Evaluation of Sensor Signal Health Using Model with Uniform Noise

Lenka Pavelková and Ladislav Jirsa
Department of Adaptive Systems, Institute of Information Theory and Automation, Czech Academy of Sciences
Pod Vodárenskou věží 4, Prague, Czech Republic
{*pavelkov, jirsa*}*@utia.cas.cz*

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Abstract: The paper proposes a method for evaluating a condition of a noisy sensor signal. The presented algorithm provides a binomial opinion on the sensor signal health including uncertainty by considering (i) user given bounds and (ii) measurement uncertainty. The obtained results can be utilised directly for a single sensor or in a hierarchical structure describing an industrial system of interest where sensors comprise one of the basic building block. There, each block provides a binomial opinion about its health including uncertainty. These opinions are combined using the rules of probabilistic logic so that a single opinion on the health of the whole monitored system is obtained.

1 INTRODUCTION

With increasing demands for safety and efficiency of complex processes, fault detection and isolation (FDI) becomes an important part of control systems in engineering applications (Hwang et al., 2010). FDI consists in binary opinion whether the system is in faulty state and indication of location and nature of the fault.

Within an industrial plant, many possible fault sources exist, e.g., sensors, actuators, hardware components. These heterogeneous fault sources inevitably place considerable demands on related FDI. The situation is yet more complicated due to different possible time developments of faults as an abrupt, a gradual or an intermittent fault. Therefore, monitoring and processing of the system as a whole results generally in a solution tailored for a particular system, combining different probability distributions of particular quantities of interest, either discrete of continuous, and having a high dimensionality. For application examples, see (Isermann, 2011).

A novel dynamic hierarchical structure based on probabilistic approach to fault detection is proposed in (Jirsa et al., 2013; Dedecius and Ettler, 2014). In the presented approach, the system of interest is decomposed into blocks, representing individual physical or logical system units. For each particular block, an opinion on its condition (health) is assessed. Subsequently, these individual information pieces are fused together in order to evaluate the health of the overall system.

The paper aims at the evaluation an opinion on the health of an above mentioned basic block. Here, the block in question corresponds to a sensor measuring an uncertain signal and user given signal bounds are considered.

The paper is organised as follows. Section 2 briefly introduces the above mentioned hierarchical structure for industrial system condition monitoring. In Section 3, a sensor signal health using user given requirements on this signal is defined and an binomial opinion on this health is constructed. Afterwards, an algorithm providing an opinion on the health of noisy signal is proposed.

Section 4 gives an example of the health evaluation of a simulated sensor signal for various types of malfunctions.

Throughout, the transposition is marked $'$.

z^* denotes a set of z-values.
z_t is the value of z at discrete-time instant $t \in t^* = \{1, 2, \ldots, T\}$, $T < \infty$.

The symbol f denotes probability (density) function (p(d)f) distinguished by the argument names. No formal distinction is made among a random variable, its realisation and a p(d)f argument.

2 HIERARCHICAL CONDITION MONITORING

A probabilistic FDI system proposed in (Jirsa et al., 2013; Dedecius and Ettler, 2014) enables to evaluate dynamically the industrial system health at any level of its functional hierarchy. The investigated industrial system is decomposed into a set of interconnected individual blocks. Each block represents an individual physical or logical system unit (e.g. sensor, actuator, communication line etc.).

To each particular block, an observer is assigned that provides a below defined binomial opinion (1) on the health of the respective block, and related uncertainty.

The basic blocks are interconnected using principles of the probabilistic logic which combines the capability of probability theory to handle uncertainty with the capability of deductive logic to exploit structure. In this way, a single opinion on the health of the whole monitored system is obtained. There, a special type of probabilistic logic called subjective logic (SL) is utilised which allows probability values to be expressed with degrees of uncertainty (Jøsang, 2001; Jøsang, 2010).

In SL, the representation of uncertain probabilities is based on a belief model. A subjective binomial opinion expresses a subjective belief of a particular subject about the truth of proposition including a degree of uncertainty. For $x \in \{0, 1\}$, a binomial opinion about the truth of proposition $x = 1$ is the ordered quadruplet

$$\omega_x = (b, d, u, a) \tag{1}$$

where
b is the belief of x being true, i.e. $b = f(x = 1)$
d is the belief of x being false, i.e. $d = f(x = 0)$
u is uncertainty, i.e., the observer is not able to decide,
a is base rate (corresponds to a prior information).
These components satisfy additivity $b + d + u = 1$ and it holds b, d, u, $a \in [0, 1]$.

