

# Concepts of Modeling and Control of Industrial Articulated Robots for Efficient, Sustainable and Safe Production

Belda, K. (a); Záda, V. (b)

a) *Institute of Information Theory and Automation AS CR, Prague, Czech Republic*

b) *Technical University of Liberec, Liberec, Czech Republic*

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## **Abstract**

The paper deals with advanced modeling and model-based control design concepts for an optimal robot motion and monitoring for safe and efficient operation. The explanation aims at concepts for the composition of suitable mathematical models of robot kinematics and dynamics. In the paper, a control design concept via model-based predictive control is introduced. The design and monitoring of recommended reference signals for safe and efficient process control are discussed. The concepts are demonstrated on six-axis multipurpose robot ABB IRB 140. This robot belongs to the considered class of industrial articulated robots-manipulators.

# Konzepte der Modellierung und Steuerung der industriellen Knickarmroboter für eine effiziente, nachhaltige und sichere Produktion

Belda, K. (a); Záda, V. (b)

a) *Institute of Information Theory and Automation AS CR, Prague, Tschechische Republik*

b) *Technische Universität Liberec, Liberec, Tschechische Republik*

*Schlagwörter:*

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## **Kurzfassung**

Der Beitrag beschäftigt sich mit den fortgeschrittenen Designkonzepten der Modellierung und modell-basierter Steuerung für die optimale Bewegung des Roboters und ihre Überwachung für den sicheren und effizienten Betrieb. Die Erklärung zielt auf Konzepte der Zusammensetzung von geeigneten mathematischen Modellen der Kinematik und Dynamik der Roboter. Im Beitrag wird das Konzept des Steuerungsentwurfs mit der modellbasierten prädiktiven Regelung vorgestellt. Der Entwurf und Überwachung der empfohlenen Referenzsignale für die zuverlässige und effiziente Prozesssteuerung werden diskutiert. Die Konzepte werden auf dem Sechssachsen-Mehrzweck-Roboter ABB IRB 140 demonstriert. Dieser Roboter gehört zur Klasse der industriellen Knickarmroboter-Manipulatoren.

# 1 Introduction

Nowadays, there exist a lot of robot constructions and structures. However, their successful use lacks suitable algorithms for the motion planning and motion control, which can naturally respect particularities and features of real robotic systems [7]. The paper deals with advanced modeling and model-based control design concepts employed for optimization of robot motion, efficient input energy distribution [5] and motion monitoring for safe and efficient robot operations [1] applied to the class of industrial articulated robots. The positioning of the individual drives of such robots provides a motion of robot end-effector. Its positioning is usually defined in Cartesian coordinate system. Due to an independent driving of individual joints and general spatial configuration, the keeping of adequate admissible ranges of kinematic and dynamic quantities has to be monitored. Precisely, the real end-effector position and torque values on appropriate robot drives have to be controlled in specific admissible ranges. Range violation may indicate presence of uncontrolled robot states or breakdowns.

Standard algorithms of control systems of industrial robots consider usually only simple kinematic transformations and do not respect dynamical interaction effects among drives or actuated joints and robot links. They are predominantly configured as non-model based. The simple kinematic models serve them only for transformations of user requirements from operational coordinate system into joint (drive) coordinate system. Such control systems consider the drives in robotic structure as individual independent systems and their interactions as external disturbances in spite of significant deterministic relations given by robot links.

Model-based control algorithms can respect the majority of interactions and couplings just via a model. Furthermore, the proposed model-based predictive control can respect additional constraints or also dynamic trajectory optimization.

The paper aims at introduction of suitable concepts for model synthesis and their particular application to the class of industrial articulated robots [7]. The application of the concepts is addressed with respect to the maintaining of efficient sustainable and safe production. It is followed by optimal model-based control design and its application as an energy-efficient tool of control design and as a generator of reference admissible values of control actions serving safety-monitoring algorithms.

