Quantification of Information Uncertainty for the Purpose of Condition Monitoring

Pavel Ettler[1] and Kamil Dedecius[2]
[1]*COMPUREG Plzeň, s.r.o., Nádražní 18, 306 34 Plzeň, Czech Republic*
[2]*Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, Pod Vodárenskou věží 4, 182 08 Prague, Czech Republic*
ettler@compureg.cz, dedecius@utia.cas.cz

Keywords: Condition Monitoring, Fault Detection and Diagnosis, Probabilistic Logic, Subjective Logic, Mathematical Modelling.

Abstract: Pervasive uncertainty of information which affects to some extent functionality of every control and information system concerns naturally the condition monitoring systems as well. Uncertainty can practically be disregarded when monitoring a single component, but it should be taken into account when compounding extensive amount of information within a hierarchical diagnostic system. When using uncertain information for expression of inner system's relations, probabilistic and namely subjective logic may do a good turn. However, the key problem remains how to quantify the uncertainty on the lowermost level of the monitoring system. The paper introduces several solutions based on inspection of either a single measured signal or a couple of correlated signals.

1 INTRODUCTION

Condition monitoring has become a rapidly developing area utilizing various approaches and methods. While at least some elements of diagnostics are involved in almost every nowadays control system, methodologies for systematic contexture of information about condition of particular system components into a comprehensive form are still evolving.

This paper aims to contribute to hierarchical assessment of system conditions (in terms of reliability), focusing on information uncertainty which may propagate among different system parts. For example, uncertainty included in a single spurious measurement may eventually lead to inappropriate decisions of the control system, yet the signal is not necessarily completely wrong.

Most state of the art solutions taking the hierarchical character of the monitored system into account are based on deterministic methods, often with heuristic decision rules. Classical field of hierarchical system monitoring are computer and telecommunication networks. Here, the existing approaches allow scalable monitoring of the network and its devices, from physical state variables (e.g. CPU temperature, fan speeds, network load) up to the state of provided services. Examples of such monitoring (and management) systems are the celebrated open-source project Nagios (Kocjan, 2008) or commercial products like IBM Tivoli Network Manager or Nexus' NexusMETER. Although their adoption to industrial systems is nontrivial if possible, they are indeed a valuable source of inspiration.

The initial results of the international consortium focusing on the design of a hierarchical monitoring system are presented. The underlying philosophy is to avoid as much heuristics from the system as possible. In this respect, the application of probabilistic and particularly subjective logic (Jøsang, 2013) for propagation of information about components condition is a promising alternative to existing methods. One of the key problems connected with this approach consists in generating inputs for the condition monitoring system based on subjective logic principles.

The paper introduces several ideas for its solution and is organized as follows: The next section sketches the system to be built, defines basic entities, formulates the problem and introduces the calculus of subjective logic. Subsequent section offers several means how to quantify the uncertainty and is followed by a section of examples and conclusions.

2 CONDITION MONITORING OF A HIERARCHICAL SYSTEM

Ideas of the paper concern a hierarchical condition monitoring system the example of which is depicted in Figure 1.

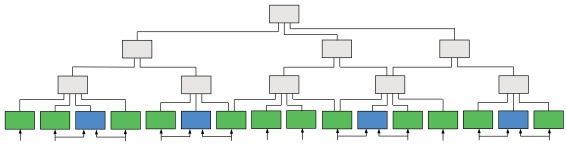

Figure 1: Example of a hierarchical condition monitoring system.

The green lowermost blocks monitor single components of the control system such as measured signals, sensors, actuators which provide feedback about their status and other hardware and possibly also software units. The blue blocks guard proper relations between pairs of correlated signals. Information about health of system components is then propagated upwards the pyramidal structure allowing to evaluate condition or health of logical subsystems and – on the uppermost level – of the entire system. The natural question, answered below, is how to combine the information about the condition of these units together.

2.1 Compounding of Information

A particular lowermost block of the system in Figure 1 provides information about the condition of the ith monitored element; say a measured signal, which can be called signal health h_i. In the simplest case, h_i can be assumed a binary variable, which can take just two values *true*|*false* or 0|1 where $h_i = 1$ represents perfect condition of the signal and $h_i = 0$ means its failure. Then, the health $h_{h_1 \wedge h_2}$ of a subsystem which relies on simultaneous operation of two signals with healths h_1 and h_2 can be expressed using the logical conjunction (AND) operator as

$$h_{h_1 \wedge h_2} = h_1 \wedge h_2 \ ,$$

where $h_{h_1 \wedge h_2}$ is evaluated according to respective truth table. Existing binary logic operators allow to respect various relations within the system. For instance, the disjunction (OR) operator can reflect redundancy of sensors; the *modus ponendo ponens* rule (MP operator) can reflect inner relations of a smart sensor, etc. However, considering health as the binary variable makes the system rather coarse from two points of view:

– It may not be obvious how to rate health of a component just 0 or 1;

– Malfunction of one component may result in evaluation of status of the whole system as "in failure" regardless the component's importance and reliability of basal information.

