Probabilistic reasoning in service of condition monitoring

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Abstract

Nowadays practically all control systems include at least some elements of maintenance support and condition monitoring. Putting them together to create a consistent and transparent system is far from being a commonplace. Our contribution presents a common effort of an international consortium, developing a novel probabilistic condition monitoring framework. The research tasks can be divided into two groups: (i) probabilistic evaluation of health of single components of the control system, such as measured signals, actuators, hardware units, and others, and (ii) hierarchical propagation and compounding of the related uncertain health information into a clear continuous notice to operators or maintenance personnel. The paper depicts possibilities of utilization of the so called subjective logic to accomplish the second task. The main objective of the methodology is to provide simplified outputs based on complex artificial reasoning. Repetitive utilization of subjective logic operators aiming to respect the pyramidal structure of the system is illustrated by simple examples. Practical aspects of the used probabilistic calculus heading to its industrial utilization are discussed.

1. Introduction

Nowadays designing industrial control systems, one can hardly omit inclusion of at least some features concerning condition monitoring and failure detection. There exist several approaches to these topics each making its rapid progress. Detection of a failure or more generally, evaluation of unit's health is the first instance of the condition monitoring process. Another question is how to treat this information, how to summarize information coming from number of units and how to report the result to process operators or maintenance personnel. Yet other factors to be taken into account are uncertainty connected to every piece of available information and different impact of condition of particular components on the health of the entire system.

Probabilistic approach, namely such focused on the use of bounded probability distributions(1), may be helpful in seeking for solutions of the above mentioned issues. This contribution concerns propagation of information within the condition monitoring system while engaging the calculus of subjective logic, the methodology belonging to the family of probabilistic logic. Main strengths of the subjective logic[2] lie in its capability of systematic treatment of information uncertainty, in existence of number of logical operators enabling to respect various dependencies within the inspected system and in its direct linkage to the beta probability distribution for which it is ideal for probabilistic modelling of presence or absence of failures.

The paper is organized as follows: the next chapter shortly depicts the system to be built while the subsequent chapter introduces the subjective logic in brief. Graphically illustrated examples of utilization of the calculus form central part of the contribution followed by conclusions and references.

2. System to be built

Hierarchical or pyramidal structure of the system to be built is diagrammatically depicted in **Figure 1**. Probabilistic information about health of particular elements of the inspected system obtained from some kind of continuous Bayesian analysis (not discussed here) enter to corresponding processing blocks on the lowermost level. The term element may cover sensors or compact measurement units, actuators which provide information about their status, hardware nodes, means of computer communication and even software running in particular nodes of the control system.

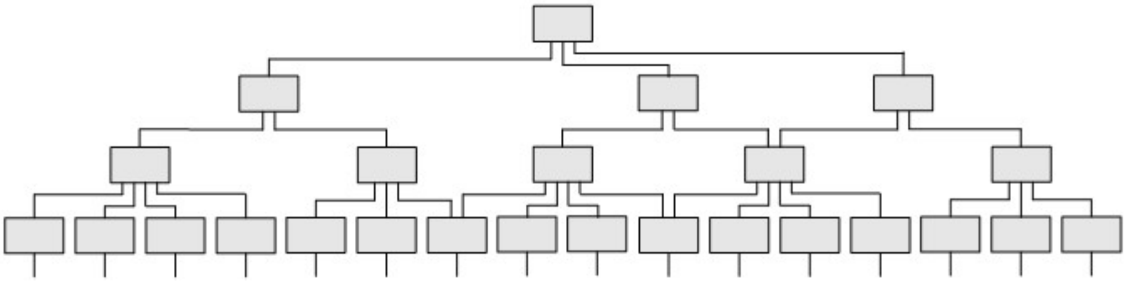

Figure 1. Schematic diagram of a pyramidal condition monitoring system

Connections of blocks of the monitoring system reflect inner relations within the inspected control system including grouping of units into subsystems. Control of a hydraulic servomechanism can be taken as an example of a subsystem: for its proper function reliable measurements of hydraulic pressures and positions are necessary, while feedback from the hydraulic actuator, i.e. measured value of its slide valve position, if available, may bring useful information. Appropriate composition of information about condition, i.e. *health* of mentioned measurements results in evaluation of health of the hydraulic subsystem. Possible redundancy of sensors and other non-trivial relations should be taken into account as well.

