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Collateral composition, diversification risk, and systemically important merchant banks

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ABSTRACT

The impact of collateral diversification by non-financial firms on systemic risk is studied in a general equilibrium model with standard production functions and mixed debt–equity financing. Systemic risk comes about as soon as firms diversify their collateral by holding claims on a big wholesale (merchant) bank whose asset side includes claims on the same producer set. The merchant bank sector proves to be fragile (has a short distance to default) regardless of competition. In this setting, the policy response, consisting in official guarantees for the merchant bank's liabilities, entails considerable government loss risk. An alternative without the need for public sector involvement is to encourage systemically important merchant banks to introduce a simple bail-in mechanism by restricting their liabilities to contingent convertible bonds. This line of regulatory policy is particularly relevant to the containment of systemic events in globally leveraged economies serviced by big international banks outside host country regulatory control.

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1. Introduction

Financial instability and crises are inseparably tied to the phenomenon of default. Crises can start with mass defaults on the micro level, as occurred in the U.S. subprime mortgage market breakdown case of 2007. They also often result in default, including by financial intermediaries, as we have seen in most manifestations of the latest financial crisis in the U.S. and Europe following the summer months of 2008. At their worst, they give rise to a vicious circle of defaults involving banks, the non-banking private sector, and the government, so that funds borrowed to prevent insolvency in one sector push the rescuer itself toward insolvency, as in the current EU periphery sovereign debt impasse. This makes default, particularly if it happens on a systemically important scale, the main adversary of prudential policy.

However, as if totally unaware of this dismal record, the available economic theories of default offer a much less dramatic picture. Agents enter into debt contracts conscious of the possibility that the payment obligation will not be honored, and

there is a whole spectrum of methods, from elementary to highly sophisticated, describing how the non-payment contingency can be reflected in the price of a debt claim. In popular terms, forewarned should be forearmed, so where are the arms of rational creditors? If default is so universally bad, why are there perfectly sensible theories telling us how the debtor chooses to default optimally, or how the creditor optimally calls an insolvency procedure in advance of a credit event (e.g. [Leland, 1994](#), or [Leland and Toft, 1996](#))? Unfortunately, economics has not yet developed a comprehensive picture of default costs and their genesis and structure, or of ways of containing them. These matters are mainly explored by practitioners. From the point of view of the latter, including policymakers, the disastrous effect of default on economic activity and welfare comes from two sources: the legal complexity of debt workout procedures, and the destruction of value, such as human capital and other assets, as a result of forced changes of ownership and control. Neither of these areas has been sufficiently investigated by mainstream financial economics, the language of which is usually employed to formulate policy. Therefore, we have little more than an informal understanding that both private and social default costs are significant enough to be acted against. This understanding has a very long tradition and may have been the principal force behind the custom, existing since ancient times, of equipping loan agreements that show a material default probability with the provision of recourse to collateral. Accordingly, without dwelling

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excessively on the question of why, the economics of debt and investment includes collateral as a standard element of its models. As an unintended consequence, since financial crises and their spillovers to the real economy are crises of risky debt, and the latter has collateral attached to it (with the objective to reduce risk), what we face are, essentially, crises of collateral markets. This observation has been gradually finding its way into the formal theory in the work of [Morris and Shin \(2004\)](#), [Brunnermeier and Pedersen \(2009\)](#), and [Geanakoplos \(2010\)](#), among others.

The objective of this paper is to examine in what ways heterogeneous collateral and the origin of its different subspecies can generate systemic risk. This is a question earlier macroeconomic models have not covered sufficiently, and it requires a more detailed specification of financial assets and contracts.

I start by formulating a model of production financing in which the Modigliani–Miller law does not hold for capital scarcity reasons. Those who have the knowledge and authority to invest (firm shareholders) do not have their own funds, whereas those who can bring investors and production opportunities together (merchant banks) first need to convince at least some of the potential investors to become their depositors as well, since there is no one else to turn to. Although firms are credit-constrained, they can find it attractive for risk management reasons to hold assets unrelated to their own business (i.e., to diversify into merchant bank claims, which serve as outside collateral). However, a firm cannot acquire enough of those claims without borrowing from some other party (commercial banks) first, and such loans are risky. This economy can only operate with leverage, and with leverage comes a systemic risk threat.

I test this construction by checking that it generates intuitively expected outcomes with regard to the reaction of credit, investment, and output to expanding leverage through outside collateral. On the downside, I find that the threat of a systemic collateralization breakdown is significant not just conceptually, but also quantitatively. A few notoriously salient stylized facts accommodated in the model are responsible for this outcome.

First, although it is an empirical fact that firms usually hold certain assets not immediately related to their own business, and put these assets up as collateral on their loans, their owners do not normally conduct sophisticated risky investments in financial markets on their own. When they decide to purchase liquid collateral other than a sight deposit, they have no choice other than to become clients of the investment banking industry. Second, investment banking tends to be oligopolistic, with significant economies of scale.¹ Despite the turbulent structural overhauls they regularly go through, mature financial centers catering to corporate clients are invariably dominated by a few big companies, for which I will employ the term merchant bank.² Third, no matter how much the merchant bank would like to fund its liabilities by a well-diversified asset portfolio, in a globalized (i.e., essentially closed) economy it cannot avoid buying liabilities connected to, ultimately, the same universe of firms whose deposit money it accepts. The chain from some firm's excess cash invested in a certificate of deposit of a merchant bank to a private equity fund holding shares in that very firm may have multiple links, but it can be invariably traced down.

¹ This property is usually explained, among other things, by diversification benefits positively related to size, the soft “closed club” human expertise of investment monitoring and information processing, or the high fixed costs involved, and sometimes also by political clout going hand in hand with network externalities.

² Our use of the term is motivated by its inclusiveness in the sense that features such as catering to the corporate sector instead of retail clients, cross-border operations, involvement in private equity investment, and substantial market power are, or were in the past, all typical of this type of financial institution. A historical overview of the subject can be found, for example, in [Craig \(2002\)](#).

Accordingly, by aggregating the merchant bank sector into one entity and inspecting that entity's balance sheet, I feel it justifiable to stylize the analysis, initially, to the case of just a few firms (I will have two in the quantitative examples of this paper) holding claims on one merchant bank who, in turn, holds a tangible portion of the equity of those same firms.

Not surprisingly, in such an environment, the aggregate productivity threshold below which default of the merchant bank occurs is much higher than the same threshold for an individual producer. The merchant bank has to pay sufficiently high deposit rates to its investors to be attractive as a collateral provider. Therefore, there is a clear bound on the merchant bank's profit regardless of competition in the industry. The situation of a commercial bank lending to the same producers is qualitatively different, as its market power depends mainly on informational exclusivity in relation to the client and is only limited by the productivity characteristics of the latter.

The merchant bank can offer claims on itself as diversified collateral to the firms only as long as it is solvent, but the solvency buffer size, i.e., the merchant bank's profit, is limited by the need to make the collateral worth something. Consequently, diversified collateral in the form of deposits (or bonds) is much more susceptible to systemic impairment than liabilities of standalone producers. Under this structure of financial services, the more one tries to diversify, the more fragile is the leverage one creates, and the harsher are the aggregate consequences.