The expected value of b is

$$E_x = b + au\,. \tag{2}$$

Here, we consider non-informative prior, i.e. $a = 0.5$.

Note that SL provides a set of operators where input and output arguments are in the form of binomial opinions. Most of the operators correspond to well-known operators from binary logic and probability calculus. Additional operators exist for modelling special situations, such as when fusing opinions of multiple observers.

3 SENSOR SIGNAL HEALTH

3.1 Health Definition

We focus on a basic block of the above mentioned hierarchical diagnostic system that represents a sensor signal y_t. This signal is measured at the discrete time instants $t \in t^* = \{1, \ldots, T\}$.

A health of y_t corresponds to how y_t meets the below defined bounds and it is described by the two-valued variable $\mathcal{H}_y$

$$\mathcal{H}_y \in \{0, 1\} \tag{3}$$

where $\mathcal{H}_y = 1$ means that y_t is healthy and $\mathcal{H}_y = 0$ is the opposite.

Here, we consider two-level user defined bounds given by intervals $[P_L, P_U]$, $[S_L, S_U]$ where

$$P_L < S_L < S_U < P_U\,. \tag{4}$$

P_L and P_U correspond to lower and upper physical or safety bounds of y_t, respectively; y_t has to never occur outside these bounds. S_L and S_U are lower and upper soft bounds, respectively; they determine required range where y_t is expected to occur under usual working conditions.

Utilising the user given bounds (4), we define that

$$\begin{array}{lll} \mathcal{H}_y = 1 & \text{for} & y_t \in [S_L, S_U] \\ \mathcal{H}_y = 0 & \text{for} & y_t \notin [P_L, P_U] \end{array} \tag{5}$$

For $y_t \in (P_L, S_L) \cup (S_U, P_U)$, we are not able to decide unequivocally about the value of $\mathcal{H}$ in this case.

3.2 Binomial Opinion on Health of Deterministic Signal

Within the above described hierarchical system, an opinion on health in the form (1) has to be assigned to each particular basic block. For a block representing sensor signal with user given bounds (4) and health assignment rules (5), the values of belief b_D and disbelief d_D in (1) are unequivocal for $y_t \in (-\infty, P_L] \cup [S_L, S_U] \cup [P_U, \infty)$. For $y_t \in (P_L, S_L) \cup (S_U, P_U)$, we has to assign it reasonably. Here, we use the linear dependence of b_D and d_D on the y_t value. The lower index D emphasizes that the value of y_t is considered to be deterministic. The shapes of b_D, d_D and u_D are depicted in Figure 1 and described in Table 1.

According to (1), it holds

$$\begin{array}{lll} b_D & = & f(\mathcal{H}_y = 1) \\ d_D & = & f(\mathcal{H}_y = 0) \\ u_D & = & 1 - b_D - d_D \end{array} \tag{6}$$

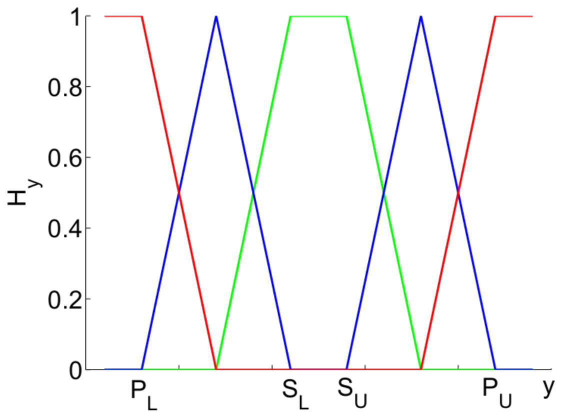

Figure 1: The course of b_D (green), d_D (red), u_D (blue) (6) for single y_t with highlighted bounds (4).

Note that there are various ways to assign b_D, d_D, u_D in the intervals (P_L, S_L) and (S_U, P_U) depending on user knowledge and requirements.

In reality, y_t is not deterministic but it is often described by a probabilistic model. Further, an algorithm is proposed that assigns b, d, u (1) to a probabilistic description of y_t using the above described courses of b_D, d_D, u_D.

Table 1: Assignment of b_D, d_D, u_D for deterministic signal y_t with user given bounds (4).