Employment of the probabilistic logic brings the possibility to represent health as a probability, i.e. a number $p(h) \in [0,1]$. Then, the above mentioned example of health of two simultaneously working sensors will read

$$p(h_1 \wedge h_2) = p(h_1)p(h_2) \ .$$

A serious limitation of probabilistic logic, and binary logic alike, is expressed in (Jøsang, 2013): It is impossible to express input arguments with degrees of ignorance as e.g. reflected by the expression "I don't know". It led the authors to the search for probabilistic distributions with limited support which can be utilized for expression of that uncertainty (Dedecius and Ettler, 2013). The winner - the beta distribution - drove to the engagement of the *subjective logic*.

2.2 Subjective Logic

Subjective logic is a comprehensive methodology for logic operations with uncertain propositions described, e.g. in (Jøsang, 2013). Essentially, the theory is based on definition of a probabilistic opinion about a proposition h in the form of a quadruplet

$$\omega_h = (b, d, u, a) \ , \tag{1}$$

where the components b, d, u, a are belief (amount of h-supporting information), disbelief (the opposite), uncertainty (amount of information insufficiency) and base rate (prior information) respectively. It must hold

$$b + d + u = 1 \ , \qquad b, d, u, a \in [0,1] \tag{2}$$

and the expected value can be expressed as

$$E_h = b + au \ . \tag{3}$$

There exists a bijective mapping between an opinion ω and the corresponding beta probability density function for non-zero uncertainty u. For $u = 0$, the function degenerates to the Dirac pdf concentrated at a point between 0 and 1 given by the belief b.

Using the terms of the subjective logic, the above mentioned example of health of two simultaneously working sensors can be expressed as

$$\omega_{h_1 \wedge h_2} = \omega_{h_1} \cdot \omega_{h_2} \ ,$$

where operator of multiplication of opinions is defined as the set of four equations for b, d, u and a (Jøsang and McAnally, 2005). There exists a full set of operators as counterparts to the binary logic and probabilistic logic operators including deduction, abduction, etc. Moreover, additional operators can be used for various types of fusion and unfusion and for the belief constraining.

The base rate a represent the prior amount of belief and can be constructed from historical data or based on experience of the user. A problem may arise when evaluating the uncertainty u, as there do not exist plain rules for its determination. Several possibilities how to quantify the uncertainty are offered in the following text.

3 QUANTIFICATION OF UNCERTAINTY

The potential of the subjective logic for the purpose of condition monitoring is inspected in (Ettler and Dedecius, 2014). The cited paper concerns propagation of information in the grey part of the system in Figure 1. The following sections present several possibilities for quantification of uncertainty on the lowermost level of the monitoring system represented by green and blue blocks. Derived methods can be considered independently or can be combined inside a single block using various operators of the subjective logic calculus.

3.1 Respecting Signal Distribution

The basic possibility for quantification of u consists in examination of the distribution of the signal $x \in \mathcal{R}$ with respect to specified a priori specified boundaries. Let $X(t)$ be a moving window in time t

$$X(t) = [x(t), x(t-1), \ldots, x(t-m)] \ ,$$

where m is the window length and let suppose that data in the window are approximately normally distributed (time indices are omitted in the following for the sake of simplicity)

$$X \sim \mathcal{N}(\mu_x, \sigma_x^2) \ ,$$

where μ_x, σ_x^2 are mean and variance of window data respectively. Then for x^+ being the positive limit for x and using the three-sigma rule, the uncertainty u can be expressed as

$$u = \begin{cases} \frac{(\mu_x+3\sigma)-x^+}{3\sigma} & \text{if } \mu_x \in [x^+, x^+ + 3\sigma_x) \\ \frac{x^+-(\mu_x-3\sigma)}{3\sigma} & \text{if } \mu_x \in [x^+ - 3\sigma_x, x^+) \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

Figure 2 illustrates the situation belonging to the second row of (4): the left plot depicts the location of the data distribution with respect to the positive boundary while the right plot shows the beta distribution corresponding to the opinion about health of the signal x as for the signal range. Taking (2) into account, $b = 1 - u$ and $d = 0$ for this case. The expected value E_h was evaluated according to (3) for a "neutral" base rate $a = 0.5$. The values of opinion components (1), the corresponding beta distribution and the expected value can be read in the right plot. Relations (4) can be easily adapted for the case of the negative boundary.

