Extending Horizon of Finite Control Set MPC of PMSM Drive with Input LC Filter using LQ Lookahead

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Abstract—Finite control set model predictive control (FS-MPC) has been shown to be a very effective approach to control of PMSM drives. FS-MPC is a very flexible tool since it can evaluate an arbitrary loss function. However, design of the appropriate loss function for the problem can be a challenge especially when the design input is visible only on the long horizon. An example where this problem becomes apparent is the main propulsion drive of a traction vehicle fed from a dc catenary. Specifically, the catenary voltage is subject to short circuits, fast changes, harmonics and other disturbances which can vary in very wide range. Therefore, the drive is equipped with the trolleywire input LC filter. The filter is almost undamped by design in order to achieve maximum efficiency and the control strategy needs to secure active damping of the filter to guarantee the drive stability. While it is possible to introduce active damping terms to the loss function, it is hard to predict its properties. In this paper, we consider decomposition of the problem to control of the LC filter and PMSM drive. We show that the resulting controllers can be elegantly combined using Bellman's principle of optimality. The resulting controller is easy to design and its performance is demonstrated in simulation and experiments on 10.7 kW drive.

I. INTRODUCTION

Control of PMSM drives is a well researched topic with many approaches ranging from classical control solutions [1] to finite control set model predictive control [2]. The finite control set model predictive control (FS-MPC) is attractive since it naturally addresses the problem of switching of the power electronics elements rather than converting continuous solution through the PWM. Simple implementation of FS-MPC together with great flexibility lead to successful application of this approach to many real problems [3]. However, one remaining open problem of FS-MPC is the design of the optimality functions and evaluation of long prediction horizons. In this paper, we show that classical LQ control design can help to address both of these problems. We will use the dynamic programming approach [4] also known as the Bellman principle of optimality.

The issue of long prediction horizons is mostly visible in demanding cases of drive control. One such case

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is control of oscillations of trolley-wire LC filter in dc catenary supplied traction drives, e.g. [5], [6]. Traction drives use an input LC filter and not only a C-filter for the following reasons: (i) an effective limitation of the trolleywire current (due to fast changes of the catenary voltage and short-circuit of the trolley-wire), and (ii) EMC issues. Thus, the LC filter is necessary for proper operation of the drive. On the other hand, the dc-link LC filter has negative impact on the traction drive stability.

The LC filter resonance can be excited by many events by the drive itself (e.g. unsuitable control commands or drive harmonics) or from the outside. One of the most common effects from outside which can excite the filter oscillations is the change of the catenary voltage which can vary very fast due to different reasons. If the catenary voltage increases then the drive still takes the constant power from the dc-link filter (fixed torque command and negligible speed change within the time interval of investigated transient phenomenon). Under some conditions, this may act as a positive feedback (or negative-resistance effect) which results in dangerous oscillations of the dclink filter. This phenomenon can be also explained using frequency characteristics of the drive.

The resonant frequency of the drive is changing e.g. with the change of the position of the vehicle within the feeding section (change of the catenary parameters with varying distance from the static substation). However, the drive resonance properties are also changing with the drive operating point (for more details see [7]). A dangerous situation also occurs when the resonance loop is composed of a two or more traction drives [8]. It is obvious that the explained phenomenon significantly impacts the stability of the traction drive and must be carefully considered during the drive design.

II. LOOKAHEAD ALGORITHMS FOR MPC

The task of optimal model predictive control is commonly defined as finding a sequence of inputs minimizing the chosen optimality criterion or loss function:

$$u_{t:t+h}^* = \arg \min_{u_{t:t+h} \in U} L(x_{t:t+h}, u_{t:t+h}, \overline{x}_{t:t+h}), \quad (1)$$

where $x_{t:t+h}$ denotes the trajectory of the system state x_t on the horizon of length h, $\sum u$ is the system input and \overline{x}_t is the reference (requested) state of the system. Set U denotes a set of all admissible control inputs u_t .

Formulation (1) is particularly well suited for control of power electronics with short prediction horizon since the number of possible control actions is small and it is possible to evaluate loss (1) for all of them in the considered control period. This is the principle of FS-MPC. An alternative to minimization of (1) over input sequence is to minimize the loss function over policy, i.e. a function generating inputs.

