \epsilon_{k,j}, \quad k \leq K, j \leq J,$$

where  $\beta_{j,k} \in \mathbb{R}^q$  are deterministic vectors,  $X_{j,k} \in \mathbb{R}^q$  are explanatory variables and  $\epsilon_{j,k}$  are mutually independent random variables each with the standard type 1 extreme value distribution (for more details, see [6]).

After some calculation, the probability that subject  $k$  chooses alternative  $j_0$  comes out as

$$p_{k,j_0} = \frac{\exp\{\beta'_{k,j_0} X_{k,j_0}\}}{\sum_{j=1}^J \exp\{\beta'_{k,j} X_{k,j}\}}. \quad (2)$$

Parameters  $\beta$  may be easily estimated by an application of standard maximum likelihood to (2). To test hypotheses about the parameters either  $t$ -tests associated with the ML estimation or likelihood ratio tests may be used.

As a measure of explanation brought by a model in comparison with

$$H_0 : p_{k,1} = p_{k,2} = \dots = p_{k,j},$$

the quantity

$$\rho = 1 - \frac{LL}{LL_0}$$

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<sup>6</sup>Note that this number depends only on the number of stocks

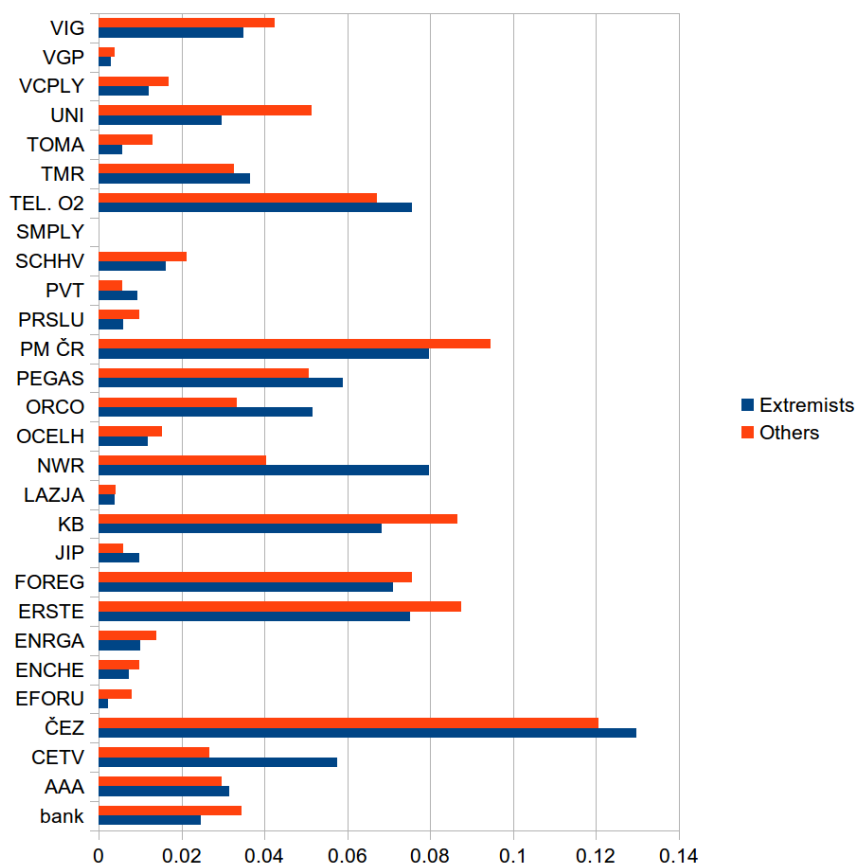


Figure 1: Relative frequencies of stocks' choice.

is often used, where  $LL$  and  $LL_0$  are the log likelihoods given the model, given  $H_0$ , respectively - note that  $\rho = 0$  given  $H_0$  and that  $\max \rho = 1$  hence  $\rho$  may be interpreted as a percentage improvement with respect to  $H_0$ .

Even if the assumptions of the multinomial logit model, implicitly including the irrelevant alternatives assumption among others, are rather limiting, the tractability, the estimability and the relative simplicity of the model speak in favour of using it at least as a useful starting point.

An additional reason for the application of the model to our problem is that it is able to describe the behaviour of participants acting according to game theory - in particular, if we assume the alternatives of the choice to be exactly the extremal portfolios and if

$H_{mm}$  : the participants act the min-max way

then, by putting  $X_{k,j} = \sigma_j$  and  $\beta_{j,k} = \beta \rightarrow \infty$ . we get

$$p_{k,j} \rightarrow \begin{cases} 1 & \text{if } j = \arg \max_l \sigma_l \\ 0 & \text{otherwise} \end{cases}$$

ie, the min-max solution. The case when  $\beta$  is finite naturally models the situation in which the participants are uncertain regarding the value of  $\sigma_j$ , see the next Section.