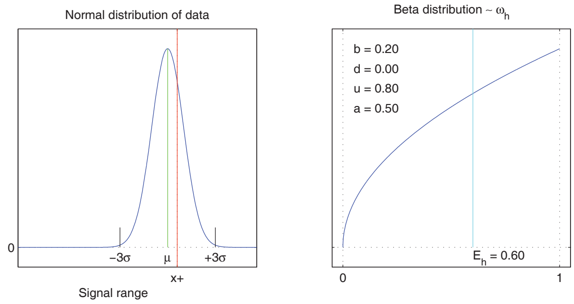

Figure 2: Evaluation of uncertainty in the proximity of signal positive limit.

Obviously, there exist other possible approximations of the data distribution – for example (Pavelková and Jirsa, 2014) considers uniform distribution and two-level boundaries while another approach (Jirsa and Pavelková, 2014) might employ Gaussian mixtures (Kárný et al., 2003) and more sophisticated distance measures.

3.2 Cautious Approach and Forgetting

There exist a class of signal failures which occur for a very short time interval though often repeatedly. It may be relatively straightforward to evaluate health h and thus b and d in the moment of failure occurrence but when it is over, the signal may seem to be perfect again. In such a case caution is advisable which can be expressed by increasing of the uncertainty in the moment of recovery from the failure.

Let $b(T) < 1$, $d(T) = 1 - b(T)$, $u(T) = 0$ be belief, disbelief and uncertainty at time T in which a failure is detected. Then, applying (3), it follows $E_h(T) = b(T)$. In time $T+1$ the failure is totally recovered and the signal might be evaluated as healthy. However, the disbelief $d(T)$ should be "transformed" into uncertainty so that the adjacent expected values do not differ

$$E_h(T+1) \approx E_h(T) \ .$$

Then, while respecting (2), considering time-invariant base rate a and substituting $b(T+1) = 1 - u(T+1)$, the uncertainty immediately after the failure can be expressed as

$$u(T+1) = \max\left[0, \min\left(1, \frac{b(T)-1}{a-1}\right)\right]$$

To allow recovery of the health in finite time, some kind of forgetting should be applied, either linear

$$u(t) = \max(0, u(t-1) + \Delta_u),\ \Delta_u \in (-1, 0)\ ,$$

where Δ_u is an empirically chosen decrement, or exponential

$$u(t) = \lambda u(t-1),\ \lambda \in (0,1)$$

where λ is a forgetting factor. The latter option was used for the treatment of outliers as depicted in Figure 4 in the section of examples.

3.3 Useful Results from Parameter Estimation

Detection of unwanted deviations between two correlated signals can be based on modelling of relation between the signals and the recursive estimation of model parameters. Then, the properties and deviations of parameter estimates or other results of model identification can be transformed into opinion about the health of the signals.

Let the relation of two signals $x_1(t)$ and $x_2(t)$ can be approximately expressed by a linear model

$$x_2(t) = P'(t)d(t) + e(t)\ ,$$

where $'$ denotes transposition and $d(t)$ is the data vector of the form

$$d(t) = [x_2(t-1), \ldots, x_2(t-n_2), x_1(t), \ldots, x_1(t-n_1), 1]$$

with $n_1 \geq 0$, $n_2 \geq 0$, P is the vector of unknown parameters and model noise is represented by the white noise $e(t) \sim \mathcal{N}(0, r)$ with unknown variance r.

The estimation in the RLS (Recursive Least Squares) manner is based on the symmetric square positive information (inverse covariance) matrix $V(t)$ of dimension $1 + \dim d$. Its recursive update reads (Peterka, 1981)

$$V(t) = \lambda V(t-1) + [x_2(t), d(t)]'[x_2(t), d(t)]\ ,$$

where $\lambda \in (0,1)$ (close to 1) is the forgetting factor allowing to track slowly varying parameters. Splitting the information matrix into blocks (time index is omitted in the following for the sake of simplicity)

$$V = \begin{bmatrix} V_{x_2} & v' \\ v & \mathcal{V}\ , \end{bmatrix}, \qquad V_{x_2} \in \mathbb{R}_+$$

reveals the least-squares estimator of parameters,

$$\hat{P} = \mathcal{V}^{-1} v\ ,$$

where $\hat{P}$ stands for the parameter estimate.