A. Dynamic programming

Optimization of the loss function (1) can be done analytically, if the loss function can be written in additive form

$$L(x_{t:t+h}, u_{t:t+h}, \overline{x}_{t:t+h}) = \sum_{\tau=t}^{t+h} l_{\tau}(x_{\tau}, u_{\tau}, \overline{x}_{\tau}),$$

then, optimum policy $u_t^* = v^*(x_t, \overline{x}_t)$ satisfies

$$V(x_t, \overline{x}_{t:t+h}) = \min_{u_t} \left\{ l_\tau(x_\tau, u_\tau, \overline{x}_\tau) + V(x_{t+1}, \overline{x}_{t+1:t+h}) \right\}.$$
(2)

Here, function $V(x_t, \overline{x}_{t:t+h})$ is known as Bellman function or cost-to-go [4]. It is evaluated in backward manner starting with chosen value $V(x_{t+h}, \overline{x}_{t+h})$. Analytical solution of (2) are known only for a limited number of models, for example linear systems with quadratic loss function (LQ). For more complex models, the optimization (2) becomes intractable and must be approximated.

B. Linear Quadratic control

Dynamic programming for linear system with quadratic loss function

$$x_{t+1} = Ax_t + Bu_t, (3)$$

$$l_t(x_t, u_t, \overline{x}_t) = x_t^T Q x_t + u_t R u_t.$$
(4)

solution with quadratic Bellman function $V(x_t, \overline{x}_t) = x_t^T \Psi_t^T \Psi_t x_t$, and optimal policy $u_t^* = -K x_t^T$. The recursion can be found using square root decomposition of the matrices $Q_t = Q_t^{\frac{1}{2}} Q_t^{\frac{1}{2}}$, $R_t = R_t^{\frac{1}{2}} R_t^{\frac{1}{2}}$. Then,

$$l_{t}(x_{t}, u_{t}) + V(x_{t+1}) = \begin{bmatrix} u_{t}^{T}, x_{t}^{T} \end{bmatrix} Z^{T} Z \begin{bmatrix} u_{t} \\ x_{t} \end{bmatrix}, \quad (5)$$
$$Z^{T} = \begin{bmatrix} R^{\frac{1}{2}} \\ Q^{\frac{1}{2}} \\ \Psi^{\frac{1}{2}}_{t+1} B & \Psi^{\frac{1}{2}}_{t+1} A \end{bmatrix}.$$

Using an arbitrary triangularization procedure, product $Z^T Z$ can be uniquely decomposed into product of triangular matrices

$$Z^T Z = Y^T Y, \qquad Y = \begin{bmatrix} Y_{uu} & Y_{ux} \\ & Y_{xx} \end{bmatrix}$$

yielding

$$l_t(x_t, u_t) + V(x_{t+1}) = (Y_{uu}u_t + Y_{ux}x_t)^T (Y_{uu}u_t + Y_{ux}x_t) + x_t^T Y_{xx}^T Y_{xx} x_t.$$
 (6)



Figure 1. Illustration of FS-MPC with LQ lookahead

which is quadratic in u_t and thus minimized for $Y_{uu}u_t + Y_{ux}x_t = 0$. The optimal policy is then

$$u_t^* = -Kx_t, \quad K = Y_{uu}^{-1}Y_{ux}$$
 (7)

and the minimum of the loss is $V_t(x_t) = x_t^T Y_{xx}^T Y_{xx} x_t$. Assignment $\Psi_t = Y_{xx}$ completes recursion of the computation (2). This basic algorithm can be analogously derived for any quadratic loss function.

C. Lookahead algorithm

Evaluation of the Bellman function for a non-linear problem is often computationally intractable and approximations were developed to address this issue. A convenient way how to combine the best of predictive control with dynamic programming is the limited lookahead policy [4]

$$u_{t:t+h}^* = \arg\min_{u_{t:t+h} \in U} \left\{ L(x_{t:t+h}, u_{t:t+h}, \overline{x}_{t:t+h}) + \tilde{V}(x_{t+h}) \right\},$$
(8)

where the prediction horizon h is short as in predictive control, and $\tilde{V}(x_{t+h})$ is an approximation Bellman function for the prediction horizon beyond t + h. From many different techniques for design of $\tilde{V}()$ we focus on the limited lookahead approach, where the Bellman function is obtained by analytical solution of a simplified model of the system. In this text, we propose to use linear Gaussian model simplification to obtain the Ballman function in the form of a quadratic function around a LQ controller. The final Algorithm for LQ lookahead FS-MPC is summarized in Algorithm 1.