Similarly, the case when

$H_{me}$  : the participants apply a mixed strategy  $(\sigma_1, \sigma_2, \dots, \sigma_J)$ .

may be emulated by assuming  $X_{k,j} = \log(\sigma_j)$  and  $\beta_{j,k} = 1$  in which case  $p_{k,j} = \sigma_j$ ; therefore, estimates of  $\beta_{k,j}$  may serve as a statistic possibly falsifying  $H_{me}$ .

## 5 Empirical Evidence

### 5.1 Extremists

In the present Subsection we deal with the 477 participants who chose extremal portfolios.

Say first that probabilities  $\sigma_\bullet$  are known only up to an additive error  $e_\bullet$  with common variance  $v$  and that  $H_{mm}$  holds true. Then

$$u_{k,j} = \sigma_j + e_{k,j}$$

which, standardized for the variance of the extreme value distribution (being  $v_e = \frac{\pi^2}{6}$ ) gives

$$\tilde{u}_{k,j} = \beta \sigma_j + \epsilon_{k,j}, \quad \beta = \sqrt{v_e/v}. \quad (3)$$

Testing  $H_{mm}$  against  $H_0$  thus reduces to testing whether  $\beta = 0$ .

The probabilities of victory  $\sigma_j$  we used in the test were computed by means of simulation: in particular, 4,000,000 simulated asset returns were drawn from multivariate normal distribution with mean and variance matrix estimated from the daily returns of the assets (with a silent assumption that the daily returns are independent in time). The victory probabilities were then evaluated by counting victories of individual extremal points.<sup>7</sup>

The results of the test of  $H_{mm}$  against  $H_0$  are as follows:

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<sup>7</sup>If the moments were exact and the distribution was indeed normal time-independent then the victory probabilities would be estimated with standard error less than 0.00005 by our computation. Due to the impreciseness of the estimates, however, our  $\sigma$ 's are imprecise, too, and, because the form of dependence of  $\sigma$ 's on the moments is complicated (its exact evaluation would require multidimensional numerical integration), we even are not able to determine the estimation error. We thus, in fact, silently assume that the participants count with the estimated distribution.

	Coefficient	Std. Error	t-ratio
PWIN	451.208	67.8915	6.64603***
$\rho$		0.00557393	observations 477
likelihood ratio		48.2814	d.f 1

Even if the test came out significant, the result is practically useless because. by (3), the standard error of  $e$  is  $\sqrt{v} = \pi^2/(6\beta) \doteq 0.02$  which is far more than the highest estimates of  $\sigma$ 's, being less than 0.01. Thus, the only conclusion we may make here is that the choice probabilities somehow, very weakly, reflect the estimated victory probabilities.

Similarly we may test  $H_{me}$ : assuming multiplicative errors  $f_{\bullet}$  this time, we get

$$u_{k,j} = \log(\sigma_j f_{k,j}) \doteq \log(\sigma_j) + \epsilon_{k,j}$$

(because only the differences matter in discrete choice, the constant term resulting from the non-linear transformation of  $f$  may be neglected). Here, however, we face the problem that about 90 estimates of  $\sigma$ 's are zero which would lead to covariates equal to minus infinity.<sup>8</sup> Therefore, when tried to overcome this by an approximation of the logarithm by a quadratic function (making our new model sup-model of the previous one). The results are as follows:

	Coefficient	Std. Error	t-ratio
PWIN	1293.9	135.179	9.57177***
PWIN2	-217748	26444.6	8.23414***
$\rho$		0.00917296	observations 477
likelihood ratio		79.4561	d.f 2

Even though the quadratic term is negative so the function has the "right" concave shape, still the explanation power of such a model is poor.

Another hypothesis could be, that

$H_r$  : people "seek risk", measured by the variance, in order to win the competition.

In order to examine this hypothesis in greater detail, we split the variance into the diagonal and the covariance parts, i.e. we assume

$$u_{k,j} = \beta_1 v_j^d + \beta_2 v_j^c + \epsilon_{k,j}, \quad v_j^d = \sum_{i=1}^n \pi_i^2 \text{var}(R_{i,i}), \quad v_j^c = \text{var}(\pi_i' R) - v_j^d.$$

The results are following:

	Coefficient	Std. Error	t-ratio
NAIVEVAR	-0.892095	0.178622	4.99431***
DIVEFFECT	13.7508	0.742238	18.5262***
$\rho$		0.0444239	observations 477
likelihood ratio		384.799	d.f 2

Contrary to the previous two models whose  $\rho$ 's were less than 1%, the  $\rho$  here is as great as 4%. Even more interestingly, if we omit the "naive" part, the  $\rho$  would not decrease too much:

	Coefficient	Std. Error	t-ratio
DIVEFFECT	12.2192	0.644341	18.9639***
$\rho$		0.0414165	observations 477
likelihood ratio		358.75	d.f 1

<sup>8</sup>This is partially due to the fact that the distribution of four stocks - LAJZA, OCELH, SCHHV and VGP - is Dirac at zero, their returns are thus dominated by the bank account which implies that no portfolio including some of these stocks and with less than 50% of the bank has chance to win.