Can an appropriate policy provide a remedy? The most immediate one (also tried many times) would be to provide an official guarantee of the merchant bank's liabilities. However, the fiscal costs may be untenable, as the Irish and Spanish examples of the near past make clear. Going back to default treatment in the earlier mainstream microeconomics, a merchant bank default would be no problem at all if its pecuniary implications were transferred one-to-one to the ultimate creditors and did not receive an institutional spin in the form of a value-destroying bankruptcy procedure. In a frictionless world, this could be achieved if the merchant bank were mandated to issue only equity as liabilities. Even so, merchant bank equity may be unsellable to firms for the reason explained in [Townsend's \(1979\)](#) costly state verification (CSV) model: the impossibility for a small shareholder to establish the appropriate value of the dividend that a big and complex merchant bank owes him. Therefore, I suggest an alternative, inspired in equal measure by [Townsend \(1979\)](#) and by the [Black and Scholes \(1973\)](#) and [Merton \(1974\)](#) treatment of risky company debt. Recall that under the Black–Scholes–Merton approach the company assets in default are transferred one to one to the creditor. The same thing happens under the debt contract considered in [Townsend \(1979\)](#). This is tantamount to the creditor becoming a shareholder. The resulting liability is a fixed-income debt instrument in good times and equity in bad times, i.e., essentially, a convertible bond. An important formal difference from the classical understanding of the latter is that its covenant makes conversion the decision of the holder. In our setting, the conversion trigger is exogenously tied to the merchant bank's solvency (the current model is sufficiently simple in this respect, so that one can assume automatic conversion whenever the bank is unable to pay the original deposit rate, without further procedural details). Essentially, our construction is a variety of the so-called contingent convertible (CoCo) bond. In our view, the most important advantage of this bond covenant is that a shareholder of a living company has a much stronger legal standing in what concerns state verification than a creditor of a defaulting company. So, the key proposition we want to exemplify with our formal exercise is that an insolvent merchant bank should not be sent into bankruptcy, but rather should exchange its fixed income liabilities for shares and then distribute whatever (little) it actually

earned among the old and new shareholders. In this way, the consequences of an adverse aggregate productivity shock will not be avoided. They will still be borne. However, in our model of merchant bank bond conversion they only have a one-to-one impact on firm owners, whereas in a pure deposit-taking merchant bank facing insolvency they are expanded. Additional losses emerge either because of a system-wide shock due to debt workout delays and destruction of value (if the merchant bank is allowed to fail) or because of a heavy fiscal burden (if official deposit guarantees are given). Accordingly, risks will be diversified as long as they are really diversifiable and not just be different labels of an aggregate risk common to everybody (as in a systemic shock case), whereas the costs of the latter will be distributed predictably among firm owners without a legal breakdown.

The results the model has provided so far can be split into three categories: predictable ones, analogs of known results in a new environment, and, finally, relatively new or unexpected findings.

Among the predictable ones, there is yet another confirmation of the conjecture that leverage in the real sector (in our model enabled by the merchant bank sector, as opposed to the better known commercial bank lending channel) increases the fragility of the financial sector. I also predictably find that official guarantees offered to collateral providers lead to overinvestment compared to regimes based on bail-in.

Next, I find that in this model, just as in others known from the literature, one needs a private equity partner in a firm co-financed by debt, as small insufficiently informed retail investors do not provide enough equity capital to support investment in equilibrium. This is the old CSV story by [Townsend \(1979\)](#) narrated in a different set-up. More generally, undercapitalized firms cannot obtain debt financing.

Last but not least, the model yields some novel policy-relevant observations. In particular

1. the financial deepening that happens through the use of outside collateral, its financing by new equity, and the accumulation of both on merchant bank sector balance sheets, is welcomed by the producer sector because it raises the distance to default, reduces the default probability, and pushes down the price of bank loans; all this happens at the expense of fragile merchant banks;
2. there exists an interval of aggregate productivity shock realizations for which producers repay debt whilst merchant banks do not. For the states of the world in this interval, a bail-in (CoCo) clause on the merchant bank debt saves the government an amount of provisional guarantees of the same order as aggregate output;
3. a properly chosen non-zero level of outside collateral, accompanied by the merchant bank CoCo debt regime, not only eliminates merchant bank failures and reduces the default frequencies of the firms, but also costs relatively little (when compared to the official guarantee regime) in terms of a modest reduction in output.

1.1. Relation to the literature

The present paper positions itself in the intersection of three research areas. The intersection of all three, to the best of our knowledge, has so far been empty, even though pair-wise overlaps exist.

The first area is the macroeconomics of financial frictions, mostly represented by the burgeoning literature on DSGE-with-financial-contracts models started by [Bernanke et al. \(1999\)](#). In terms of such attributes as random event spaces, preferences, and technologies, our model stands closer to stochastic

general equilibrium theory than to the elementary “high school algebra-amenable” constructions widespread in the financial intermediation literature. After extending to multiple periods and adjusting, wherever warranted, the risk attitudes of agents from neutral to averse, I am able to make our set-up compatible with the standing DSGE paradigm. This has the advantage of both reducing arbitrariness in agents’ decision problems (which would otherwise be necessary to attain tractability) and facilitating derivation of empirical implications, not to mention the opportunity to state the results in terms comparable with the already available DSGE models with financial frictions (see [Iacoviello, 2005](#), [Covas and Fujita, 2010](#), and [Gerali et al., 2010](#), among others).

The second strand is the literature on regulation of systemically important financial institutions (SIFIs). Among many important theoretical and econometric contributions, I would like, without any claim on exhaustiveness, to mention [Acharya \(2009\)](#) and [Acharya et al. \(2012\)](#). These papers emphasize the role of joint failure risks of many financial institutions *within the sector*. Expanding this view, I believe it is necessary to also consider joint failure risks coming to the sector *from the outside* (financial intermediaries’ partners that appear at once on the sector’s asset and liability sides). The post-crisis regulatory state of the art with regard to SIFIs is codified by several Basel 3 documents, such as [BCBS \(2011\)](#). As to illiquidity (in our model, it is assumed to be the central problem for business partners of a big merchant bank undergoing resolution), a standing regulatory consensus ([BCBS, 2010](#)) has been achieved with regard to buffers and other monitored indicators. Progress in identifying principal illiquidity sources is limited. Most space in the regulatory discussion concerning SIFIs is occupied by standard capital requirements, i.e., equity satisfying the Tier 1 capital definition. [BCBS \(2011\)](#) is very cautious with respect to alternatives to common stock such as contingent debt. This is opposed by some scholars, who are quite vocal about the necessity of protecting taxpayers against costly bank failures with more incentive-robust alternatives to Tier 1 capital which would return loss absorption obligations to specific investors instead of spreading them around ([Bulow and Klemperer, 2013](#); [Calomiris and Herring, 2012](#)). The regulatory approach discussed in the present paper also relies on contingent capital and bail-in, pointing at the next area of related research.

The third, contingent convertible bond line of literature is traditionally anchored in asset pricing theory and derivatives research, since the proposed forms of contingent bank capital have always been understood by their proponents as marketable instruments. Therefore, the original theory of convertible bond pricing ([Black and Scholes, 1973](#)) was not directly related to bank regulation. With time, the idea of using bond-to-stock conversion mechanisms for bank risk containment became increasingly attractive, so that proposals of different varieties of contingent bonds, including the CoCos I refer to here, began to proliferate. (See [Flannery, 2005](#), as one of the pioneers of this line of thought, or [Pennacchi et al., 2012](#), for an example of “precautionary,” i.e., convertible in a pre-emptive way to resolution, contingent debt; an overview of these and other alternatives can be found in [Murphy et al., 2012](#).) I have deliberately made this paper simple and stylized enough to avoid having to solve the various incentive and dynamic inconsistency problems that arise when contingent convertible instruments exist for multiple periods and interact with pre-existing equity markets. Contractual fine-tuning of merchant bank liabilities to reflect their pricing in secondary markets is a topic of future research.

In addition to the three previously discussed ones, there are two other areas of financial economics that provided impulses for the present study: corporate asset structure and optimal management of large risky investment portfolios.