The on-line part of the algorithm is identical to a standard FS-MPC approach with additional quadratic loss function. The key difference is in design of this additional term. Its coefficients $(Y_{uu} \text{ and } Y_{ux})$ are not directly tuned by the designer. They result from the iterative optimization of the LQ procedure. Iterating the recursion (2) until a steady state solution is equivalent to optimization of the predictive control on infinite horizon length.

This design procedure can be easily extended for online adaptation of the LQ Bellman function. The resulting algorithm would be similar to the cascade control, where the output of the LQ controller provides inputs for the FS-MPC controller. The unique feature of such approach is that the LQ controller provides not only reference values but also penalization matrices for the FS-MPC controller. Algorithm 1 Design procedure of the FS-MPC with LQ lookahead.

Off-line (LQ):

- 1) Design approximate linear model of the controlled system (or its important subsystem) with matrices A, B.
- 2) Choose quadratic loss function l_{LQ} with penalization matrices Q, R.
- 3) Run dynamic programing algorithm to obtain the core of the Bellman function Y_{uu}, Y_{ux}, Y_{xx} . This can be done in potentially many points of linearization.
- 4) Validate results of the optimized closed loop. If not successful GOTO 2.

On-line (FS-MPC):

- 1) For all potential inputs $u_t^{(i)} \in U, i = 1, ..., I$ evaluate $l_i = \{l_{MPC}(x_{t+1}, u_t) + (Y_{uu}u_t^{(i)} + Y_{ux}x_t)^T(Y_{uu}u_t^{(i)} + Y_{ux}x_t)\},$
- 2) Choose input function with the minimum loss l_i .

DC Catenary



Figure 2. Equivalent circuit of the dc catenary supplied traction drive with input LC filter

III. Control of PMSM drive with input LC Filter

The equivalent circuit of the drive used for the description of the LC filter resonance is shown in Fig. 2. The voltage-source converter and an ac motor can be replaced by an equivalent current source which models the filter load.

A. Model of the controlled system

State equations of the PMSM drive are:

$$\frac{di_{d,t}}{dt} = -\frac{R_s}{L_{sd}}i_{d,t} + \frac{L_{sq}}{L_{sd}}i_{q,t}\omega_t + \frac{1}{L_{sd}}u_{d,t},\tag{9}$$

$$\frac{di_{q,t}}{dt} = -\frac{R_s}{L_{sq}}i_{q,t} - \frac{\Psi_{pm}}{L_{sq}}\omega_t - \frac{L_{sd}}{L_{sq}}i_{d,t}\omega_t + \frac{1}{L_{sq}}u_{q,t}, \quad (10)$$

$$\frac{d\omega}{dt} \approx 0,\tag{11}$$

where $i_{d,t}$, $i_{q,t}$ are the currents in rotating (d-q) reference frame of the drive; ω_t is the electrical rotor speed; L_{sd} and L_{sq} are stator inductances; R_s is a stator resistance, Ψ_{pm} is the flux linkage excited by permanent magnets on the rotor, and $u_{d,t}$, $u_{q,t}$ are coordinates of the input vector. The associated LC filter can be modelled as:

$$\frac{di_{l,t}}{dt} = -\frac{R_f}{L_f}i_{l,t} + \frac{1}{L_f}(U_{T,t} - U_{c,t}), \qquad (12)$$

$$\frac{dU_{c,t}}{dt} = \frac{1}{C_f} (i_{l,t} - i_{z,t}), \tag{13}$$

$$\frac{dU_T}{dt} = 0, (14)$$

where R_f , L_f and C_f are the resistance, inductance and capacitance of the LC filter; $i_{l,t}$ is the catenary current, U_T is the catenary voltage and U_c is the voltage on the dc-link filter capacitor; i_z is the current consumed from the dc-link capacitor by the voltage-source converter.