$y_t \in$	$b_D =$	$d_D =$	$u_D =$
$(-\infty, P_L\rangle$	0	1	0
$(P_L, \frac{P_L+S_L}{2}\rangle$	0	$\frac{P_L+S_L-2y_t}{S_L-P_L}$	$\frac{2y_t-2P_L}{S_L-P_L}$
$(\frac{P_L+S_L}{2}, S_L\rangle$	$\frac{2y_t-P_L-S_L}{S_L-P_L}$	0	$\frac{2S_L-2y_t}{S_L-P_L}$
(S_L, S_U)	1	0	0
$\langle S_U, \frac{S_U+P_U}{2})$	$\frac{S_U+P_U-2y_t}{P_U-S_U}$	0	$\frac{2y_t-2S_U}{P_U-S_U}$
$\langle \frac{S_U+P_U}{2}, P_U)$	0	$\frac{2y_t-S_U-P_U}{P_U-S_U}$	$\frac{2P_U-2y_t}{P_U-S_U}$
$\langle P_U, \infty)$	0	1	0

3.3 Uniform Model of y_t

Considering the given hard bounds on y_t, we use a probabilistic model with uniformly distributed noise for y_t description. This model is described in detail in (Pavelková and Kárný, 2013).

Here, a simple version of this model, a static model, is used for the description of a time evolution of y

$$y_t = K + e_t \tag{7}$$

where K is an unknown constant and e_t is an uniform white noise e_t, i.e. $f(e_t) = \mathcal{U}(-r, r)$, $r > 0$. The equivalent description y_t by pdf is

$$f(y_t) = \mathcal{U}(K - r, K + r) = \mathcal{U}(L, U), \tag{8}$$

where $L = K - r$, $U = K + r$. To estimate parameters K and r, Bayesian maximum a posteriori (MAP) estimation of K and r on sliding window Δ is performed. The MAP estimation converts to a problem of linear programming.

Note that for the purpose of signal health monitoring, the static model is fully acceptable. Here, Δ plays the role of a forgetting. Thus, possible changes of K from required value can be detected.

3.4 Binomial Opinion on $\mathcal{H}$ for Uniformly Distributed y_t

To evaluate ω_H for uniformly distributed y_t, $f(y_t) = \mathcal{U}(L, U)$, and respecting the given bounds, the following technique is used. The b, d and u in (1) are obtained computing expected values of b_D, d_D and u_D (given by rules in Table 1) on the interval (L, U), respectively.

belief

$$b = \mathcal{E}(b_D)_L^U = \int_L^U b_D f(y_t) \mathrm{d}y \tag{9}$$

$$= \int_{a_1}^{b_1} \frac{2y - P_L - S_L}{S_L - P_L} \frac{1}{U - L} \mathrm{d}y + \int_{a_2}^{b_2} \frac{1}{U - L} \mathrm{d}y + \int_{a_3}^{b_3} \frac{S_U + P_U - 2y}{P_U - S_U} \frac{1}{U - L} \mathrm{d}y$$

$$= \frac{(b_1 - a_1)(b_1 + a_1 - P_L - S_L)}{(U - L)(S_L - P_L)} \chi(b_1 > a_1) + \frac{b_2 - a_2}{(U - L)} \chi(b_2 > a_2) + \frac{(b_3 - a_3)(S_U + P_U - b_3 - a_3)}{(U - L)(P_U - S_U)} \chi(b_3 > a_3)$$

where
$a_1 = \max(L, \frac{P_L+S_L}{2})$
$b_1 = \max(a_1, \min(S_L, U))$
$a_2 = \max(L, S_L)$
$b_2 = \max(a_2, \min(S_U, U))$
$a_3 = \max(L, S_U)$
$b_3 = \max(a_3, \min(\frac{S_U+P_U}{2}, U))$
$\chi(.)$ is an indicator function
Note that $\max(.)$ in b_i boundaries guarantee zero integral value (i.e. identical lower and upper bounds) as long as this part is not included in (L,U)

disbelief

$$d = \mathcal{E}(d_D)_L^U = \int_L^U d_D f(y) \mathrm{d}y \qquad (10)$$

$$= \int_{c_1}^{d_1} \frac{1}{U-L} \mathrm{d}y + \int_{c_2}^{d_2} \frac{P_L + S_L - 2y}{S_L - P_L} \frac{1}{U-L} \mathrm{d}y$$
$$+ \int_{c_3}^{d_3} \frac{2y - S_U - P_U}{P_U - S_U} \frac{1}{U-L} \mathrm{d}y + \int_{c_4}^{d_4} \frac{1}{U-L} \mathrm{d}y$$
$$= \frac{d_1 - c_1}{(U-L)} \chi(d_1 > c_1)$$
$$+ \frac{(d_2 - c_2)(P_L + S_L - d_2 - c_2)}{(U-L)(S_L - P_L)} \chi(d_2 > c_2)$$
$$+ \frac{(d_3 - c_3)(d_3 + c_3 - S_U - P_U)}{(U-L)(P_U - S_U)} \chi(d_3 > c_3)$$
$$+ \frac{d_4 - c_4}{(U-L)} \chi(d_4 > c_4)$$