Firms that hold liquid assets in parallel with using bank loans are a well known phenomenon. This issue was studied theoretically in the context of a credit-constrained neoclassical economy by Woodford (1990), and there has been substantial theoretical and empirical literature in the same vein since then (see, for example, Bacchetta and Benhima, 2010, for further references). Diversification leading to the opposite of its initial goal, i.e., risk concentration, has been quantitatively examined by, for example, Ibragimov et al. (2011) and a host of earlier papers cited therein. However, these models are almost purely probabilistic and have but a rudimentary economic structure (i.e., no distinction between agent roles or between equity and debt, etc.). In our approach, the diversification curse is accommodated in a standard choice-theoretic environment of a production economy.

The rest of the paper is organized as follows. I describe the model in Section 2. Section 3 reports the results of numerical experiments with different merchant bank liability regimes. Section 4 discusses the implications of these simulation results in more detail and concludes.

2. Model

2.1. The economy

The agents of this economy are firm shareholders, firm managers, workers, commercial banks, and merchant banks. Each firm is, actually, a continuum of mass one of small identical producers who take all prices relevant for them as given. Accordingly, the attributes of each micro-producer are a micro-shareholder, a micro-manager, and a micro-worker, all forming continua of small identical agents of mass one as well. It is possible to discuss representative agents in all three categories. Hereinafter, I will deal with two separate “macro-firms” representing one of the two mentioned continua, with one shareholder, one manager, and one worker in each being typical elements of the corresponding large sets.

There are also two large commercial banks, each servicing the whole “macro-firm” (for a bank, all its client micro-firms are identical, hence the credit terms are identical, too), and one large merchant bank servicing all firms. Investment opportunities include firm stock (available to the merchant bank), bank loans (available to the commercial banks), and claims on the merchant bank in deposit form (available to the firms’ shareholders).³

A worker sells one unit of labor to his firm. Firm managers hire labor and borrow from commercial banks. The latter pay funding costs for the credit they grant. The loan proceeds are split between wage expenditure and purchase of physical capital in excess of the quantity provided by the shareholder. This quantity comes from an exogenous stock endowment owned by the initial (I will also use the term incumbent) shareholder. The latter can either use the whole endowment as an investment in physical capital or divert part of it to purchase other available assets. Shareholder wealth is measured in the same units as physical capital and is transferrable between the latter and other assets, at no cost. The incumbent shareholder can also issue new stock in his firm and sell it. I assume (on the grounds of missing specific skills) that neither managers nor incumbent shareholders are able to engage in asset trade on

their own. Instead, they buy claims issued by expert intermediaries who, in turn, are able to trade among themselves, invest in outside assets, and purchase newly issued stock in the firms. The role of expert intermediary community in our model is taken by a representative merchant bank. The latter can invest in the world market (outside the examined economy) at a fixed positive rate, as well as purchase private equity partnerships in both firms, with funds raised as deposits. As a result, the merchant bank accumulates assets which, as it may erroneously believe, can serve as risk diversifiers.

There are two periods. In the first, labor hiring and pre-paying, borrowing, and investment decisions are made, and in the second, the production output is sold and the revenue distributed between the borrowers and the lenders, and other investment returns paid out.

The producing firm has a Cobb–Douglas production function

$$Af(k, m) = ALk^\alpha m^{1-\alpha}, \quad (1)$$

in which k is physical capital, m is labor, L is a private total factor productivity (TFP) component, and A is an aggregate TFP component. One should think of situations in which A is a random variable with known distribution, whereas L is either a simple scaling constant (our benchmark) or a firm-specific parameter with each of a large set of small firms identified by their individual L values. In general, the TFP risk defined in this way is a mixture of aggregate (A) and idiosyncratic (L) uncertainties. Two corner cases are purely firm-level (A constant, L varies) and purely systemic (L constant, A varies) risks. In this paper, I analyze the implications of the latter case for economies in which collateral and equity provision by the financial system (merchant banks) is prepared mainly to address the opposite case, i.e., diversification of firm-specific shocks.

Capital is released after the end of the production cycle, but its transformation from a producer-specific to a generally usable state is costly. For each quantity k leaving the production facility one gets $(1 - t(k))k$ units for further use. The structure of the capital transformation function t is as follows:

$$t(k) = \delta + \tau(k), \quad (2)$$

where the positive constant δ is the conventional depreciation rate and the strictly increasing function τ ($\tau(0) = 0$, $\tau(k) > 0$ for all $k > 0$) stands for increasing “capital dismantling” costs. That is, τ can be considered a reverse of the traditional capital installation cost function. If the firm defaults (see later), $(1 - t(k))k$ is added to the collateral seized by the lender; if it survives, this term is a part of the shareholder revenue (“EBIT”). Thus, EBIT consists of the sum $Af(k, m) + (1 - t(k))k$ and one other term to be described below. I have added the term $\tau(k)$ to the usual constant capital depreciation rate to account for the difference between firm-specific and general collateral, which is important both conceptually and quantitatively.⁴

I assume a competitive labor market with labor force supply normalized to unity for each firm (there are two macro-firms, so that one has to assume some form of firm-specific skills; then, m becomes more a variety of human capital than classic unskilled labor). Labor market competitiveness means that workers are paid the marginal product of labor as their wage, and the wage expenditure is subtracted from the firm revenue. To avoid dealing with

³ More precisely, one needs to talk about time deposits, CDs, or bonds with fixed maturity, since standard demand deposits leave space for a run on the merchant bank in the event of its suspected insolvency. But the possibility of a run goes against the spirit of contingent capital requirements, which we want to apply to the merchant bank. So, deposits in our model will be understood as claims with fixed maturity (in the second period of the model) with no possibility of early withdrawal. We keep the term deposit for reasons of economy of language.

⁴ It turns out that, under linear capital depreciation, one would not be able to exclude spurious high-risk investment equilibria with a very small probability of survival and a very high lending rate, but still a tiny positive expected value of after-interest earnings (a variant of the well-known “gambling for resurrection”). Such equilibria are outside the focus of the present paper, so we eliminate them by introducing convex capital dismantling costs.

wage settlements in a defaulting firm, I assume that the whole wage bill is paid in advance in period 1, for which purpose the firm borrows the whole amount, denoted b^m , from its “house” bank (working capital loan).

The labor market does not play any significant conceptual role in this model, but it is necessary for calibration purposes. With a single-input production function, one would obtain unrealistically high marginal products of capital as well as interest rate levels, and also have difficulties generating reasonable default rates.

Remark 1. The present version is in two periods. In a multi-period variant, interpretation of m as skilled labor (firm-specific human capital) could be used to augment the default costs in welfare terms with the corresponding loss of accumulated human capital. This feature might add rationale to the policy of trying to reduce the default frequency.

Remark 2. Having both equity and debt investment financing is important when we want to consider the case of limited (or, at least, highly elastic) supply of equity capital. That this intention has good grounds can be validated ex post in our setting if one considers a standard stock market populated by traditional small moderately risk-averse equity investors. Then it turns out that, in many situations, such a market, acting on the usual limited information about producer technology, is only able to provide a portion of the capital needed (cf. Section 3.2). The rest must be available as an exogenous foundation stock, a private partnership, or a bank loan. In other words, quite often, there does not exist an equilibrium based predominantly on a publicly traded stock able to complement a small level of private equity participation. These are the cases where a merchant bank can fill the gap.

Remark 3. This paper functionally separates commercial and merchant banking, even though, in most of the developed world outside the U.S., the prevailing model has until recently been that of a universal bank. For one thing, the current regulatory trend worldwide counts on maintaining strict Chinese walls between the two activities. For another, I simply define distinct optimization problems for the two lines of business on the level of their management, but do not rule out a hypothetical single ultimate shareholder of a joint holding company in control of both.