3. Subjective logic

Subjective logic is a type of probabilistic logic that explicitly takes uncertainty and belief ownership into account. Arguments in subjective logic are subjective opinions about propositions[3]. A binomial opinion applies to a single proposition, and can be represented by a beta distribution. Properties of the subjective logic are in a simplified way interpreted in the following.

3.1 Binomial opinions

Let the health of a system unit (an element, subsystem or the whole system) be defined as a value in interval [0,1] and let it be represented by a binomial random variable. For this purpose and with the intent to include the uncertainty into the reasoning, one can exploit the binomial opinions about the proposition *x* as a quadruple

$$\omega_x = (b, d, u, a), \tag{1}$$

where *b* is the belief that the specified proposition is true, *d* is disbelief, i.e. the belief that the specified proposition is false, *u* is uncertainty and *a* is the base rate, corresponding to prior probability.

For the sake of consistency, it must hold

$$b + d + u = 1, \qquad b, d, u, a \in [0,1]. \tag{2}$$

There exist an interesting graphical illustration of mutual relations among quadruple's components with the aid of *binomial opinion triangle*[4]. It is necessary to introduce here just the probability expectation value of an opinion which is defined as

$$\mathrm{E}_x = b + au. \tag{3}$$

Important asset of the approach consists in the possibility to represent the opinion by a beta distribution with two nonnegative parameters α, β. The beta distribution of a random variable *x* can be characterized by the probability density function (pdf)

$$f(x; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1} \tag{4}$$

where $B(\alpha, \beta)$ is output of the beta function used as a normalization constant to ensure that the total probability integrates to 1. The expected value of *x* can be obtained from the relation

$$\mathrm{E}[x] = \frac{\alpha}{\alpha + \beta}. \tag{5}$$

There exist a bijective mapping[4] between opinion ω and corresponding beta pdf for non-zero uncertainty *u*:

$$\alpha = 2\left(\frac{b}{u} + a\right), \qquad \beta = 2\left[\frac{d}{u} + (1-a)\right].$$

For *u = 0*, the function degenerates to the Dirac pdf concentrated at a point between 0 and 1 given by the belief *b*.

3.2 Subjective logic operators

Nowadays calculus of the subjective logic contains comprehensive set of 15 operators. Most of the operators represent generalisations of binary logic and probability operators, e.g. addition ≈ union, subtraction ≈ complement, multiplication ≈ conjunction (AND), comultiplication ≈ disjunction (OR), complement ≈ negation (NOT), etc. Moreover, there exist unique operators such as cumulative fusion/unfusion, belief constraining, etc. Existence of the operators and the above mentioned mapping enable indirect logical operations with beta distributions.

3.2.1 Logical multiplication and other practical operators

Logical multiplication – a counterpart to the Boolean operator of conjunction (AND) – is the candidate for the most frequent operation used for the purpose of the condition monitoring. It is defined by following equations[(5)]:

$$b_{x\wedge y} = b_x b_y + \frac{(1-a_x)a_y b_x u_y + (1-a_y)a_x b_y u_x}{1-a_x a_y} \tag{6}$$

$$d_{x\wedge y} = d_x + d_y - d_x d_y \tag{7}$$

$$u_{x\wedge y} = u_x u_y + \frac{(1-a_y)b_x u_y + (1-a_x)b_y u_x}{1-a_x a_y} \tag{8}$$

$$a_{x\wedge y} = a_x a_y . \tag{9}$$

Among other operators which are intended to be used within the health evaluating system belong logical comultiplication, logical deduction and others the definition of which can be found in[(4)].

4. Practical use of the calculus

4.1 Configuration of examples

Figure 2. shows elementary configuration of system blocks which will be used hereafter to illustrate the use of the most frequently used subjective logic operators. ω_x and ω_y stand here for input binomial opinions about healths of subordinate blocks while $\omega_{x\sim y}$ represents the output opinion resulting from applying an operator symbolized by ~.