where
$c_1 = L$
$d_1 = \max(c_1, \min(P_L, U))$
$c_2 = \max(L, P_L)$
$d_2 = \max(c_2, \min(\frac{P_L+S_L}{2}, U))$
$c_3 = \max(L, \frac{S_U+P_U}{2})$
$d_3 = \max(c_3, \min(P_U, U))$
$c_4 = \max(L, P_U)$
$d_4 = \max(c_4, U)$

uncertainty

$$u = \mathcal{E}(u_D)_L^U = \int_L^U u_D f(y) \mathrm{d}y \qquad (11)$$

$$= \frac{2}{U-L} \int_{e_1}^{f_1} \frac{y - P_L}{S_L - P_L} \mathrm{d}y + \frac{2}{U-L} \int_{e_2}^{f_2} \frac{S_L - y}{S_L - P_L} \mathrm{d}y$$
$$+ \frac{2}{U-L} \int_{e_3}^{f_3} \frac{y - S_U}{P_U - S_U} \mathrm{d}y + \frac{2}{U-L} \int_{e_4}^{f_4} \frac{P_U - y}{P_U - S_U} \mathrm{d}y$$
$$= \frac{(f_1 - e_1)(f_1 + e_1 - 2\,P_L)}{(U-L)(S_L - P_L)} \chi(f_1 > e_1)$$
$$+ \frac{(f_2 - e_2)(2\,S_L - f_2 - e_2)}{(U-L)(S_L - P_L)} \chi(f_2 > e_2)$$
$$+ \frac{(f_3 - e_3)(f_3 + e_3 - 2\,S_U)}{(U-L)(P_U - S_U)} \chi(f_3 > e_3)$$
$$+ \frac{(f_4 - e_4)(2\,P_U - f_4 - e_4)}{(U-L)(P_U - S_U)} \chi(f_4 > e_4)$$

where
$e_1 = \max(L, P_L)$
$f_1 = \max(e_1, \min(\frac{P_L+S_L}{2}, U))$
$e_2 = \max(L, \frac{P_L+S_L}{2})$
$f_2 = \max(e_2, \min(S_L, U))$
$e_3 = \max(L, S_U)$
$f_3 = \max(e_3, \min(\frac{S_U+P_U}{2}, U))$
$e_4 = \max(L, \frac{S_U+P_U}{2})$
$f_4 = \max(e_4, \min(P_U, U))$

3.5 Algorithm

Here, an algorithm is summarised how to obtain an opinion on the health of noisy signal under user given bounds P_L, S_L, S_U, P_U

Initialisation:

- set bounds (4)
- construct the b_D, d_D, u_D according to Table 1
- set Δ, i.e. length of window for $f(y)$ estimation
- set $t = 0$

On-line phase:

1. set $t = t + 1$
2. update measurements – add y_t
3. IF $t > \Delta$, THEN remove $y_{t-\Delta-1}$
4. estimate L, U in (8)
5. consider parts of b_D, d_D, u_D bounded by L, U
6. compute b, d and u according to (9), (10) and (11), respectively
7. IF $t < T$, GO TO 1.

4 EXPERIMENTS

A series of experiments is performed to illustrate the proposed approach. We use a simulated data. Bounds (4) are given as follows. $S_L = 2.5$, $S_U = 3.5$, $P_L = 0$, $P_U = 6$.

4.1 Slowly Varying Parameters

The sensor signal is simulated by the model (8) with K varying from 3 up to 8.5, $r = 0.75$. The parameter estimation is performed on the window $\Delta = 25$. Data course including bounds and b, d, u assignment are depicted in Figure 2.

4.2 Abrupt Fault

The sensor signal is simulated by the model (8) with K changing abruptly from the value 3 up to value 6 at the time instant $t = 100$, $r = 0.75$, see the left part of Figure 3. The parameter estimation is performed on the window $\Delta = 25$. After the first fault datum arrives, the value of b rapidly decreases but it does not fall down completely because of the past correct data considered within constant memory length Δ. An alternative is to set $\Delta = 1$ after b decrease. Then, Δ is successively increased by 1 at each step up to the original level. Figure 4 depicts both cases.