For the real-time application, propagation of the information matrix should be realized in the form of matrix factorization of some kind, e.g. $V^{-1} = LDL'$ where L, D are lower triangular and diagonal matrices respectively. This suppresses numerical difficulties associated with potentially ill-conditioned matrices.

Unexpected changes of parameter estimates can indicate oncoming failure of the signal. Parameter estimates variance should then be transformed into uncertainty u as illustrated in Figure 5 in Section 4.3.

Another possibility for quantification of uncertainty consists in the use of associated estimation results such as the quality of the model expressed as the estimate of variance r

$$\hat{r} = \frac{V_{x_2} - v'\hat{P}}{\kappa_t}\ ,$$

where κ_t represents number of data samples in time t, or the estimated parameter variance

$$\hat{\sigma}^2_{p_i} = (\mathcal{V}^{-1})_{ii}$$

where $(\mathcal{V}^{-1})_{ii}$ denotes the element of $\mathcal{V}^{-1}$ on position (i,i).

A problem may occur with the unknown scale of the above mentioned measures. Some form of the logistic or another sigmoid function can be used to transform the measure into the requested interval $u \in [0,1]$.

4 EXAMPLES

4.1 Situation Near the Signal Boundary

The simulated example depicted in Figure 3 illustrates the method from Section 3.1.

A noisy signal crosses its positive boundary $x^+ = 10$ in the left plot. Signal was constructed as the positive trend

$$x(t) = 0.001 \cdot t + 5 + e(t)$$

where $e(t) \sim \mathcal{N}(0,1)$. The moving average regarded as the mean $\mu_x(t)$ of the signal was being evaluated from the moving window of length $m = 100$ together with the standard deviation $\sigma_x(t)$. The right plot shows three components of the opinion $\omega_h(t)$ about signal health and its expected value $E_h(t)$. *Neutral* base rate $a = 0.5$ was considered as time-invariant. It can be seen that the highest point of u coincides with the moment in which the moving average crosses the boundary x^+.

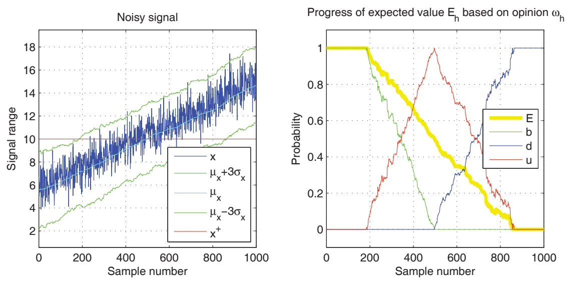

Figure 3: Progress of ω_h and E_h for the simulated noisy signal crossing its positive limit.

4.2 Influence of Outliers

Figure 4 illustrates the situation when outliers were detected in the real measured signal. The example comes from the metal-processing industry: the signal in the left plot represents thickness deviation of the metal strip entering a rolling mill.

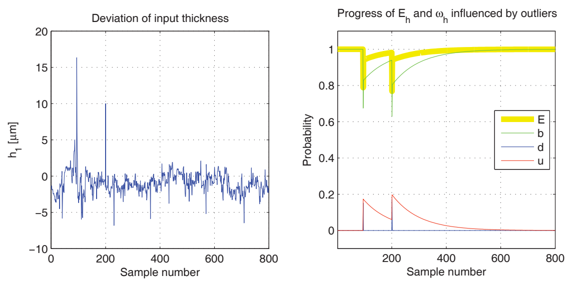

Figure 4: The real signal with outliers and progress of related ω_h and E_h.

Two positive outliers which were probably caused by a dirt on the strip surface influenced expected value E_h of the signal health in the right plot. It can be seen how the disbelief which was increased in the moment of failure was transformed into uncertainty afterwards. Exponential forgetting enabled gradual recovery of health.

Similar method can be used when the FFT (Fast Fourier Transform) analysis detects sudden peak corresponding to the momentary periodic disturbance. Again, incurred uncertainty can be gradually diminished by the exponential forgetting.

4.3 Comparison of Parameter Estimates

Left plot in Figure 5 shows two correlated real signals: slide-valve position of the hydraulic servo valve and its control signal. The servo valve controls flow of oil into hydraulic actuator of a rolling mill. Peaks on the signal correspond to wanted jerking of the servo valve causing slight positional changes of the actuator during rolling.