The paper is organized as follows. Section 2 introduces Denavit-Hartenberg [3] and exponential [4] concepts and their suitable application to a composition of mathematical model of kinematics of articulated robots. These concepts are applied

by specific refined generalized formulas providing safe straightforward synthesis of direct and inverse structure kinematics of given robots. Section 3 presents concepts of the modeling robot dynamics and section 4 deals with a model-based control design. Here, the model-based concept is considered as a powerful tool either for design of reference or for real process control. Section 5 outlines promising application ways of introduced concepts towards efficient sustainable and safe production. In the section 6, there is an illustrative example, which demonstrates introduced concepts of modelling and model-based control for real industrial articulated robot IRB 140 from ABB Company [2].

## 2 Concepts for modeling of robot kinematics

Usual procedure for composition of a kinematic model for spatial articulated robots and other link mechanical systems employs Denavit-Hartenberg concept (D-H concept). Other possible concept takes advance of exponential calculus. The both concepts will be briefly outlined in the following subsections. They consider features of the class of six axis articulated robots, i.e. six actuated joints specified by joint angles  $\varphi_i$  (drive space); six degrees of freedom of the robot end-effector specified by its known reference position  $r$  and orientation  $R^3$  (operational space).

### 2.1 Denavit-Hartenberg concept

The composition of the robot kinematics via D-H concept [3] uses a compact expression of a translation  $T$  ( $3 \times 1$  submatrix) and a rotation  $R$  ( $3 \times 3$  submatrix) of consecutive  $i-1$  and  $i$  links forming the robot structure

$$T_{i-1}^i = \begin{bmatrix} \cos \varphi_i & -\cos \alpha_i \sin \varphi_i & \sin \alpha_i \sin \varphi_i & a_i \cos \varphi_i \\ \sin \varphi_i & \cos \alpha_i \cos \varphi_i & -\sin \alpha_i \cos \varphi_i & a_i \sin \varphi_i \\ 0 & \sin \alpha_i & \cos \alpha_i & e_i \\ \hline 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R & T \\ \hline 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where  $e_i$  is offset along  $z$ -axis of  $i-1$  coordinate frame to the common normal of considered consecutive frames (translation  $e_i$ );  $\varphi_i$  is angle about  $z$ -axis of  $i-1$  coordinate frame oriented from  $i-1$  to  $i$   $x$ -axis (rotation  $\varphi_i$ );  $a_i$  is length of the com-

mon normal (translation  $a_i$ );  $\alpha_i$  is angle about common normal from  $i-1$  to  $i$  z-axis (rotation  $\alpha_i$ ). It represents two screw displacements illustrated in Figure 1.

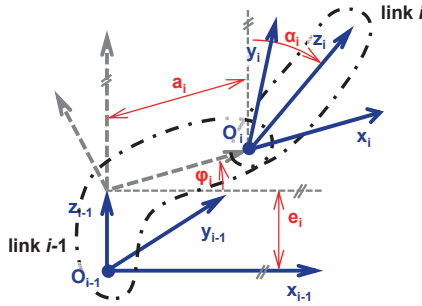


Figure 1: Four disposition parameters of two consecutive links (Denavit–Hartenberg parameters)

Then, the direct kinematics is described by a sequence of the appropriate transformation matrices (1) and their derivatives. The procedure of inverse kinematics uses the transformation matrices supplemented with analytic geometry in space. The procedure starts from known operational description, which defines position  $r$  (Cartesian coordinates) and orientation  $R^3$  (orientation coordinates) of the robot end-effector, tool or gripper fixed to the robot. By this knowledge, the joint angles  $\varphi_{i=1,2,3}$  (positioning robot subsystem = the first three joints/drives) are determined. It is given by the possibility to recompute the position of end-effector considering end-effector orientation to the center position of robot wrist, which already determines angles  $\varphi_{i=1,2,3}$  sufficiently. Then, the application of these angles in appropriate transformational matrices leads to equations for remaining angles  $\varphi_{i=4,5,6}$  determining orientation subsystem (robot wrist and end-effector) of the articulated robot.