2.2. Borrowing, collateral, and default

Physical capital is financed by both equity and bank debt. If q is the amount available as equity (the equity market will be defined separately), then

$$k = q - v + b^k. \tag{3}$$

Here, b^k is the amount borrowed to co-finance physical capital purchase. We have already introduced another component of bank debt, b^m , needed to pay wages. Thus, the total loan size is $b = b^k + b^m$.

The remaining term on the right-hand side of (3), v , is the amount set aside by the controlling shareholder as a source of additional collateral in excess of $(1 - t(k))k$. This quantity (diversified collateral) is invested outside the firm to generate a buffer formally unrelated to the company’s own production. (Note that “unrelated to” does not always mean “independent of,” since under systemic events the dependence comes about.) When $v = 0$, the only collateral the firm has comes from its own output and (dismantled) physical capital. When $v > 0$, the collateral is augmented by $(1 + i^0)v$, where i^0 is the rate of return that can be earned on v in financial markets via the merchant bank. In a surviving firm, $(1 + i^0)v$ is a part of its revenue. I set the maximum allowed value of v equal to q in order to exclude cases of unlimited leverage out of bank-lent funds. When $v = q$, the firm only finances physical capital out of its bank

loan while spending the totality of its equity capital on collateral diversification. Such behavior, if shared by all producers, generates the maximum admissible degree of leverage in the economy.

The firm pays the shareholders dividends defined as

$$y(A) = \max\{Af(k, m) + (1 - t(k))k + (1 + i^0)v - (1 + r)(b^k + b^m), 0\}, \tag{4}$$

under the constraint $b^k = k - q + v$. In a defaulting firm, $Af(k, m) + (1 - t(k))k + (1 + i^0)v$ is treated as collateral seized by the bank. This definition of collateral is a synthesis of the classical Black–Scholes–Merton one (Black and Scholes, 1973; Merton, 1974), later taken over by Bernanke et al. (1999) and supported by Townsend’s (1979) CSV analysis, and the Kiyotaki and Moore (1997) concept, also widely used in models created by Geanakoplos (cf. Geanakoplos, 2010, and references to his earlier papers therein). The “Merton part” is formed by the output plus released physical capital, $Af(k, m) + (1 - t(k))k$. The term $(1 + i^0)v$ is the “Geanakoplos part,” potentially liquid but subject to random swings in value. In a multi-period model, this part would be the source of the collateral cycle and, given a systemic event, the debt deflation effect.

The firm either survives or defaults depending on the realized total factor productivity A . Survival is equivalent to the firm’s EBIT exceeding its debt service:

$$Af(k, m) + (1 - t(k))k + (1 + i^0)v \geq (1 + r)(b^k + b^m). \tag{5}$$

This happens if and only if the realized A exceeds the threshold value

$$A^d = \frac{(1 + r)(b^k + b^m) - (1 - t(k))k - (1 + i^0)v}{Lk^\alpha m^{1-\alpha}}. \tag{6}$$

If the realized A is below A^d , the firm defaults and the bank seizes EBIT, whereas the firm shareholders get nothing. There are situations in which A^d is negative (typically, this means very strongly capitalized firms in an environment of low lending rates), in which case survival is a certainty.

2.3. Investment and labor-hiring decisions

Let us denote the p.d.f. of the aggregate TFP factor A by φ and introduce the notation

$$\Phi^+(A) = \int_A^{+\infty} \varphi(S) dS, \quad \Psi^+(A) = \int_A^{+\infty} S\varphi(S) dS, \quad \text{for } A \geq 0.$$

That is, $\Phi^+(A^d)$ is the survival probability of the firm and $\Psi^+(A^d)$ is the expected TFP of surviving firms. Another piece of notation to be used in the sequel is

$$\theta(A) = \frac{\Psi^+(A)}{\Phi^+(A)}$$

i.e., the average TFP value of a firm conditioned on it exceeding A .

For future use, I also introduce the notation Φ for the cumulative distribution of A (i.e., $\Phi^+(A) = 1 - \Phi(A)$) and Ψ^- for the expected TFP of defaulting firms (i.e., $\Psi^-(A) = \bar{A} - \Psi^+(A)$, \bar{A} being the unconditional mean of A).

I assume a hired manager remunerated in proportion to the firm’s dividend (i.e., the manager receives 0 if the firm defaults). This assumption is made to avoid complications with agency problems between the shareholder and the manager. Also for the sake of simplification, I assume manager risk-neutrality.

The manager takes the level of equity q , the diversified collateral v , the lending rate r , and the wage level as given and decides upon labor hiring and investment in physical capital k (which, for him,

becomes equivalent to setting the size of the bank loan). Due to risk-neutrality, the chosen k and m levels must satisfy the first-order conditions

$$\Psi^+(A^d)f_k(k, m) = \Phi^+(A^d)[r + t(k) + kt'(k)], \tag{7a}$$

$$\Psi^+(A^d)f_m(k, m) = \Phi^+(A^d)(1 + r)w, \tag{7b}$$

where w is the wage, paid, as was agreed, out of the bank loan in advance of production (which is why (7b) contains the lending rate factor $1 + r$). Accordingly, $b^m = wm$ and

$$\theta(A^d)f_m(k, m)m = (1 + r)b^m. \tag{7c}$$

That is, b^m is the present value of the (survival-conditional) labor share.

In the case of Cobb–Douglas, as well as any other constant-returns-to-scale (such as CES) production, (7c) allows one to eliminate the labor market variables from further calculations completely. Recall that we normalize the labor input to unity, thereby pinning the wage level down.

2.4. Bank loans

Jointly, production decisions (7) determine the demand $B(r)$ for loans (parameters on which B depends besides r are omitted for simplicity). On the credit supply side, a commercial bank is assumed to enjoy market power over the borrower (e.g. due to a borrower hold-up problem of the Diamond–Rajan type, cf. Diamond and Rajan, 2000, as the firm cannot credibly communicate its productivity type to outsiders). The base funding cost for the bank is denoted by i . In addition, we endow the credit supply side with some realistic elasticity, which proves important when it comes to calibration. Specifically, we assume that there is also a non-linear component of the funding cost, e.g. a quadratic of the form

$$\frac{a}{2} \left(\frac{B(r) - v - y_0}{q} \right)^2.$$

The latter is added to the linear component $(1 + i)B(r)$ and puts an additional brake on borrower leverage expansion in excess of some exogenous reference level, mimicking the popular loan-to-value/debt-to-equity indicators used by regulators. Here, we have set the driving variable of this brake as the ratio of the debt in excess of the diversified part of the collateral plus a reference output, y_0 , over the equity value. The exogenous parameters appearing in the above expression, namely, y_0 and a positive constant a , originate in macroprudential regulation.⁵

I will denote by hats the variables (such as physical capital and production level) chosen optimally by the borrowing firm. A risk-neutral bank announces r taking into account the loan demand, its funding costs, and the equity value of the loan applicant. Altogether, the bank maximizes the expected profit from the loan given by

$$\Psi^-(\hat{A}^d)\hat{f} + \Phi(\hat{A}^d)[(1 - t(\hat{k}))\hat{k} + (1 + i^0)v] + \Phi^+(\hat{A}^d)(1 + r)B(r) - (1 + i)B(r) - \frac{a}{2} \left(\frac{B(r) - v - y_0}{q} \right)^2. \tag{8}$$

⁵ Technically, the cost term in question offers additional degrees of freedom for model calibration purposes, giving rise to plausible default frequency and risk premium/credit spread values. Both would be unrealistically low if one assumed linear funding costs.

2.5. Choice of collateral diversification

It makes sense to consider an exogenously fixed level of outside collateral, v , first and discuss mechanisms by which agents may coordinate on a particular value later. For several reasons, the determination of the v -size is not a unilateral optimization decision that can be taken by anyone in control of the firm.