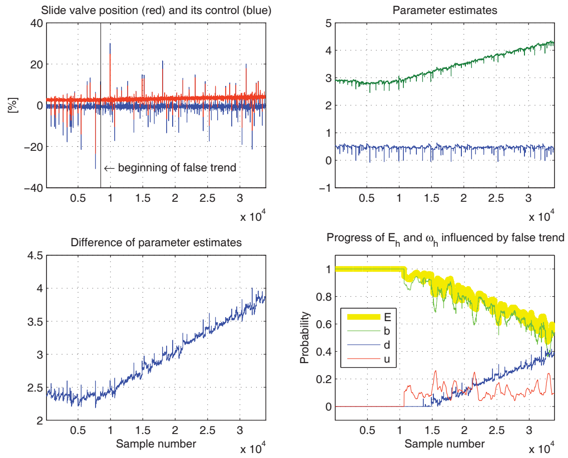

Figure 5: Progress of ω_h and E_h for the couple of related signals.

In normal situation, relation of both signals can be expressed by the model

$$x_2(t) = p_1 x_1(t-1) + p_2 + e(t) ,$$

where $x_1(t)$ represents the control signal and $x_2(t)$ the controlled variable. The parameters p_1, p_2 should be constant except for small variations caused by the simplification imposed by the model. An artificial trend added to one of the signals simulates a creeping failure in the hydraulic circuit. Beginning of the trend is depicted by the vertical line in the upper left plot in Figure 5. The trend was catched by one of the parameter estimates (upper right plot in the same figure) and the difference of parameters (lower left plot) was transformed into decreased health in the lower right plot. The three-sigma rule was used to quantify uncertainty of information about the failure.

5 CONCLUSIONS

There were introduced several methods for quantification of information uncertainty when evaluating condition/health of a measured signal or a pair of signals. Such knowledge about particular components of the inspected control system can help substantially in evaluation of health of a whole system if particular pieces of information are compounded by the means of the subjective logic.

Examples taken for this paper belong to the research being accomplished in the framework of the international project aiming to develop a practical condition monitoring system based on probabilistic treatment of information. Algorithms and methods are being developed in the Matlab environment using its existing OOP (Object Oriented Programming) features

(Ettler and Puchr, 2013). Development of the production version of the system in the appropriate OOP framework is in progress now. The system will be tested in a selected metal processing plant.

ACKNOWLEDGEMENTS

The research project is supported by the grant MŠMT 7D12004 (E!7262 ProDisMon).

REFERENCES

Dedecius, K. and Ettler, P. (2013). Overview of bounded support distributions and methods for Bayesian treatment of industrial data. In *Proceedings of the 10th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2013)*, pages 380–387, Reykjavík, Iceland.

Ettler, P. and Dedecius, K. (2014). Probabilistic reasoning in service of condition monitoring. In *Proceedings of the 11th International Conference on Condition Monitoring and Machinery Failure Prevention Technologies CM2014/MFPT2014*, Manchester, UK.

Ettler, P. and Puchr, I. (2013). Utilization of Matlab classes to streamline experimental software. In *Proceedings of the International Conference Technical Computing Prague (TCP 2013)*, Prague, Czech Republic.

Jirsa, L. and Pavelková, L. (2014). Testing of sensor condition using gaussian mixture model. In *Proceedings of the 11th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2014)*, Vienna, Austria.

Jøsang, A. (2013). *Subjective logic (draft)*. University of Oslo. Available on-line at http://folk.uio.no/josang/papers/subjective_logic.pdf/.

Jøsang, A. and McAnally, D. (2005). Multiplication and comultiplication of beliefs. *International Journal of Approximate Reasoning*, 38(1):19–51.

Kárný, M., Böhm, J., Guy, T. V., and Nedoma, P. (2003). Mixture-based adaptive probabilistic control. *International Journal of adaptive control and signal processing*, 17(2):119–132.

Kocjan, W. (2008). *Learning Nagios 3.0*. Packt Publishing.

Pavelková, L. and Jirsa, L. (2014). Evaluation of sensor signal health using model with uniform noise. In *Proceedings of the 11th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2014)*, Vienna, Austria.

Peterka, V. (1981). *Bayesian Approach to System Identification In P. Eykhoff (Ed.) Trends and Progress in System Identification*. Pergamon Press, Eindhoven, Netherlands.