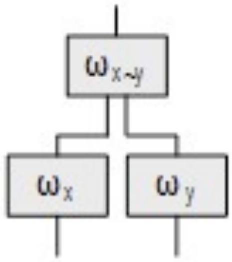

Figure 2. Depiction of elementary relations within the hierarchical condition monitoring system. Symbol ~ stands for the operation accomplished between input opinions ω_x and ω_y

In the following, the use of selected operators is demonstrated on four pairs of binomial opinions which were adjusted to cover at least some of possible typical situations. Thanks to the bijective mapping between opinions and beta distributions, the examples are illustrated by the means of pdfs. The set of manually adjusted inputs which come from blocks on the lower level, consists of four pairs of opinions:

1. $\omega_x = (0.65, 0.0, 0.35, 0.1)$, $\omega_y = (0.6, 0.0, 0.4, 0.5)$ – no disbelief in both inputs
2. $\omega_x = (0.3, 0.2, 0.5, 0.1)$, $\omega_y = (0.2, 0.3, 0.5, 0.5)$ – high amount of uncertainty
3. $\omega_x = (0.3, 0.6, 0.1, 0.5)$, $\omega_y = (0.29, 0.61, 0.1, 0.9)$ – almost no uncertainty
4. $\omega_x = (0.8, 0.2, 0.0, 0.5)$, $\omega_y = (0.6, 0.4, 0.0, 0.5)$ – no uncertainty

4.2 Example of the use of logical multiplication

Logical multiplication which corresponds to the Boolean conjunction (AND) should be used in situation when the health of the block depends on simultaneous operation of the subordinate blocks. Results of testing set of opinions are shown in **Figure 3** in the form of beta pdfs. Subplots in the Figure should be read row wise.

case 1: Expected value after the multiplication of opinions with zero input disbeliefs lies approximately in the middle of the interval [0,1] in spite of relative high input beliefs, which is partially caused by the low prior probability of input *x*: $E_{x\wedge y}(1) = 0.548$.

case 2: Resulting pdf converges to 0 because of irresoluteness between belief and disbelief for both inputs, relative high uncertainty and low prior probability of input *x*: $E_{x\wedge y}(2) = 0.1575$.

case 3: Almost certain inputs follow prevailing disbeliefs which are contradicted by high prior probability of input *y*: $E_{x\wedge y}(3) = 0.133$.

case 4: This case shows how pdfs of both inputs and output degenerate to Dirac distributions due to absence of uncertainty. The result corresponds to natural expectation that mean value (expectation) of the output should be lower than those of both inputs: $E_{x\wedge y}(4) = 0.48$.