It can be easily demonstrated that a hired manager who takes the equity capital q of the firm as given would prefer no collateral diversification at all. Namely, by increasing v from zero to q , one obtains increasing total output, but a decreasing expected dividend. This is a consequence of higher debt levels under higher v , cf. (3): whereas physical capital k is determined “technologically” by the manager according to (7a), there is less equity to finance it if q is diverted toward v . Consequently, the firm must borrow more and the debt service component of output goes up. The negative effect on dividends is a consequence of higher debt service. Accordingly, there is potential for a conflict between the controlling shareholder and the manager.

The preferences of the commercial bank with respect to the v level of its borrower depend on its degree of sophistication. If the bank, in the same way as the firm manager with whom it negotiates the loan, takes the equity capital value as given, then raising v from zero to a small positive level has a first-order positive effect on both the loan demand and the survival probability. These are factors that make the lending bank encourage the use of outside collateral by the borrower. However, if the bank “knows the model” to the same degree as the firm shareholders, i.e., expects additional capital to be raised to finance v -purchases, it is also aware of the downward pressure on loan demand due to the lower default risk, and lower resulting interest rates, making its expected profit lower in equilibrium. Such a bank would be unlikely to support collateral diversification.

Another vantage point from which the level of outside collateral v can be evaluated is that of a ruler who cares about total output but not necessarily its distribution between shareholders and debt-holders. By this, I mean a regulator who takes the earlier defined institutional constraints of production financing (i.e., the existence of shareholders, managers, and commercial and merchant banks, and their decision sets) as given, but is able to mandate the upper bound of admissible v values.⁶ For such an agent, as the results concerning expected output under different fixed v levels, as reported in Section 3.1, indicate, positive values of v would normally also be preferable to no outside collateral at all. On the other hand, if the regulator overlooks the systemic consequences of collateral funds being invested in the same type of assets (via the merchant bank in our case), she runs the risk of magnifying a systemic crisis which might emanate from, say, an adverse shock to aggregate TFP. Actually, such a regulatory oversight can easily occur since, whilst collateral in the form of the firm’s physical assets is generally regarded as highly illiquid, window-dressing v can create a powerful illusion of collateral liquidity. In this paper, the ruler preferences are not formally defined. Qualitatively, I feel confident to assume that a standard policymaker would value both high output and low aggregate losses to default, but at the current juncture I do not ask what the exact weights should be.

The agent that unambiguously gains from collateral diversification is the controlling shareholder, provided he chooses the

⁶ Accordingly, this ruler is neither omniscient nor omnipotent, i.e., it corresponds to a conventional government body with information comparable to that of the biggest private sector players, but with the authority to set rules. Later, this restricted social planner will be additionally endowed with the power to determine the type of merchant bank liabilities, cf. Section 3.3.

preferred quantity of ν in advance of all other decisions in the first period.

The incumbent shareholder has a different decision set than the manager. He takes the production decisions of the latter as given, but is free to issue new shares in excess of his own stock q^h to finance the acquisition of ν units of outside collateral. The new shareholders' contribution, q^p , compensates for funds originally diverted from physical capital purchases. Therefore, the equity capital available to the firm does not have to fall by ν , as the manager perceives it. Although the incumbent shareholder must now give up a fraction $q^p/(q^p + q^h)$ of the expected dividends, he still benefits from a lower default probability and better credit conditions for the firm. So, he would prefer non-zero collateral diversification. However, there is more than one way to decide what exact value of ν will be chosen.

The optimal level of ν does not just depend on the quantity of new equity capital issued to finance its acquisition. It also differs depending on whose perspective one takes: that of the incumbent shareholders, the new shareholders, or some coalition inside the changed shareholder set. For simplicity (see also the discussion in the next subsection), we let all the shareholders act cooperatively when choosing q and ν .

In sum, the firm owner who finances outside collateral acquisition by issuing new shares ($\nu \leq q^p$) prefers at least a small level $\nu > 0$ to $\nu = 0$. The exact size of the marginal benefit of raising ν depends on the controlling rights distribution within the firm and the equilibrium conditions of merchant bank financing (see the equilibrium taxonomy in Section 2.7). The analytical expressions for particular cases are not important for the qualitative discussion I pursue here and are therefore omitted. By showing the comparative statics of different fixed ν levels, the numerical exercises of Section 3.1 allow one to assess the quantitative gains for the incumbent shareholder. Section 3 also provides the optimal ν value from the representative shareholder perspective in the pure self-financed outside collateral case ($\nu = q^p$).

2.6. Merchant banks, equity partnerships

The basic arrangement to be considered here for the merchant bank is that of taking deposits from both firms. These deposits constitute its liability side. On the asset side, the merchant bank acquires shares of the same two firms in the form of a partnership or private equity participation. One should remember that the abstract merchant bank construction here impersonates the whole global investment banking sector. Inside this aggregate construction with its consolidated balance sheet, individual institutions hold claims on others from the same set, so that the balance sheets of the constituent parts are strongly interconnected. Shin and Shin (2011) argue that growth of these non-core bank liabilities (which also include foreign liabilities in the same non-core group) indicate a nascent credit bubble. This view can be made consistent with our own if we agree that a high weight of non-core bank liabilities is just the reverse side of concentrating non-financial corporate sector non-core (outside collateral in our terms) assets within one highly specialized branch of the financial industry, which is represented by the merchant bank in the model.

Being a big company, the merchant bank acquires a stock sufficient to influence the marginal product of capital in any firm it buys into. For simplicity, we assume a risk-neutral merchant bank, as it would be natural to expect from a manager of a large enterprise. In any case, the risk attitudes of merchant banks are not our prime concern here.

The firm is controlled by two agents: the holder of the foundation stock, which we consider an exogenous initial endowment, and the merchant bank purchasing a partnership. One can think

of many variants as to how the stock is split between the two, for example depending on their relative negotiating power. Namely, the optimal size of the private partnership from the viewpoint of the foundation stock holder is normally smaller than the optimal size from the perspective of the merchant bank (incoming partner). In order not to complicate matters with the issue of bargaining between shareholder incumbents and newcomers, we assume throughout that the two are always able to agree on the partnership size that maximizes the producer's expected profit when the amount and cost of credit (the commercial bank loan size and the lending rate) are given. This is what would happen if the representative shareholder played a symmetric information simultaneous-move game with the firm manager (recall that the latter, in turn, is assumed to take the equity capital size as given).

I assume that the merchant bank has just one other investment opportunity besides equity partnerships in the two firms. This outside investment has the form of a homogeneous asset paying a net return i^0 on a unit of investment. Since, in order not to complicate matters with the merchant bank's risk management decisions, we will deal with risk-neutral merchant banks in this paper, it is irrelevant whether i^0 is deterministic or stochastic. So, we take it to be a mean net return. Recall that the merchant bank and the incumbent shareholder take the borrowing decision of the manager as given. The initial stock q^h given exogenously, and taking into account the first-order conditions (7) of the production input optimization, they should jointly optimize the size of the merchant bank's private partnership, q^p , to satisfy the following simple first-order condition:

$$\Phi^+(A^d(q^h + q^p))(1 + r) = 1 + i^0. \quad (9)$$

Here, the default threshold A^d defined in (6) is considered a function of total equity capital $q^h + q^p = q$ (recall that physical capital is given by $k = b^k + q - \nu$, ν has been pre-defined by the shareholder, cf. Section 2.5, $b = b^m + b^k$ is chosen by the manager, and b^m is pinned down by (7b)).

When there are just two ex ante identical firms, the ν value of one becomes the q^p value of the other, and vice versa. In this paper, we restrict attention to this symmetric case.