## 2.2 Exponential concept

The exponential concept arises from the properties of exponential calculus [4]. It represents different description of screw displacements. The orientation between  $i-1$  and  $i$  coordinate frames is represented by a rotational matrix composed via exponential functions as follows

$$\widehat{R}_{i-1}^i = e^{u^i \varphi_i} e^{u^i (\alpha_i - \alpha_{i-1})} \tag{2}$$

where  $u^i$  is appropriate orthogonal unit vectors describing an orientation of  $i$  coordinate frame,  $\alpha_i - \alpha_{i-1}$  is the twist angle between  $i-1$  and  $i$  coordinate frame.

Then, the relative position of consecutive coordinate frames is defined analogically

$$P_{i-1}^i = a_i \bar{R}_{i-1}^i u_1^i + d_i \bar{R}_{i-1}^i u_3^i \quad (3)$$

where  $a$ ,  $d$ ,  $\bar{R}$  and  $u$  denote the link length, the link offset from longitudinal link axis, the  $i$ -coordinate frame orientation matrix and appropriate unit vectors of the frame. Ordered configuration of relative positions gives an appropriate solution of direct kinematics. Inverse kinematics follows solution in the analogy to the D-H concept. However, it naturally employs exponential-based transformational matrices.

### 3 Concepts for modeling of robot dynamics

The modeling of robot dynamics represents composition of pure equations of motion [7], [8]. Due to the same number of actuated joints (six drives) and degrees of freedom in 3D Cartesian space of six axis articulated robots, usual way of composition of equations of motion is based on Lagrange's equations of second type

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = \bar{u} \quad (4)$$

Their application forms the dynamic model, which can be usually written as follows

$$H(q)\ddot{q} + \left( \frac{1}{2} \dot{H}(q, \dot{q}) + S(q, \dot{q}) \right) \dot{q} + \bar{g}(q) = \bar{u} \Rightarrow \ddot{q} = f(q, \dot{q}) + g(q)\bar{u} \quad (5)$$

where individual terms  $H$ ,  $S$ ,  $g$  represent inertia matrix, its partial derivative by coordinates  $q$  and partial derivative of potential energy by  $q$  as well. In (5), the indicated form represents the simulation model (set of ordinary differential equations) as a substitution of real system i.e. real robot. However, for multistep control design, that model form can be suitable rearranged as follows

$$\ddot{q} = f_0(q, \dot{q}) + f_g(q, \dot{q}) + g(q)\bar{u} \Rightarrow \ddot{q} = f_0(q, \dot{q}) + u \quad (6)$$

Resultant model in (6) works with term  $f_0$  independent on effects of gravitation, which are added into control action  $u$ . That model can be decomposed and re-written in usual continuous-time state-space formula and discretized:

$$\dot{x} = A(x)x + B(x)u \Rightarrow x_{k+1} = A_k x_k + B_k u_k \tag{7}$$

where state vector  $x$  is defined as follows  $x = [q, \dot{q}]^T = [\varphi_{1-6}, \dot{\varphi}_{1-6}]^T$ . It means that joint (drive) coordinates  $\varphi_i$  corresponding to six axes of the articulated robot structure are taken into account as generalized coordinates  $q$  for the Lagrange's equations (4) and resulting models in the form of ordinary differential equations (5) and (6).

The continuous state-space model can substitute the real robot in the simulation or be used after discretization for discrete model-based control design. Note that the control actions obtained with the simplifying rearrangement indicated in (6) have to be recomputed into real original actions  $\bar{u}$  in (5) applied on robot drives. Real end-effector position  $y$  relating to its reference  $r$  follows from direct kinematics.

### 4 Optimal model-based control design

Considered model-based control design (Figure 2) represents approach with global control action or motion optimization. It is provided just by appropriate mathematical models of considered robots.