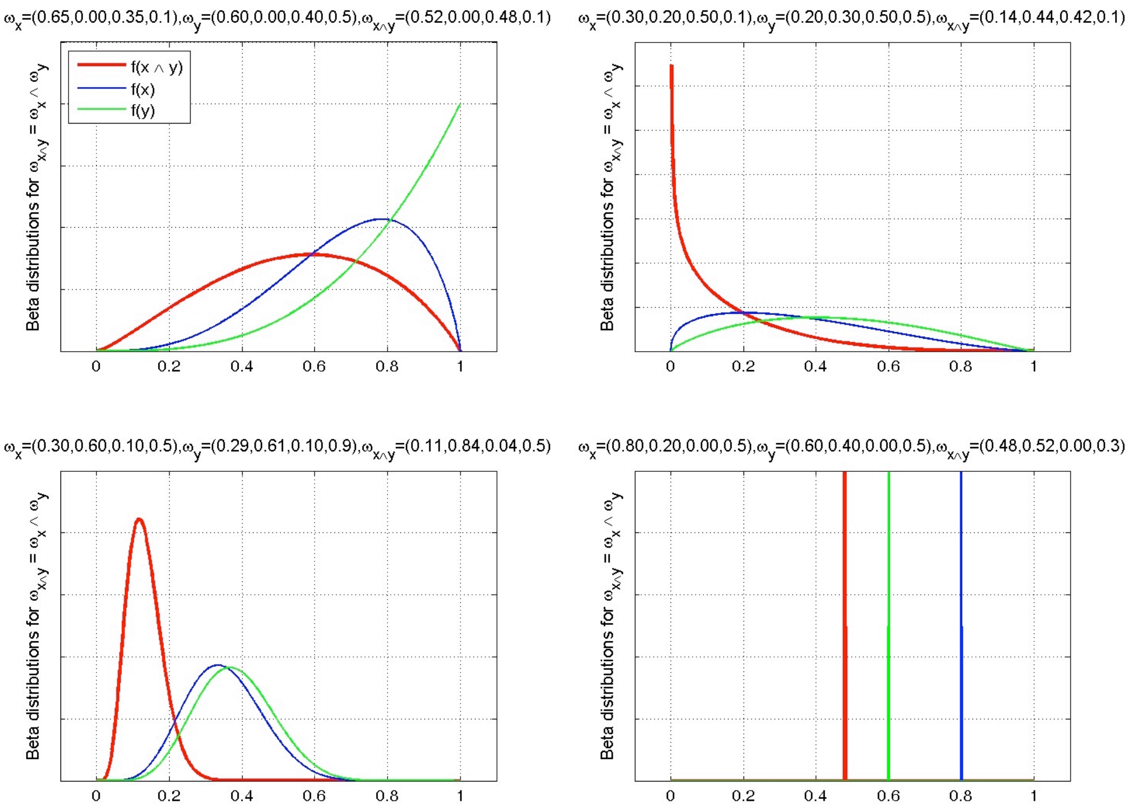

Figure 3. Multiplication of opinions: examples of beta distributions of original opinions ω_x, ω_y and resulting $\omega_{x\wedge y}$

4.3 Example of the use of logical comultiplication

Logical comultiplication which corresponds to the Boolean disjunction (OR) stands for the situation when subordinate blocks are both interchangeable and independent, i.e. proper operation is ensured if the first or the second block work well. Corresponding beta pdfs for four tested cases can be seen in **Figure 4**.

case 1: Prevailing beliefs of both inputs pull the output pdf to 1 overtrumping the low *a* of the first input. Expected value $E_{x\vee y}(1) = 0.944$ labels the output as almost healthy.

case 2: Unconvincing inputs and low prior expectation of the first input allow only moderate shift of the output towards 1: $E_{x\vee y}(2) = 0.661$.

case 3: Comultiplication of almost equal inputs differing only in the high prior probability of the second one correspond to the expectancy of moderately increased health of the output: $E_{x\vee y}(3) = 0.602$ with no uncertainty.

case 4: Again, zero uncertainties mean Diracs instead of beta functions. The result is not surprising – the Dirac pdf located at $E_{x\vee y}(4) = 0.92$.