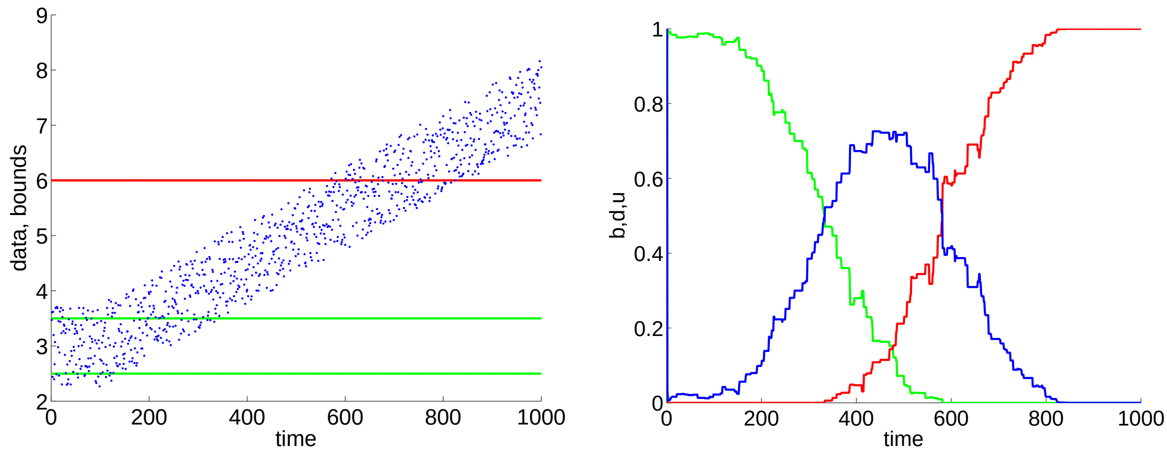

Figure 2: Simulation of slowly varying parameters. On the left side are depicted simulated data (blue), soft bounds (green) and upper physical bound (red). On the right side is ω_H assignment with b in green, d in red, u in blue.

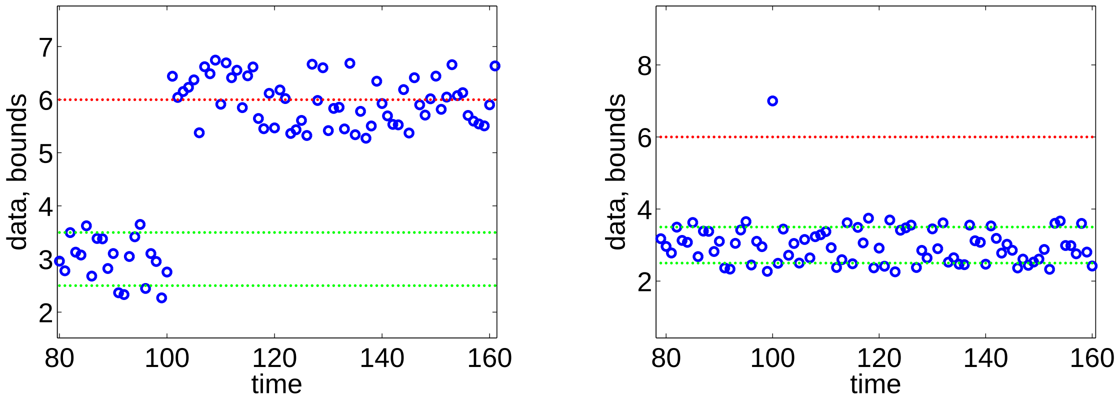

Figure 3: Simulated data (blue) including soft bounds (green) and physical bounds (red) – abrupt fault (on the left side) and outlier (on the right side)