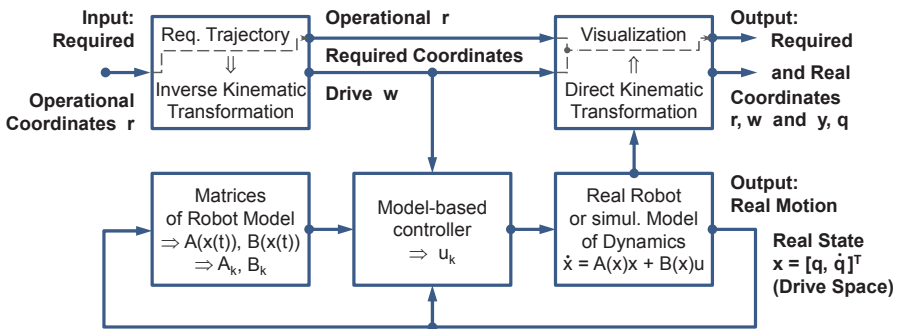


Figure 2: General scheme of model-based control

## 4.1 Model-based predictive control

The predictive control is a powerful and flexible discrete model-based approach to control design [5]. It exploits specific equations of predictions and quadratic cost function, which is optimized from control actions point of view. A general form of equations of predictions is as follows

$$\hat{q} = f + G u, \quad \hat{q} = [\hat{q}_{k+1}, \dots, \hat{q}_{k+N}]^T, \quad f = \begin{bmatrix} CA_k \\ CA_k^2 \\ \vdots \end{bmatrix}, \quad G = \begin{bmatrix} CB_k & \dots & 0 \\ CA_k B_k & CB_k & \vdots \\ \vdots & \vdots & \ddots \end{bmatrix} \quad (8)$$

where state-space matrices  $A_k$  and  $B_k$  are considered to be constant within each individual prediction horizon  $N$ . The predictions (8) involve the model of the robot dynamics (6) in its discrete form (7). In the case of articulated robots, the eq. (8) expresses functional predictions of the joint angles  $\varphi_i$ ,  $i = 1, 2, \dots, 6$  (i.e. drive coordinates  $q$ ) relative to unknown control actions (torques) on individual actuated joints (drives). Subsequently, the appropriate control actions  $u_k$ ,  $k = 1, 2, \dots, N$  are computed by minimization of the cost function (9), in which  $Q_{yw}$  and  $Q_u$  mean weighting parameters and  $w$  is a reference vector of drive coordinates  $q = [\varphi_{1-6}]^T$ .

$$J_k = \sum_{j=1}^N \{ (q_{k+j} - w_{k+j})^T Q_{yw}^T Q_{yw} (q_{k+j} - w_{k+j}) + u_{k+j-1}^T Q_u^T Q_u u_{k+j-1} \} \quad (9)$$

The minimization of the quadratic cost function leads to optimal (energetically efficient) control actions. It represents economic solution in the view of energy consumption in comparison to conventional approaches such as decentralized PID/PSD control structures. Furthermore, the multistep character of the predictive control design given by the prediction horizon  $N$  provides safe smooth transitions among different motion segments e.g. abscissa and arc segments or two abscissa segments with sharp connection angle. The smoothness of the transitions can be tuned via weighting parameters  $Q_0$ .

Besides real-time control action computation, the indicated procedure can be applied to off-line robot motion optimization from motion dynamics point of view and as well as a specific admissible torque-range generator for on-line safety monitoring procedures.



## **5 *Towards efficient sustainable & safe production***

The outlined concepts of modeling and model-based control have a wide range of application. From efficient sustainable production point of view, they serve for optimal motion planning and optimal input energy distribution leading to wear reduction of individual robot drives and whole robot construction in general [1]. They can be applied also in the situations, when the robot or machine tool has to work in very fast production cycles under full load.