Because the explanatory power is still low and the differences between the individual stocks are still unexplained, the next step was to seek stock traits which would be able to explain the participants' choices in addition to the diversification effect. To this end, we assume that

$H_t$  : a participant gets utility from certain traits of the stocks

i.e.

$$u_{k,j} = \beta v_j^d + \sum_i \gamma_i t_{i,j} + \epsilon_{k,j} \quad t_{i,j} = \sum_\nu \pi_{j,\nu} \tau_{\nu,i}$$

where  $\tau_{\nu,i}$  is the  $i$ -th trait of the  $\nu$ -th stock. The traits we take into account include the information about individual stocks provided by the Prague stock exchange on their website plus several additional traits which are deducible from historical data being available on the website in a graphical form:

**LOGMK** logarithm of market capitalisation, measuring the size of the firm

**PE** price earning ratio

**PEMISSING** a dummy being one in case that the PE is not available on the website

**MAJORITY** a stake of a major owner

**DIVIDENDRET** dividend return in the previous year

**TRADEABILITY** equal to one, if the stock belongs to more liquid stocks (displayed as "selected stocks" on the website)

**TREND6M** trend from the last half year

**TRENDLONG** long trend, measured by the relative position of the current price to the average of the highest and the lowest prices from the last year

**TRADEFREQ** percentage of days in which the price changed

**ZEROTRADES** equal to one if the variance of the stock is zero (see above)

**VOLATILITY** volatility of the stock

All the traits had been standardized, the results of the estimation are following:

	Coefficient	Std. Error	t-ratio
LOGMK	0.569593	0.183298	3.10748**
PE	-0.395217	0.271275	1.45689
PEMISSING	0.142912	0.140483	1.01729
MAJORITY	0.0341098	0.537776	0.0634276
DIVIDENDRET	-0.00561123	0.717448	0.0078211
TRADEABILITY	0.895469	0.154135	5.80963***
TREND6M	2.08681	0.165754	12.5898***
TRENDLONG	0.225133	0.138736	1.62275
TRADEFREQ	-0.502935	0.126327	3.98122***
ZEROTRADES	-0.763352	0.772684	0.987922
VOLATILITY	-30.0713	16.1025	1.86749
DIVEFFECT	9.14242	1.09444	8.35354***
$\rho$	0.121835	observations	477
likelihood ratio	1055.33	d.f	12

It is obvious that this model brings much better explanation than the "risk" one.

The last in the chain of models we studied was the one assuming

$H_c$  : a participant get a constant utility for each stock

i.e.

$$u_{k,j} = \beta v_j^d + \sum_{\nu} \pi_{j,\nu} \eta_{\nu} + \epsilon_{k,j}$$

where  $\eta_{\nu}$  is the utility from the stock  $\nu$ , whose results are

	Coefficient	Std. Error	t-ratio
AAA	0.530561	0.658	0.806324
CETV	0.160249	0.725612	0.220847
ČEZ	4.74244	0.543707	8.72241***
EFORU	-8.03221	2.1953	3.65882***
ENCHE	-3.59581	1.16197	3.09457**
ENRGA	-2.50735	0.943191	2.65837**
ERSTE	1.23782	0.666734	1.85655
FOREG	2.47644	0.574739	4.3088***
JIP	-3.10638	1.01899	3.04849**
KB	1.84364	0.628468	2.93355**
LAZJA	-5.74459	1.49506	3.84238***
NWR	2.03821	0.629318	3.23876**
OCELH	-1.9931	0.909482	2.19146*
ORCO	1.51898	0.612075	2.48169*
PEGAS	2.60494	0.601844	4.32827***
PM ČR	3.42789	0.588588	5.82393***
PRSLU	-4.28001	1.15181	3.71589***
PVT	-2.94068	0.944044	3.11499**
SMPLY	-1.01016	0.859054	1.1759
TEL. O2	3.10432	0.583214	5.32277***
TMR	1.6007	0.638862	2.50556*
TOMA	-4.35303	1.22469	3.55439***
UNI	0.681223	0.684628	0.995027
VCPLY	-1.88229	0.915875	2.05518*
VGP	-6.63777	1.53987	4.31061***
VIG	0.35951	0.651651	0.55169
DIVEFFECT	8.71744	1.31269	6.6409***
$\rho$	0.137125	observations	477
likelihood ratio	1187.77	d.f	27