The systems interact via current equation:

$$i_z U_c = \frac{3}{2}(i_d u_d + i_q u_q).$$
 (15)

B. FS-MPC of the PMSM drive

Control of the PMSM drive with FS-MPC is a well studied problem [2]. For torque control of the drive, we may choose loss function as

$$l_{PMSM,t} = i_{d,t}^2 + (i_{q,t} - \bar{i}_{q,t})^2 + \chi(|i| > i_{max}), \qquad (16)$$

where χ is high penalty for violation of current constraints. The constraint are typically formulated in the phase currents, i.e. $i_{a,t} > i_{max}$ coordinates.

However, loss function (16) does not consider stability of the LC filter and can be lead to unstable behavior of the drive. Various active damping extensions of the loss function has been proposed, e.g. in [9], [10]. In this approach, we design active damping term using LQ lookahead.

C. Control of the LC filter

Note that the model of the LC filter (12)–(14) is linear with state $x_{f,t} = [i_{l,t}, U_{c,t}, U_{T,t}],$

$$\frac{dx}{dt} = \underbrace{\begin{bmatrix} -\frac{R_f}{L_f} & -\frac{1}{L_f} & \frac{1}{L_f} \\ \frac{1}{C_f} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{A_{cf}} \underbrace{\begin{bmatrix} i_{l,t} \\ U_{c,t} \\ U_T \end{bmatrix}}_{B_{cf}} + \underbrace{\begin{bmatrix} 0 \\ -\frac{1}{C_f} \\ 0 \end{bmatrix}}_{B_{cf}} i_{z,t}, \quad (17)$$

where $i_{z,t}$ is considered as controlled input of the LC filter. Which can be linearized for sampling time Δt as follows:

$$\begin{aligned} x_{f,t+1} &= A_f x_t + B_f i_{z,t}, \\ A_f &= \exp(A_{ef} \Lambda t) \end{aligned} \tag{18}$$

$$A_f = \exp(A_{cf}\,\Delta t),\tag{18}$$

$$B_f = \int_0 \exp(A_{cf} \sigma) B_{cf} d\sigma \qquad (19)$$

The control aim is to achieve damped oscillations, we intend to minimize quadratic loss

$$l_{LC} = i_l^2 q_l^2 + (U_c - U_T)^2 q_c^2 + i_z^2 q_z^2.$$
 (20)

where q_l, q_c and q_z are chosen penalizations;

Following the general algorithm of LQ design (Section II-B) with the assumption of Bellman



Figure 3. Properties of the closed loop for LQ controller with different penalizations. Left: damping ratio; **Right**: resonance frequency.

function $V_t = x_t^T \Psi_t^T \Psi_t x_t$, the function to minimize is (5) with i_z in the role of u_t , and

$$Z = \begin{bmatrix} q_z & 0\\ 0 & Q_C\\ \Psi_{t+1}B_E & \Psi_{t+1}A_E \end{bmatrix}, Q_C = \begin{bmatrix} q_l & 0 & 0\\ 0 & q_c & -q_c \end{bmatrix},$$

After triangularization (6), the loss function have form

$$I_{rollout} = (Y_{uu}i_{z,t} + Y_{ux}x_{f,t})^T (Y_{uu}i_{z,t} + Y_{ux}x_{f,t}), \quad (21)$$

IV. SIMULATIONS

The proposed control approach was tested on a system with parameters:

$$R_s = 0.28, \qquad L_s = 0.003465, \\ L_f = 0.006, \qquad C_f = 0.004, \\ R_f = 0.01, \qquad \Psi = 0.1989, \qquad (22) \\ \Delta t = 25e^{-6}$$

In the first experiment, we compare properties of the LC filter. Substituting (22) into (17)–(18), we obtain linear model with eigenvalues poles at [1, 0.9999, 0.9999], i.e. at the stability boundary. Any disturbance can thus cause undamped or even unstable oscillations.