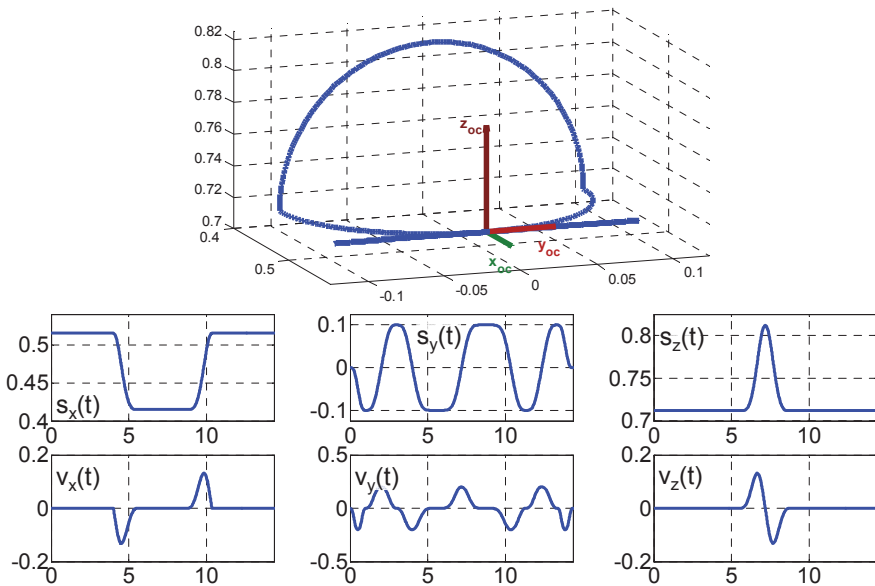
The model-based predictive control design due to its multi-step optimization can generate only really necessary magnitudes of torques of drives even near to their physical limits. Further substantial application of the presented concepts consists in a motion monitoring for safe and secure robot operations. It covers monitoring of kinematic quantities – tracking quality of the motion trajectory, avoiding obstacles; and monitoring of dynamic quantities – comparison of designed (ideal) control actions with really realized actions by individual drives. In this application, each noticeable deviation from planned reference motion or designed control actions indicates undesired robot states or behavior, which can lead to serious damages of the robot, products or industrial injuries.

## **6 *Illustrative example***

Illustrative example demonstrates the behavior of the articulated robot ABB IRB 140 (Figure 4) at the motion along one selected 3D testing trajectory (Figure 3). There are considered three different loads here. The loads were consecutively fixed on the wrist flange representing robot end-effector. The number of actuated (driven) axes of the robot is six as well as a number of its degrees of freedom. However, in the example, only the first three axes are considered respecting mass distribution of the robot structure. The testing trajectory consists of arc and abscissa segments as it is documented by the list of plain NC Code of the trajectory [6] in Table 1.

**Table 1:** Plain NC Code of 3D testing trajectory

001:	N010	G19						
002:	N020	G00	X515	Y-100	Z712			
003:	N030	G00	X515	Y100	Z712			
004:	N040	G00	X515	Y0	Z712			
005:	N050	G02	X415	Y-100	Z712	I-100	J0	K0
006:	N060	G02	X415	Y100	Z712	I0	J100	K0
007:	N070	G02	X515	Y0	Z712	I0	J-100	K0
008:	N080	G00	X515	Y-100	Z712			
009:	N090	G00	X515	Y100	Z712			
010:	N100	G00	X515	Y0	Z712			

**Figure 3:** 3D testing end-effector trajectory with appropriate kinematic quantities (xyz components of positions  $s(t)$  [m] and velocities  $v(t)$  [ $\text{ms}^{-1}$ ] designed via acceleration polynomial of 5<sup>th</sup> order [6])

In the example, relaxed, unloaded and loaded wrist flange is considered; specifically: relaxed flange with weight  $m_f = m_r - 5\text{kg}$ ; unloaded flange  $m_f = m_r$  (zero load); and loaded flange  $m_f = m_r + 5\text{kg}$ . The value 5Kg corresponds to rated maximum load of the given robot. In the Figure 5 on the top, there are time histories showing comparison of appropriate control actions for positioning robot subsystem just for mentioned different loading.