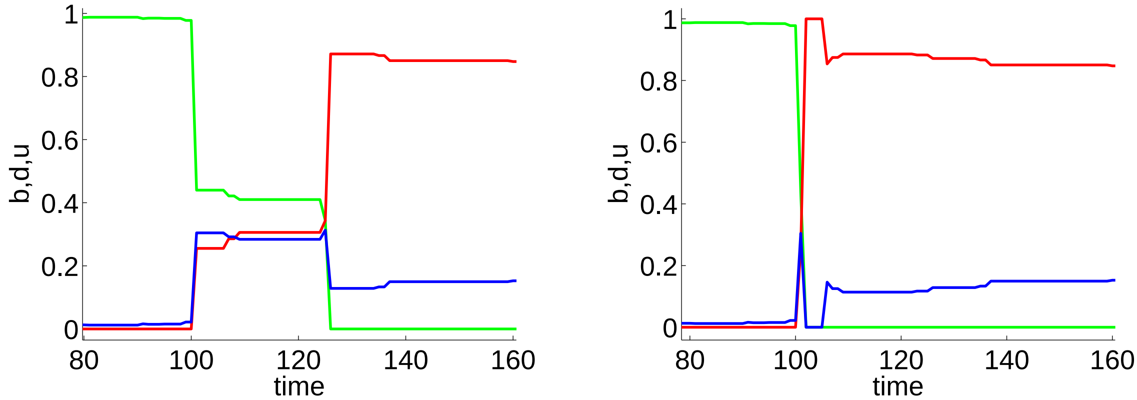

Figure 4: Simulation of abrupt fault. On the left side is ω_H assignment for constant $\Delta = 25$ with b in green, d in red, u in blue. On the right side is ω_H assignment for varying Δ with b in green, d in red, u in blue.

4.3 One-shot Outlier

The sensor signal is simulated by the model (8) with $K = 3$, $r = 0.75$, see the right part of Figure 3. An outlier $y_{100} = 7$ is simulated at the time instant $t = 100$. The parameter estimation is performed on the window $\Delta = 25$. Similarly to previous example, an outlier causes a rapid decrease of b. The utilisation of

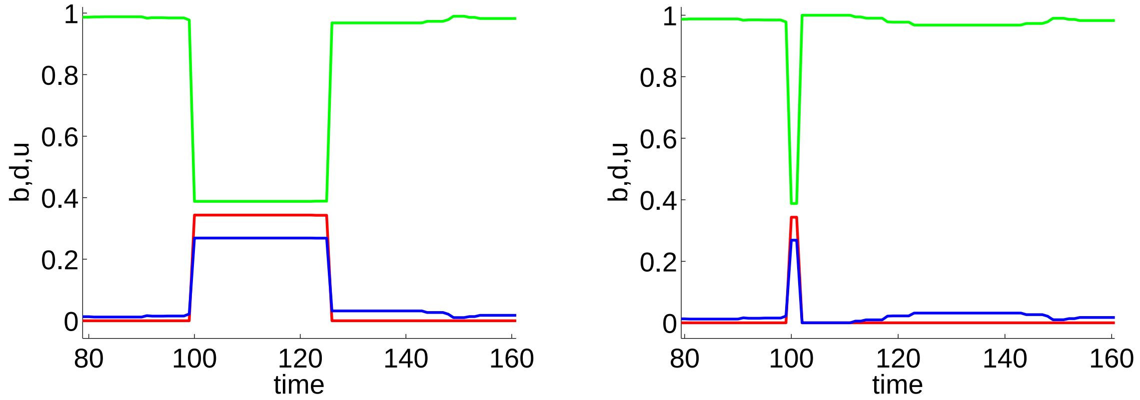

Figure 5: Simulation of an outlier. On the left side is ω_H assignment for constant $\Delta = 25$ with b in green, d in red, u in blue. On the right side is ω_H assignment for varying Δ with b in green, d in red, u in blue.

varying window (described in the previous example) forwards the return to the regular state. The results are depicted in Figure 5.

4.4 Discussion

Three frequent types of sensor signal fault were simulated. The sliding window Δ enables to track slow parameter changes. A higher Δ causes the smoother courses of b, d, u but in the case of sudden decrease of b, the estimation algorithm cannot distinguish immediately between the abrupt fault and mere outlier, see left parts of Figures 4 and 5. Here, the setting of $\Delta = 1$ and its successive increasing up to the original value improves the algorithm performance.

5 CONCLUDING REMARKS

The algorithm for assignment of a binomial opinion on the uncertain sensor signal was developed considering user given bounds. The proposed method seems to be effective. It enables to detect both main sensor signal faults – abrupt and gradual.

A uniform description of involved signal leads to simple and straightforward solution. The resulting algorithm with static signal model requires only the signal bounds to be defined and a memory length to be set. For the purpose of a sensor signal health monitoring, the static model is fully acceptable. Possible changes from the required value are detected thanks to the moving window Δ used during the parameter estimation.

Alternative evaluation of uncertainty using model with Gaussian noise is proposed in (Ettler and Dedecius, 2014).

Further research will focus on model with alternative noise description, e.g. a trapezoidal one, that will describe the sensor signal more precisely.

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