Since the LQ design yields linear controller, the properties of the closed loop can be tested using standard linear systems theory. This can be helpful for tuning of the penalization terms q_c , q_l and q_{iz} . We may design controllers for different penalization values and check the closed loop properties. Properties of the closed loop of the LC filter for $q_{iz} = 1$ and a range of $q_c = \langle 0.1, 10 \rangle q_l = \langle 0.1, 10 \rangle$ are displayed in Figure 3 via damping ratio and resonance frequency. Note that these properties are not guaranteed for the final control system, however, these properties may guide tuning of the penalizations.

Iterations of the LQ recursion for penalization matrices [1, 1, 1] is displayed in Figure 4. The iterations start at t + h, i.e. optimization on horizon of length 100 would yield controller coefficients displayed at 900 on the x axis. Since the coefficients stabilize after 700 iterations, resulting controller is optimal on horizon of length 700 and more.

Application of the LQ term (21) in the FS-MPC was tested on a step change of the catenary voltage from 200



Figure 4. Convergence of the LQ controller and coefficient Y_{uu} on horizon of length h = 1000.

V to 170V. The requested current i_q was kept constant at 20A for the whole time of the simulation. If the drive was controlled by the FS-MPC controller (16), the reference current was followed exactly, however, the drop of the catenary voltage resulted in undamped oscillations (Figure 5 left). Including the LQ term (21) with $q_l = 3, q_c = 10$ and $q_{iz} = 0.1$ to the FS_MPC loss function resulted in suppressed tracking of the initial reference a little, however, it secured damped oscillations of the capacitor voltage after the drop of the catenary voltage.

V. Experiments

A laboratory prototype of the PMSM with the input LC filter with the same parameters as in the simulation (22) was used to verify the approach experimentally. The control algorithm was implemented in TMS320F28335 signal processor supporting floating point calculations. The experiments ware designed to approximate the worst case scenario of operating conditions of a traction drive fed from a DC-catenary.

A drop of catenary voltage from 80 V to 60 V was simulated, with constant electrical rotor speed of f_{me} =15Hz. The drive was operated in torque control mode and the requested current \bar{i}_q = 20A was kept constant during the experiment.

If the drive controller does not use any active damping method, the behavior of the drive under the tested conditions may results in undamped (or very lightly damped) oscillations of the input LC-filter as demonstrated in Fig. 6. This behavior would result in emergency shutdown of the drive.

Next, we evaluate performance of two approaches to active damping. Both of them are based on extension of the basic FS-MPC controller with loss function (16). The first approach is based on introduction of a simple 'damping' term in the loss function

$$l_{UC,t} = l_{PMSM,t} + (U_{c,t+1} - U_{c,filtered})^2 Q_{uc}, \qquad (23)$$

where $U_{c,filtered}$ is filtered capacitor voltage and Q_{uc} is a penalization of the damping term. This method is closely related to the approach of [11]. The second approach is the proposed quadratic term (21) designed by the LQ



Figure 5. FS-MPC control of PMSM drive without (left) and with the LQ term (right). **Top row**: current and voltage of the LC-filter. **Bottom row**: reference and measured currents in the PMSM drive.



Figure 6. Catenary voltage drop. FS-MPC without LC-filter stabilizing, constant el. rotor speed of $f_{me}{=}15{\rm Hz}$, ch1: i_d (35A/div), ch2: i_q (35A/div), ch3: U_t (50V/div), ch4: U_c (25V/div),.

lookahead approach (Algorithm 1). Naturally, performance of both approaches depend on the chosen penalizations. We tried to tune the penalization matrices of both approaches to obtain similar behavior.

In the first comparison, we tuned the penalizations to achieve light damping of the capacitor voltage. Results of both compared approaches are displayed in Figure 7. The results of both methods are almost identical, proving suitability of the simple penalization such as (23) for this task.

In the second comparison, we tuned the penalizations to achieve fast damping of the capacitor voltage, Figure 8. While the LQ lookahead approach was able to stabilize the transient and achieve stable steady state performance, the conventional approach was able to suppress the harmonic oscillation but did not achieve stable steady state performance. Different tuning can suppress the amplitude of the instability in Figure 8 however, the LQ lookahead approach always provides more stable steady state behavior. Note that the fast damping tuning increased torque ripple (see in Fig. 8) compared to the light damping in Figure 7. Thus, the resulting behavior is always a trade-off between the torque ripple and the filter capacitor voltage ripple.