Here we see that the explanatory power did not increase much in comparison with the previous model, so we may admit that  $H_t$  is able to explain the participants' choices to some extent, taking the significant coefficients as possible factors explaining the participants' behaviour.<sup>9</sup>

## 5.2 Remaining Participants

The behaviour of the participants not choosing extremal portfolios may be analysed similar way. However, additional question arises: what is the set of alternatives here? From the matter of fact,

<sup>9</sup>One may object here that *any* vector standing for traits could come out significantly. However, if we take random numbers instead of the traits, the resulting  $\rho$  is 0.076 on average with a standard error 0.015 which proves the objection false.

the set is infinite, in which case the discrete choice models could not be applied. Therefore, we made an additional assumption that only portfolios with weights taking values in a certain finite subset of  $[0, 1]$  are the alternatives. Even given this simplification, however, the set of alternatives turns out to be extremely huge. Therefore, we decided to approximate the denominator of (2) by an integral, which we consequently evaluated by means of Monte Carlo.<sup>10</sup>

Because the probability of victory is zero for all the non-extremal portfolios, we omit the first two models from the previous Subsection, and the chain of the model will be as follows:

$H_r$  without naive part:

	Coefficient	Std. Error	<i>t</i> -ratio
DIVEFFECT	24.2769	0.43612	55.6658***
$\rho$	0.0507012	observations	2222
likelihood ratio	2599	d.f	1

$H_t$  (including the diversification effect):

	Coefficient	Std. Error	<i>t</i> -ratio
LOGMK	-0.0333417	0.117292	0.284261
PE	-2.9273	0.170304	17.1887***
PEMISSING	0.97319	0.085496	11.3829***
MAJORITY	-0.158435	0.29394	0.539004
DIVIDENDRET	0.767688	0.387857	1.97931*
TRADEABILITY	-0.00332881	0.0818279	0.0406806
TREND6M	3.17522	0.0958563	33.1248***
TRENDLONG	-0.80461	0.101747	7.90792***
TRADEFREQ	0.538875	0.0844264	6.38278***
ZEROTRADES	-16.7061	0.41519	40.2373***
VOLATILITY	-169.245	9.91285	17.0733***
DIVEFFECT	1.39461	0.834849	1.67049
$\rho$	0.208218	observations	2222
likelihood ratio	10673.5	d.f	12

$H_c$  (including the diversification effect):

<sup>10</sup>In the present preliminary paper we allowed the portfolios in the set to have more than 10 positive weights, even if it was not allowed by the rules of the competition. We regard the accommodation of this fact as the necessary next step in our research.

	Coefficient	Std. Error	<i>t</i> -ratio
AAA	-0.699784	0.370038	1.89111
CETV	-0.61192	0.491754	1.24436
ČEZ	1.01176	0.275866	3.66759***
EFORU	-13.0467	0.607241	21.4853***
ENCHE	-7.84003	0.550969	14.2295***
ENRGA	-2.45436	0.477254	5.14266***
ERSTE	0.69641	0.389015	1.79019
FOREG	0.809344	0.309536	2.6147**
JIP	-23.3353	0.726183	32.1342***
KB	0.374025	0.357288	1.04685
LAZJA	-44.0061	0.894984	49.1698***
NWR	0.0260313	0.395202	0.0658684
OCE LH	-2.70495	0.474067	5.70583***
ORCO	-0.0864965	0.355688	0.243181
PEGAS	-0.0400821	0.304455	0.131652
PM ČR	0.867288	0.279767	3.10004**
PRSLU	-8.80391	0.545008	16.1537***
PVT	-24.7978	0.694069	35.7281***
SMP LY	-0.774537	0.428156	1.80901
TEL. O2	0.315307	0.301567	1.04556
TMR	0.11573	0.344331	0.3361
TOMA	-4.43616	0.504323	8.79627***
UNI	0.740131	0.314619	2.35247*
VCPLY	-2.21295	0.463615	4.77324***
VGP	-46.8473	0.901979	51.9384***
VIG	0.22218	0.351757	0.631629
DIVEFFECT	-0.361743	1.02811	0.351852
$\rho$	0.235593	observations	2222
likelihood ratio	12076.8	d.f	27

At the first look, the results are similar to the "extremists" case, with the important exception that, contrary to extremists, the diversification effect is insignificant here suggesting less degree of the "risk to win" approach in comparison to the extremists. However, it is also possible that this difference is caused solely by the fact that the portfolios here consist of more stocks here - to solve this problem seems to be the one of the next steps of our research.

## 6 Conclusion

We analysed a rather general case of a portfolio competition. As the behaviour of players in an actual game of this type is apparently inconsistent from the game-theoretical point of view, we applied a discrete choice model in order to explain the participant's choices by certain stock traits, several of which we found significant.

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