Granger Causality for Ill-Posed Problems:

Methods, Ideas and Application in Life Sciences

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Abstract

For detecting causal relationships between variables $x_j, j = 1, ..., p$ the concept of the so called multivariate Granger causality has been proposed. Based on the intuition that the cause should precede its effect, in Granger causality one says that a variable x_i can be potentially caused by the past versions of the involved variables $x_j, j = 1, ..., p$. Then, in the spirit of the statistical approach and using a linear model for the causal relationship, we consider the following approximation problem:

$$x_t^i \approx \sum_{j=1}^{p} \sum_{l=1}^{L} \beta_l^j x_{t-1}^j, \; t = L+1, \ldots, T. \tag{1}$$

where L is the so called maximal lag, which is the maximal number of the considered past versions of the variables. The coefficients β_l^j can be determined by the least squares method. As in the statistical approach, one can now fix the value of the threshold parameter $\beta_{tr} > 0$ and say that

$$x_j \text{ has a causal influence on } x_i \text{ if } \sum_{l=1}^{L} |\beta_l^j| > \beta_{tr}. \tag{2}$$

It is well known that for a big number of variables p, the causality network obtained from the approximation problem (1) is not satisfactory. First of all, it cannot be guaranteed that the solution of the corresponding minimization problem is unique. Another issue is connected with the number of the causality relationships that are obtained from (1). This number is typically very big, while one expects to have a few causality relationships with a given gene. To address this issue, various variable selection procedures can be employed. In this paper we review variable selection procedures applied to multivariate Granger causality. On a practical example of a gene regulatory network we illustrate, how our method called Graphical Lasso Granger method with 2-levels-thresholding overcomes the commonly used methods.