2.7. Equilibrium

In the baseline model, there are three agent categories that are given decision variables to maximize profit: firm management optimizes labor and bank credit quantities, commercial banks optimize the lending rate given the credit demand schedules of the firms, and the merchant bank optimizes equity partnership sizes in both firms. The rest (workers and incumbent firm shareholders) are passive.

The markets whose simultaneous clearing we focus on are two for each firm: bank credit and private equity partnerships. (The word "private" assumes that there is no separate price to be considered for the latter, as opposed to the regular credit price for commercial loans.) Thus, we deal with a static general equilibrium model. In this paper, we restrict attention to symmetric equilibria with two identical firms. Accordingly, there are three endogenous variables to be determined in equilibrium: the equity partnership size, the bank loan size, and the loan interest rate.

In addition, to develop intuition about the aggregate consequences of the outside collateral option, we consider restricted equilibria in which the partnership size is limited by an exogenous upper bound. The bound becomes a parameter with which these restricted equilibria are labeled. It can vary between zero and the endogenous partnership size of the baseline equilibrium.

I also discuss an extension in which small retail equity investors are offered a certain number of firm shares in the secondary equity

market. The latter either entirely (complete outside equity financing) or partially (incomplete outside equity financing) replaces the private equity partnership of the merchant bank. In the incomplete financing variant, the merchant bank still offers deposits to firms as a source of diversified collateral, although the feasible size is generally smaller than in the baseline. Secondary market investors are risk-averse expected terminal wealth maximizers who are given a choice between the firm stock and an outside risky asset with returns imperfectly correlated with the earlier defined aggregate TFP variable.

The formal definitions are as follows.

Definition 1 ((Baseline private equity equilibrium)). The equilibrium is a vector $[b, r, q, v]$ in which

- the loan size b chosen by the firm manager optimizes the expected firm dividend given the lending rate r , the available equity capital q , and the outside collateral v , i.e., it satisfies (7a) and (7b) with labor input $m=1$, physical capital equal to $k = b - b^m + q - v$, and b^m satisfying (7c);
- the lending rate r maximizes the expected commercial bank profit (8) given the loan demand by the firm, \hat{b} , satisfying (7);
- the equity partnership q^p is chosen by the merchant bank so that the total equity capital $q = q^h + q^p$ maximizes the expected firm profit after interest, with its debt service selected by the firm manager taken as given;
- the merchant bank finances equity partnership acquisitions entirely by firm deposits: $q^p = v$.

Definition 2a ((Restricted private equity equilibrium, merchant bank deposit financing)). The equilibrium restricted by the outside collateral size \bar{v} as the only source of merchant bank financing is a vector $[b, r, q]$ in which

- the loan size b chosen by the firm manager optimizes the expected firm dividend given the lending rate r , the available equity capital q , and the outside collateral \bar{v} , i.e., it satisfies (7a) and (7b) with labor input $m=1$, physical capital equal to $k = b - b^m + q - \bar{v}$, and b^m satisfying (7c);
- the lending rate r maximizes the expected commercial bank profit (8) given the loan demand by the firm, \hat{b} , satisfying (7);
- the merchant bank finances equity partnership acquisitions entirely by firm deposits: $q^p = \bar{v}$.

Definition 2b ((Restricted private equity equilibrium, outside merchant bank financing)). The equilibrium restricted by the outside collateral size \bar{v} , but with the merchant bank able to access external sources of financing, is a vector $[b, r, q]$ in which

- the loan size b chosen by the firm manager optimizes the expected firm dividend given the lending rate r , the available equity capital q , and the outside collateral \bar{v} , i.e., it satisfies (7a) and (7b) with labor input $m=1$, physical capital equal to $k = b - b^m + q - \bar{v}$, and b^m satisfying (7c);
- the lending rate r maximizes the expected commercial bank profit (8) given the loan demand by the firm, \hat{b} , satisfying (7);
- the equity partnership q^p is chosen by the merchant bank so that the total equity capital $q = q^h + q^p$ maximizes the expected firm profit after interest, with its debt service selected by the firm manager taken as given;
- the merchant bank finances the portion of equity partnership acquisitions in excess of \bar{v} by borrowing amount q^o in world markets: $2q^p = 2\bar{v} + q^o$.

Definition 3 ((Secondary equity market equilibrium, incomplete financing)). The equilibrium parameterized by the secondary market capitalization size q^e is a vector $[b, r, q, v, p, x^e]$ in which

- the loan size b chosen by the firm manager optimizes the expected firm dividend given the lending rate r , the available equity capital q , and the outside collateral v , i.e., it satisfies (7a) and (7b) with labor input $m=1$, physical capital equal to $k = b - b^m + q - v$, and b^m satisfying (7c);
- the lending rate r maximizes the expected commercial bank profit (8) given the loan demand by the firm, \hat{b} , satisfying (7);
- a representative outside stock investor maximizes the expected utility of second-period wealth given the private equity holdings $q^h + q^p$, the debt service chosen by the firm manager, the secondary market stock price, p , and returns on alternative assets, by purchasing x^e shares in the firm;
- the equity partnership q^p is chosen by the merchant bank so that the total equity capital $q = q^h + q^e + q^p$ maximizes the expected firm profit after interest, with its debt service selected by the firm manager, as well as the secondary stock market capitalization, q^e , both taken as given;
- the secondary stock market clears: $px^e = q^e$;
- the merchant bank finances equity partnership acquisitions entirely by firm deposits: $q^p = v$.

Definition 4 ((Secondary equity market equilibrium, complete financing)). The equilibrium is a vector $[b, r, q, p, x^e]$ in which

- the loan size b chosen by the firm manager optimizes the expected firm dividend given the lending rate r and the available equity capital q , i.e., it satisfies (7a) and (7b) with labor input $m=1$, physical capital equal to $k = b - b^m + q$, and b^m satisfying (7c);
- the lending rate r maximizes the expected commercial bank profit (8) given the loan demand by the firm, \hat{b} , satisfying (7);
- a representative outside stock investor maximizes the expected utility of second-period wealth given the private equity holdings q^h equal to the incumbent equity holder's participation (the merchant bank is absent), the debt service chosen by the firm manager;
- the secondary market stock price, p , and returns on alternative assets, by purchasing x^e shares in the firm, the secondary stock market capitalization, q^e , selected by the incumbent firm shareholders, maximizes the expected firm profit after interest, with its debt service as selected by the firm manager taken as given, by supplying the appropriate number of new shares;
- the secondary stock market clears: $px^e = q^e$.

The situations covered by Definitions 1–4 are discussed in more detail in the next section. Note that the definitions do not mention the legal status of the merchant bank's liabilities. In the following, we experiment with different state-contingent provisions of the latter on the scale between standard bonds and standard equity. These give rise to different "editions" of the equilibria defined in this subsection.

3. Calculated equilibria

3.1. Baseline equilibrium with government guarantees for the merchant bank

In this section, we calculate real and financial fundamental values for selected equilibrium varieties as listed in Section 2.7. The economy is calibrated with possibly standard values of the parameters one needs to pin down the functional forms. So, the capital

Table 1

Economic fundamentals in a symmetric equilibrium with fully guaranteed deposits in the merchant bank.

Notes: The foundation equity capital of each of the two identical firms is $q^h = 2.7$. The outside investment rate of return is 5%, the same as the merchant bank's own deposit rate. The commercial bank cost of funds is 4%. Data are shown for one of the two identical firms. LGD = Loss Given Default. The last column shows the optimal private equity participation size.

The first two columns show economic fundamentals in the restricted private equity partnership equilibrium cases (ν restricted to 0 and 0.1) of Definition 2a – merchant bank deposit financing. The last column shows values for the baseline private equity partnership equilibrium (Definition 1). All three equilibria are the editions corresponding to a full government guarantee of merchant bank deposits (see Section 2.7).

