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Academy of Sciences of the Czech Republic
Institute of Information Theory and Automation

RESEARCH REPORT

Lenka Pavelková

Fuel consumption optimization

Modification of optimality criterion in the case of speeding risk

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ÚTIA AV ČR, P.O.Box 18, 182 08 Prague, Czech Republic
Tel: (+420)266052422, Fax: (+420)286890378, Url: http://www.utia.cas.cz, E-mail: utia@utia.cas.cz

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1 Introduction

The report describes some experiments related to the project Ekodrive, TAČR TA0103012. This project concerns with fuel consumption optimisation under condition of keeping the recommended speed. The presented approach is based on data currently measured on a driven vehicle and on external observations. Using adaptive optimal control algorithms under Bayesian methodology, a compromise between fuel consumption minimisation and keeping the recommended speed is reached [2].

During control, a speeding risk can occur, i.e. situation when designed control leads to a vehicle speed higher than speed limit. This problem is solved by switching to an alternative (non-optimal) regulator and/or by using a deterministic logic control block whenever a speed limit overrun is predicted.

In this report, an alternative approach is proposed. It uses an original optimal controller designed in [2] but the optimality criterion is extended. The extension adds one terms that penalises vehicle speed increments. Also, the construction of a state-space model required control design differs.

2 Fuel consumption optimisation with extended optimality criterion

2.1 Problem description

The fuel optimisation task is formulated as the following servo problem: Design the control values expressing how much the gas pedal should be pressed so that to push the fuel consumption towards its setpoint and the vehicle speed as close as possible to the recommended speed. The currently used recommended speed is provided by experts for a known route when the driver respects the rules of an economical drive and existing speed limits.

2.2 Model

We consider a "driver-vehicle" closed loop which in discrete time instants $t \in \{1, \ldots, T\}$ produces the following observed variables: an output vector y_t, that is influenced by a control input u_t and by an external variable v_t. The involved system is described by a 2nd order Gaussian ARX model

$$y_t = B_0 u_t + A_1 y_{t-1} + B_1 u_{t-1} + D_1 v_{t-1} + A_2 y_{t-2} + B_2 u_{t-2} + D_2 v_{t-2} + \epsilon_t. \tag{1}$$

where
scalar input u_t corresponds to a pressing the gas pedal;
output y_t consists one by one of the following variables - fuel consumption, speed of car, engine torque, engine speed, distance traveled from the last measurement whereas $y_{t;1}$ and $y_{t;2}$ are optimized to be as close as possible to the below defined setpoint w_t while the other entries are not optimised;
v_t is a road altitude, $v_t = v_{t-1} + \bar{\varepsilon}_t$.

The setpoint for output values is w_t, we consider it to be modelled by $w_t = w_{t-1} + \varepsilon_t$.

For the control design, ARX model (1) is transformed into the state-space model

$$x_t = P z_t + e_t = P_u u_t + P_x x_{t-1} + e_t \tag{2}$$

where
$x_t = [u_t, u_{t-1}, y_t, y_{t-1}, v_t, v_{t-1}, w_t]'$, $z_t = [u_t, x_{t-1}]'$,

$$P_u = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \\ B_0 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, P_x \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ B_1 & B_2 & A_1 & A_2 & D_1 & D_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}, e_t = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \epsilon_{\mathbf{t}} \\ \mathbf{0} \\ \bar{\varepsilon}_{\mathbf{t}} \\ \bar{\varepsilon}_{\mathbf{t}} \\ \varepsilon_{\mathbf{t}} \end{bmatrix},$$

2.3 Optimality criterion

A control is designed that minimises the criterion

$$\tilde{J}_t = (y_t - w_t)'\Omega(y_t - w_t) + (y_t - y_{t-1})'\tilde{\Omega}(y_t - y_{t-1}) + (u_t - u_{t-1})'\Lambda(u_t - u_{t-1}) \tag{3}$$

where set-point w_t is obtained from real drives - $w_{t;1}$ correspond to the measured car speed and $w_{t;2}$ to 85% of the reached consumption. The entries $w_{t;3}$, $w_{t;4}$ and $w_{t;5}$ correspond to the non-optimized output entries and they are set to zeros.

Note that in comparison with previous experiments, the criterion is extended by the term penalising output increments. This extension aims to treat such situations where the speed is near the limit and a danger exist that that the limit will be exceed. The tuning of output increment penalisation allows to prevent speed limit overrun and simultaneously it can prevent oscillations that can occur by switching between two controllers or by imposing control input from logical block.