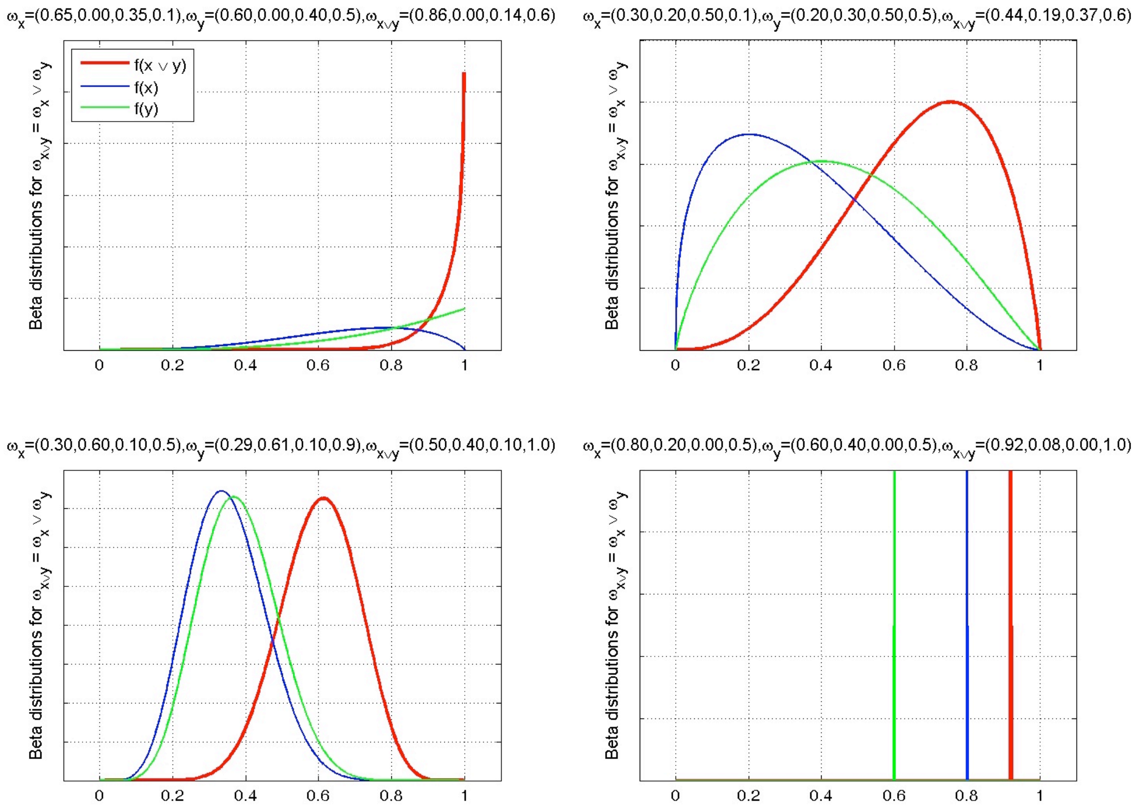

Figure 4. Comultiplication of opinions: examples of beta distributions of original opinions ω_x, ω_y and resulting $\omega_{x\vee y}$

4.4 Example of the use of logical addition

Logical addition which corresponds to the Boolean union differs from the preceding case - now the healths of both subordinate blocks are added thus imperfection of one block can be saved by proper condition of the second one. Corresponding beta pdfs for four tested cases can be seen in **Figure 5**.

case 1: There is no surprise that addition pushes the result to one without uncertainty: $E_{x\cup y}(1) = 1.0$.

case 2: The same effect as in the first case – shape of the pdfs changed in favour of 1: $E_{x\cup y}(2) = 0.80$.

case 3: Result of this addition is interesting in comparison to corresponding comultiplication – expected value is almost the same but uncertainty disappeared: $E_{x\cup y}(3) = 0.566$.

case 4: Expectable result – result is certain and equal to 1: $E_{x\cup y}(4) = 1.0$.