ν	0	0.1	0.404991
Lending rate	0.0757712	0.0732556	0.069355
Physical capital	13.6654	13.7408	13.656000
Total equity capital	2.7	2.8	3.104991
Average gross output	17.1668	17.2473	17.156800
Working capital loans	2.55836	2.51444	2.408040
Total loans	13.5394	13.5711	13.379800
Debt service	14.56529659	14.5652591	14.30776
Survival probability, firm	0.901305	0.928249	0.981900
Default threshold, firm	0.319907	0.272727	0.143834
Expected dividend	2.64068	2.81098	3.277320
Expected merchant bank profit	0	-0.00921571	0.00445542
Default threshold, merchant bank	0.98492915	0.98295585	0.623454
LGD of merchant bank	0	5.00129	27.3482

share α in the production function is the conventional 1/3, the interest rate on deposits is 4%, and the global risk-free rate is 5%, reflecting the generically higher interest rate level outside the OECD countries. The remaining parts of the model that need calibrating are non-standard terms in producer and commercial bank costs, as defined in (2) and (8), respectively. That calibration was selected with the objective of generating plausible equilibrium values of lending rates and default frequencies. All calculations were conducted in Mathematica®.

Let us start with the case in which the merchant bank pays the agreed deposit rate regardless of the performance of its equity portfolio. For instance, this behavior can be rationally expected from it by the firm shareholders (who decide about the deposit amounts) if the government provides a full guarantee. That is, we examine, so to say, an “Irish” type of policy.

Given the outside return rate i^0 and the commercial bank cost of funds i , the simple symmetric (i.e., with two identical firms and TFP A being the common aggregate productivity shock) equilibria (both baseline and restricted, i.e., given by Definitions 1 and 2) of our model are fully characterized by pairs of lending rate r and merchant bank partnership size q^p variables jointly satisfying Eqs. (7a) and (9). The baseline equilibrium has fundamentals collected in the last column of Table 1 (all values are for one of the two identical representative firms). For comparison, in two additional columns I also show values of economic fundamentals in two cases of restricted equilibrium (Definition 2) when collateral diversification is restricted downwards away from the baseline equilibrium: one with no collateral diversification ($\nu=0$) and another with low collateral diversification ($\nu=0.1$).

Apparently, total output is not particularly affected by the diversified funds approaching their optimal size. On the other hand, the survival probability increases and the TFP default threshold decreases. This can be attractive from the viewpoint of firm shareholders, and lends strong support to the use of diversifying financial intermediary services.

An important thing to observe about the results shown in Table 1 is the merchant bank's performance. Whereas the individual firm default probabilities are less than 2% (a little higher if collateral diversification size is restricted) even when their TFP shocks are perfectly correlated, as we assume in this example, the merchant bank makes a negative profit even under a small deviation from the average TFP of unity. This fragility can be somewhat reduced when the merchant bank is allowed to raise the size of its partnership to the optimal level, but still remains incomparable with those of its

client firms: the latter safely survive when their common TFP falls to the level of 0.5, whereas the merchant bank becomes insolvent.

Insolvency of the merchant bank means that the loss must be taken by the government that provided the deposit guarantee. The expected size of the official loss conditioned on the aggregate TFP falling below the merchant bank survival threshold is shown in the last column of Table 1. Although it starts at a low level when collateral diversification and the implied leverage are low themselves (because the merchant bank's balance sheet size is proportional to the leverage), it reaches levels comparable to the economy's aggregate output as soon as collateral diversification moves toward the baseline equilibrium (with endogenously determined merchant bank balance sheet size) of the last column. If guarantees must be funded by additional tax revenue, the private sector's net loss from collateral diversification behavior would likely exceed its benefits from optimal capital structure.

If collateral diversification entails such big tail risks for the public sector, can firms do without it, in the hypothetical case of policies striving for a complete ban on outside collateral? The model suggests that the attractions of collateral diversification behavior can be quite strong. One reason is the already mentioned reduction of default frequency in sectors that diversify. Another is even more fundamental and has to do with scarce equity capital.

3.2. Retail stock market financing

Our next example concerns a pair of cases, covered by Definitions 3 and 4 of the previous section, in which the foundation equity is lower than the $q^h = 2.7$ value considered earlier. Let us allow for the existence of a standard market in the firm's shares, in which traders are small, are risk averse with negative exponential utility of final wealth, and have alternative investment opportunities besides the discussed firm stock, with an imperfect correlation of returns. The important thing is that these investors do not know the firm production function, just the statistics of its TFP, average revenue, and costs, i.e., they see the dividend defined in (4) as an affine function $Af + g$ truncated at zero due to limited liability at default, with no insight into the structure of f and g . Being small, they do not internalize the effect of their investment on the firm's earnings (as opposed to the merchant bank with its private equity position). As a first step, we would like to know what amount of equity capital is this set of traders able to provide in equilibrium.

The results for the case of two identical firms in a symmetric equilibrium (i.e., $\nu = q^p$) are shown in the first column of Table 2. We

Table 2
Economic fundamentals in the presence of secondary equity market.
Notes: q^h is the foundation equity capital of each of the two identical firms. The outside investment rate of return is 5%, the same as the merchant bank's own deposit rate. The commercial bank cost of funds is 4%. Data are shown for one of the two identical firms. q^e is the secondary stock market capitalization, x^e is the number of shares sold in the secondary market, and p is the share price.
Column 1, as well as the left sub-columns of columns 2 and 3, show economic fundamentals in the complete secondary market equity financing equilibrium (Definition 4). Values in the right sub-columns of columns 2 and 3 are for the incomplete secondary market equity financing equilibrium (Definition 3). The column 4 values are for restricted private equity financing, with the merchant bank investing excess funds outside (Definition 2b). All equilibria are the editions corresponding to a full government guarantee of merchant bank deposits (see Section 2.7).

q^h	1.0286351	2	2	2	
v	0	1	1.06316	3.06023	
	Minimum q^h for which equity finance suffices		$v = q^p$, i.e., no outside equity needed	$v = q$, max allowed	
Lending rate	0.0675991	0.0683201	0.0683651	0.069773404	
Physical capital	13.7016000	13.551	13.5417	13.2554	
Total equity capital	3.0679157	3.06337	3.06316	3.06023	
Average gross output	17.2055000	17.0446	17.0347	16.7286	
Working capital loans	2.3910200	2.38065	2.38001	2.36008	
Total loans	13.0247000	13.8683	13.9217	15.6155	
Debt service	13.9051580	14.8157836	14.873458	16.705047	
Survival probability, firm	0.9835150	0.982852	0.98281	0.981516	
Default threshold, firm	0.1379920	0.140426	0.140576	0.145183	
Expected dividend	3.3029400	3.28166	3.28035	3.23994	
Merchant bank profit		0.04251625	0.04444839	0.01849962	
Secondary equity market financing	Complete	Complete	Incomplete ($q^p - v$)	Complete	Incomplete ($q^p - v$)
q^e	2.0392800	1.06337	0.0633700	1.06316	0
p	2.0392800	2.33000	2.63417	2.32895	2.65199
x^e	1	0.456384	0.0240583	0.456497	0

see that the firm cannot be completely financed in the secondary stock market, i.e., there is a minimum positive value of foundation capital q^h for which both equity and credit markets clear. This is a variation of the classical CSV theme: investors without inside knowledge of the firm can provide only so much equity. The needed minimum q^h for the chosen stock market parameters is shown in the column heading. As soon as the available foundation stock is lower, public traders are not enough, one needs additional private equity to get the firm operating, and the merchant bank becomes indispensable. In circumstances of scarce private equity, leverage through collateral diversification becomes attractive from the private sector perspective no matter what the public authority knows or thinks about the attached risks.