VI. CONCLUSION

In this paper, we proposed to extend the horizon of optimization of the FS-MPC using LQ lookahead. This is a special case of the limited lookahead approach known from optimal control. We proposed to use the LQ control since it is well understood, and can be efficiently computed even for long horizons. The approach was demonstrated on control of a PMSM drive with an input LC filter. The LQ lookahead was applied only for the model of the LC filter which is linear with time invariant parameters. The resulting filter has a cascade structure with LQ controller providing setpoint for the FS-MPC algorithm. Even in this simplistic setup the resulting algorithm was able to damp oscillations of the LC filter. However, we expect that the methodology has much wider use and can be applied to a wider class of problems.

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References

- R. Krishnan, Electric motor drives: modeling, analysis, and control, vol. 626. Prentice Hall, NJ, 2001.
- [2] J. Rodriguez and P. Cortes, Predictive Control of Power Converters and Electrical Drives. Wiley - IEEE, Wiley, 2012.



Figure 7. Experimental result of catenary voltage drop performance tuned for light damping. FS-MPC with the LQ term (left) and conventional active damping (right), constant el. rotor speed of f_{me} =15Hz, ch1: i_d (35A/div), ch2: i_q (35A/div), ch3: U_t (50V/div), ch4: U_c (25V/div).



Figure 8. Experimental result of catenary voltage drop performance tuned for fast damping. FS-MPC with the LQ term (left) and conventional active damping (right), constant el. rotor speed of f_{me} =15Hz, ch1: i_d (35A/div), ch2: i_q (35A/div), ch3: U_t (50V/div), ch4: U_c (25V/div),.

- [3] J. Rodriguez, M. Kazmierkowski, J. Espinoza, P. Zanchetta, H. Abu-Rub, H. Young, and C. Rojas, "State of the art of finite control set model predictive control in power electronics," *Industrial Informatics, IEEE Transactions on*, vol. 9, no. 2, pp. 1003–1016, 2013.
- [4] D. Bertsekas, Dynamic Programming and Optimal Control. Nashua, US: Athena Scientific, 2001. 2nd edition.
- [5] B. Delemontey, C. Iung, B. Jacquot, B. De Fornel, and J. Bavard, "Nonlinear decoupling of an induction motor drive with input filter," in *Control Applications*, 1995., Proceedings of the 4th IEEE Conference on, pp. 1004–1009, IEEE, 1995.
- [6] S. Sudhoff, K. Corzine, S. Glover, H. Hegner, and H. Robey Jr, "Dc link stabilized field oriented control of electric propulsion systems," *Energy Conversion, IEEE Transactions on*, vol. 13, no. 1, pp. 27–33, 1998.
- [7] T. Glasberger, M. Janda, and Z. Peroutka, "Frequency domain evaluation of a traction pmsm drive with direct torque control including field weakening area," in *Power Electronics and Applications (EPE), 2013 15th European Conference on*, pp. 1–8, IEEE, 2013.
- [8] B.-H. Bae, B.-H. Cho, and S.-K. Sul, "Damping control strategy for the vector controlled traction drive," in *Proc. 9th Eur. Conf. Power Electron. and Appl*, 2001.
- [9] M. Rivera, P. Correa, J. Rodriguez, I. Lizama, J. Espinoza, and C. Rojas, "Predictive control with active damping in a direct matrix converter," in *Energy Conversion Congress and Exposition*, 2009. ECCE 2009. IEEE, pp. 3057–3062, 2009.
- [10] C. Rojas, M. Rivera, J. Rodriguez, A. Wilson, J. Espinoza,

F. Villarroel, and P. Wheeler, "Predictive control of a direct matrix converter operating under an unbalanced ac source," in *Industrial Electronics (ISIE), 2010 IEEE International Symposium on*, pp. 3159–3164, IEEE, 2010.

[11] S. Mastellone, G. Papafotiou, and E. Liakos, "Model predictive direct torque control for mv drives with lc filters," in *Power Electronics and Applications*, 2009. EPE '09. 13th European Conference on, pp. 1–10, 2009.