Optimality criterion is in the form $x_t' Q x_t$ with

$$Q = \begin{bmatrix} \Lambda & -\Lambda & 0 & 0 & 0 & 0 & 0 \\ -\Lambda & \Lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega + \tilde{\Omega} & -\tilde{\Omega} & 0 & 0 & -\Omega \\ 0 & 0 & -\tilde{\Omega} & \tilde{\Omega} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Omega & 0 & 0 & 0 & \Omega \end{bmatrix}$$

2.4 Control design

The suboptimal certainty equivalence control strategy on the horizon of length N is designed using technique from [1]. This strategy works with point parameter estimates, uncertainties are omitted. The resulting recursive formula for control is

$$u_{t+1} = -(P_u'(Q + S_N)P_u)^{-1} P_u'(Q + S_N)P_x x_t \quad (4)$$

with
$S_N = P_x'(Q + S_{N-1})P_x - (P_u'(Q + S_{N-1})P_x)'((P_u'(Q + S_{N-1})P_u)^{-1})'(P_u'(Q + S_{N-1})P_x)$,
$S_0 = 0$

3 Experiments

An illustrative experiment is performed using the control on horizon $N = 5$. Data and a software vehicle simulator are provided by Škoda Auto, for more info see [2]. In Figures 1 - 4, the results for original and extended optimality criteria are depicted. Table 1 summarises the obtained results.

Table 1: Comparison of fuel consumption and vehicle speed for original control and control with extended optimality criterion

	average speed [km/h]	% of recommended speed	consumption [l/100km]
original control	70.9	100.2	4.88
extended criterion	69.2	96.5	4.73

Obtained results indicates that optimisation with extended optimality criterion give comparable results and a slightly smoother velocity course.

References

[1] M. Kárný, A. Halousková, J. Böhm, R. Kulhavý, and P. Nedoma. Design of linear quadratic adaptive control: Theory and algorithms for practice. *Kybernetika*, 21, 1985. Supplement to Nos. 3, 4 , 5, 6.

[2] E. Suzdaleva, I. Nagy, L. Pavelková, and T. Mlynářová. Double optimization of fuel consumption and speed tracking. In *Proceedings of the 11th IFAC International Workshop on Adaptation and Learning in Control and Signal Processing*, pages 305–310, Caen, France, July 3–5 2013.