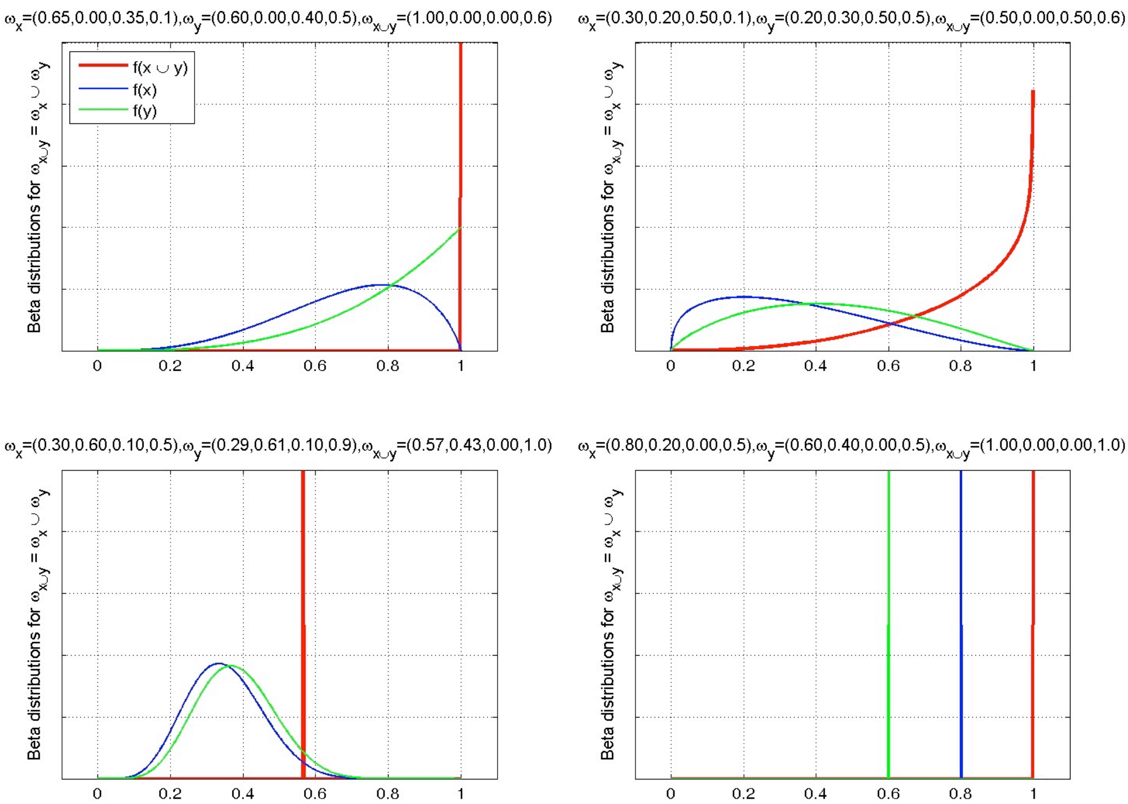

Figure 5. Addition of opinions: examples of beta distributions of original opinions ω_x, ω_y and resulting $\omega_{x \cup y}$

4.5 Other logical operations

Coverage of subjective logic operators supports even more complicated relations within the system such as cooperation of a smart sensor with the raw measurement unit which may lead to the use of logical deduction or backward evaluation of health of an individual from the pair of sensors (logical division and co-division), etc.

4.6 Propagation of information

As already mentioned, information about health of system elements is composed according to system's inner relations to represent logical subsystems and further upwards up to the uppermost block which provides information about the health of the entire system. Drop of health of a single element can influence related blocks up to the top as demonstrated in **Figure 6**.

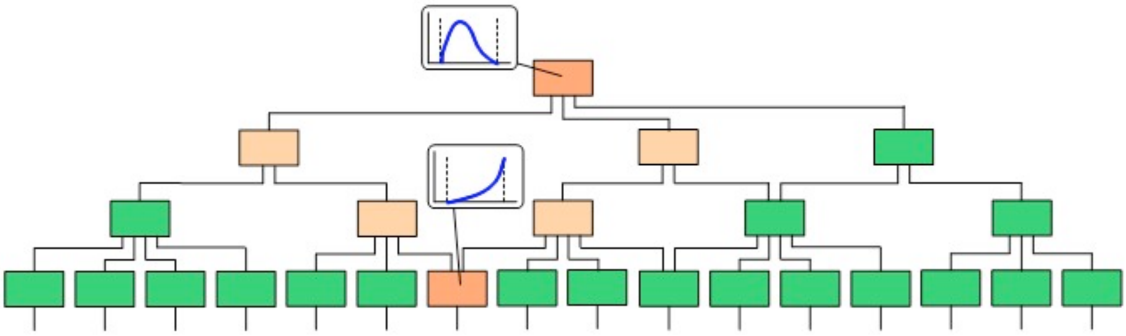

Figure 6. Upward propagation of information about impaired health of a system component. Detailed information is available for each block of the system also in the form of the beta distribution

Remaining task consists in transformation of the opinion on health of the entire system into clear report to the operators, e.g. in the form of traffic lights or clear textual announcement. Then, inspection of the system downwards the pyramid leads to the cause of the problem.

5. Conclusions

The paper concerns application of the calculus of probabilistic and more specifically subjective logic for the purpose of treatment and contexture of uncertain information about health of components and subsystems of an industrial control system. The approach is being elaborated and tested within an international cooperation focussed on research of the probabilistic condition monitoring. One of the interesting features of the subjective logic consists in its close relation to beta distribution which may be utilized for visualisation of the uncertainty amount. Main goal of the project consists in experimental application of the developed system in a metallurgical plant.

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