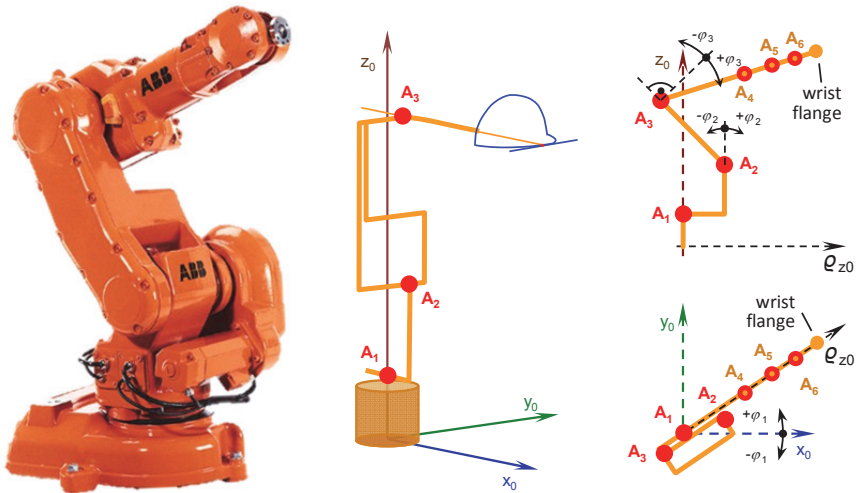


Figure 4: ABB robot IRB 140; 3D wireframe model with testing trajectory; angles' definition of positioning robot subsystem: front view: angles  $\varphi_2, \varphi_3$ ; top plan view: angle  $\varphi_1$

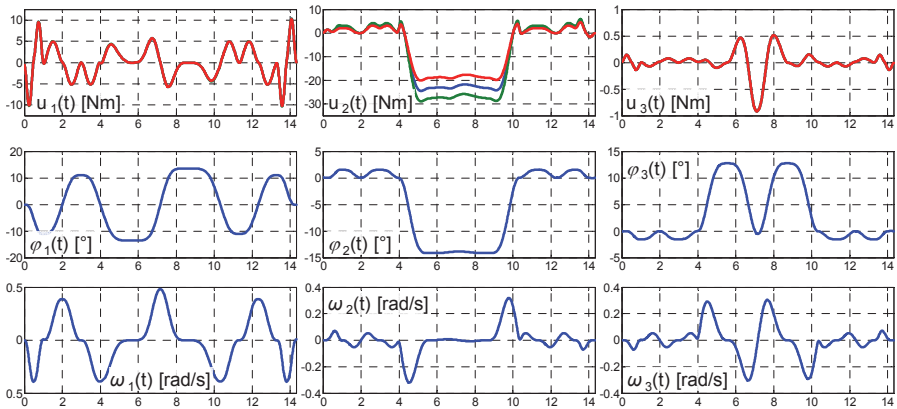


Figure 5: Top: time behavior of first three joint angles (positioning robot subsystem) for three different loads; middle: appropriate joint angles  $\varphi_1, \varphi_2, \varphi_3$ ; bottom: joint angular speeds  $\omega_1, \omega_2, \omega_3$

The biggest differences in Figure 5 are obvious at the robot axis 2 (joint angle  $\varphi_2$ , control action  $u_2$ ). It is caused by character of the trajectory, in which the robot arm leans back and the drive of axis 2 has to counterbalance such motion. The showed control actions were computed by model-based predictive control introduced in 4.1.

The rest time histories in the middle and at the bottom of the Figure 5 represent behavior of the first three joint angles. They correspond to commented control actions. In regard to an efficient sustainable and safe production, the selected example demonstrates computation of specific admissible ranges of control actions.

## 7 Conclusion

The contribution introduces advanced modeling and control concepts for industrial robots as promising way for efficient sustainable and safe production. The outlined concepts for direct and inverse kinematics; dynamics with separated gravitation effects and predictive control were realized for the industrial robot ABB IRB 140.

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