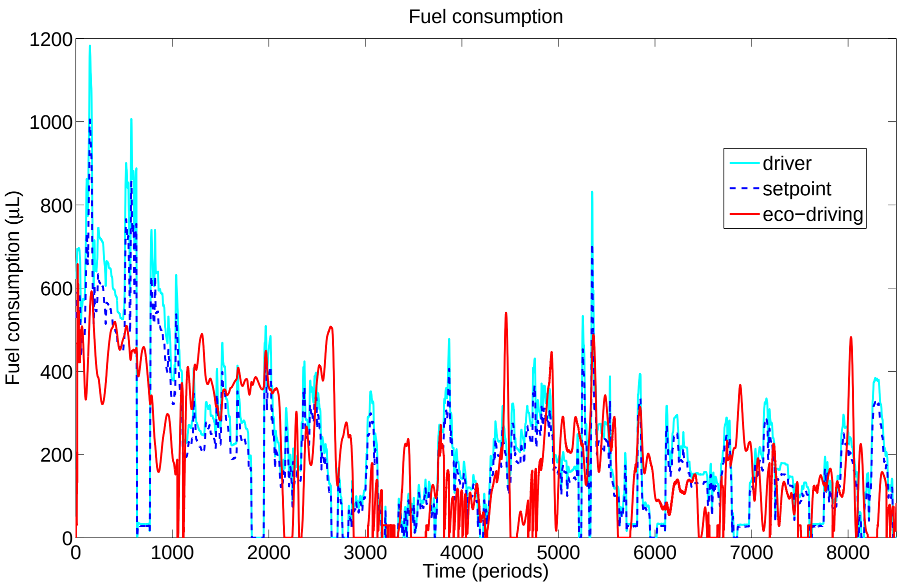

Figure 1: The fuel consumption optimization - original control

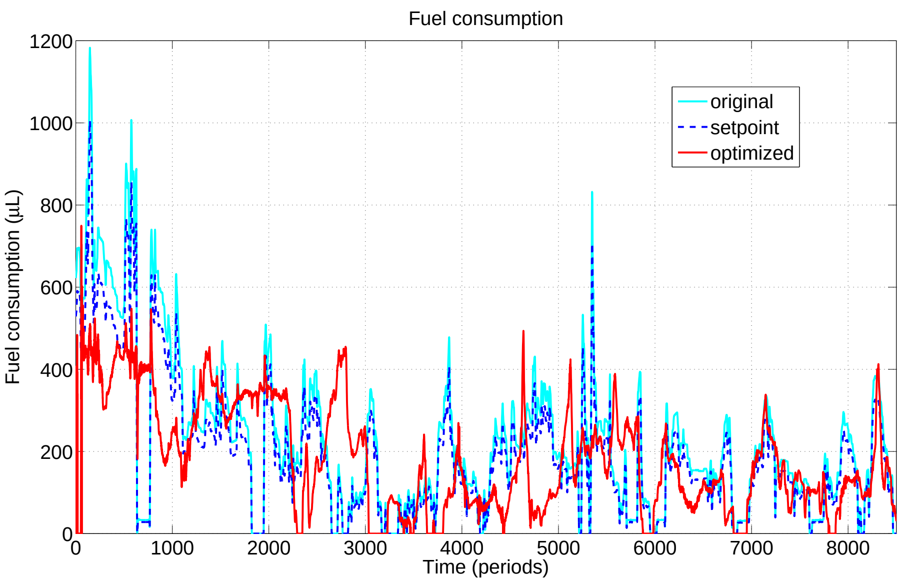

Figure 2: The fuel consumption optimization - extended criterion

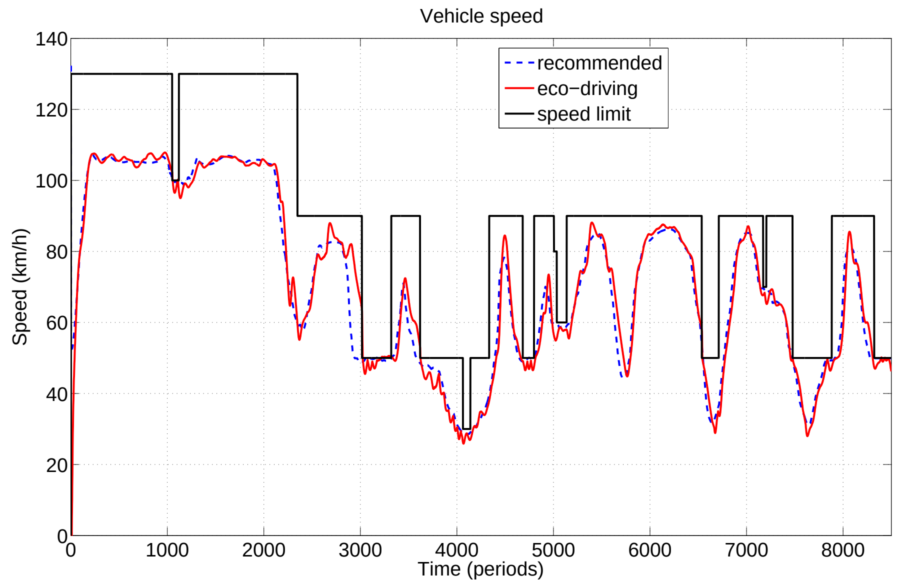

Figure 3: The speed tracking - original control

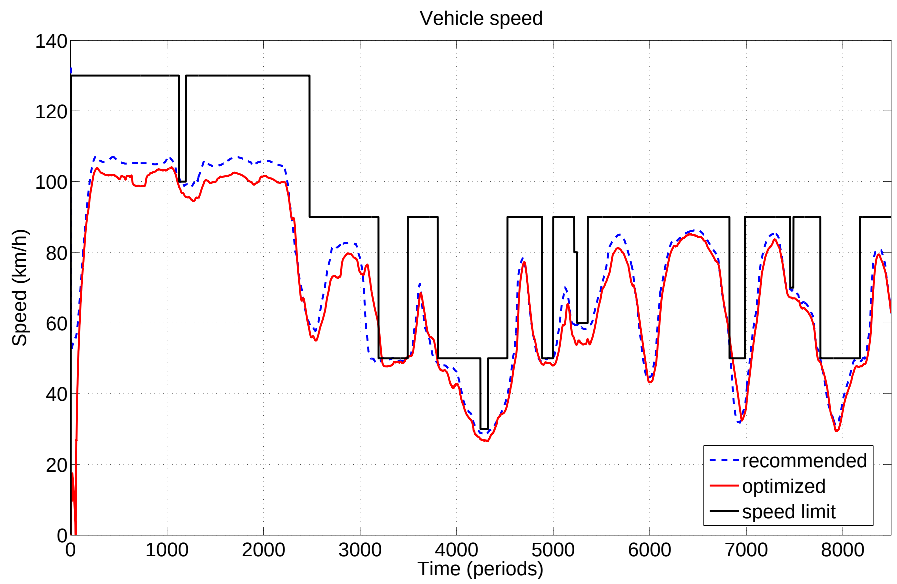

Figure 4: The speed tracking - extended criterion