The first column of Table 2 was calculated under the natural assumption that there are no private equity partnerships beside the foundation stock (i.e., $v = 0$). I call this case of stock market financing complete (cf. Definition 4). If the number of publicly traded shares is normalized to unity (the number in the last line), the penultimate two (equal) numbers of the same column give the total stock market financing and the share price. Next, let us allow for non-zero participation of the merchant bank (positive v , cf. Definition 3) in the presence of the same stock market. Since from the stock market trader perspective, there is no difference between equity provided in the form of foundation stock and a private equity partnership (due to the assumed joint optimality behavior of inside shareholders, expressed by (9), only the sum $q^i = q^h + q^p$ matters), we fix the value $q^h = 2$ for definiteness. Then, one can raise the value of v from zero to some level at which the outside stock market becomes redundant, i.e., the optimal level of equity capital $q^* = q^h + v^*$. The second and third columns of Table 2 describe the corresponding equilibria for the intermediate case of $v = 1$ and the maximum v level compatible with secondary stock trading (the exact number shown in the column heading).

Actually, the firm can now choose between raising private and public equity capital. In the lower part of Table 2 we show two corner alternatives: all-public (complete stock market financing) and residual (called incomplete in Definition 3) public stock trading.

Both alternatives are non-trivial only in intermediate cases (since $x^e = 0$ when $q^p = q^* - q^h$ and likewise $x^e = 1$ when $q^p = 0$). We see that for $v = 1$, publicly traded stock comprises less than 50% of shares in the Complete case and less than 2.5% in the Incomplete case. For obvious reasons, residual public trading results in a higher stock price than all-public trading.

Naturally, the size of the possible partnership is not limited to the value $q^* - q^h$. It can grow further, as we agreed in Section 2.2, up to the total equity level, which becomes an endogenously determined quantity. This is the case of the entire foundation capital spent on diversified collateral, whereas own production is funded by commercial bank loans. Formally, we have the restricted private equity financing equilibrium of Definition 2, but with the restriction level raised to cover the whole equity value. In fact, the amount of deposits amassed by the merchant bank is now much bigger than required for optimal equity participations. Therefore, we assume for simplicity that the merchant bank invests excess funds outside the economy at the same rate as those it pays to the firms (formally, the size of the outside loan, as mentioned in Definition 2, last bullet, becomes negative), i.e., it makes no profit on this part of its portfolio. All profits it can make in expectation come from private equity partnerships. However, with growing deposit size, servicing this liability becomes increasingly expensive, so that the expected profits fall, whereas the merchant bank default threshold in terms of aggregate TFP becomes precariously close to the average TFP value (of unity in our examples). That is, the resulting “crazy” leverage serviced by the merchant bank goes hand in hand with extreme fragility of the latter, which the regulator should prevent by all available means.

3.3. Merchant bank liabilities: guaranteed deposits vs. common equity vs. contingent capital

I go now to the third example, which concerns a change in the status of the merchant bank's claims. As mentioned in the introduction, it may be unfeasible, even though desirable in principle, to restrict merchant bank liabilities to common equity. So, I try out

Table 3

Economic fundamentals when merchant bank debt is convertible into equity.

Notes: The foundation equity capital of each of the two identical firms is $q^h = 2.7$. The outside investment rate of return is 5%, the same as the merchant bank's own deposit rate. The commercial bank cost of funds is 4%. Data are shown for one of the two identical firms. LGD = Loss Given Default. The three columns show economic fundamentals in three different editions of the baseline private equity partnership equilibrium (Definition 1 of Section 2.7).

$q^h = 2.7$	Merchant bank deposits officially guaranteed	Merchant bank liabilities in equity form only	Merchant bank deposits converted into equity when insolvent
v	0.404991	0.402676	0.598661
Lending rate	0.069355	0.069351	0.0697665
Physical capital	13.656000	13.6569	13.5779
Total equity capital	3.104991	3.102676	3.298661
Average gross output	17.156800	17.1577	17.0733
Working capital loans	2.408040	2.40809	2.40342
Total loans	13.379800	13.3807	13.2969
Debt service	14.30776	14.308665	14.224578
Survival probability, firm	0.981900	0.981904	0.981523
Default threshold, firm	0.143834	0.14382	0.145161
Expected dividend	3.277320	3.27744	3.26608
Expected merchant bank profit	0.00445542	0	0.117892
Merchant bank profit under unit TFP	0.00366253	0	0.00498064
Default threshold, merchant bank	0.623454	0	0.546639
Expected revenue on diversified collateral	0.42746826	0.425357475	0.414181
LGD of merchant bank	27.3482	0	0

a hybrid solution that mandates conversion into equity only when the merchant bank becomes insolvent. In this CoCo liability regime, the firms do not have to solve the CSV problem in a high-earning merchant bank. On the other hand, they participate in the debt workout as bona fide shareholders when the merchant bank is in distress, meaning that, in bad times, they simply receive what little the economy (including the firm itself) in aggregate was able to earn, without the additional losses associated with merchant bank dissolution under a standard bankruptcy procedure.

When I say “bad times,” I mean an intermediate outcome between failure of the merchant bank and failure of the firms. (When aggregate TFP falls below the corporate default threshold A^d , as defined by (6), everybody's earnings are zero except for the commercial banks.) As could be seen in the last column of Table 1, reproduced as the first column in Table 3, the TFP default threshold of the merchant bank is much higher, so there is a whole range of TFP-realizations under which the firms can operate, i.e., repay their loans, even if the merchant bank cannot honor its deposit rate payments.

Complete quantitative results are shown in Table 3 (for better oversight, I only discuss the baseline equilibrium of Definition 1). Beside the first column carried over from Table 1, in the second column I show the hypothetical case of the merchant bank issuing liabilities in the form of equity only. Apparently, the change of legal status of the merchant bank's liabilities has a very modest impact on major fundamentals (the interest rate, credit, investment, and average output), at the same time as it eliminates, by construction, the huge conditional liability of the government associated with the merchant bank deposit guarantee. However, as mentioned earlier, if pure equity funding of the merchant banking sector is unfeasible (for example, for CSV and other asymmetric information-related reasons), the third column shows a compromise with deposits transformed into equity only when the merchant bank does not earn enough to pay the deposits out in full. Under this contractual change, too, most economic fundamentals move only slightly. There is marginally less investment, lower expected output, and a rise in the lending rate of a couple of basis points. The survival probability of both firms imperceptibly decreases, whereas the TFP default threshold imperceptibly increases. A somewhat more tangible change is visible in the quantity of diversified collateral (it is roughly 30% higher under convertible than under guaranteed deposits). Also the default threshold of the merchant bank is visibly lower (by about 14%). Actually, when deposits are convertible,

default as such is not required, so it is better to talk about the liability transformation threshold. The expected profit of the merchant bank is also higher in the conversion case than under official guarantees (note that profit is zero by construction in an equity-funded merchant bank). Most importantly, the merchant bank LGD, comparable to the size of the economy-wide physical capital aggregate, now disappears, as does the associated contingent claim on the official bailout fund.

4. Discussion and conclusion

We defined a production economy in which efforts to diversify productivity risk on the producer (micro) level result in elevated systemic (macro) risk due to the mechanism by which collateral is transformed into private equity partnerships and concentrated in one sector of the financial industry (merchant banks) with a highly fragile balance sheet.

Merchant banks do not have to be fully competitive. They may pay fixed interest allowing for an economic profit, but still be fragile because what they pay is tied to what their depositors receive as prudential buffers. So, higher/lower buffers mean safer/riskier equity participations in the merchant bank portfolio, but have to be provided by the merchant bank itself in the form of interest payments to the same set of agents. The systemic merchant bank in this setting is not just a gainful enterprise, but also a device holding together the equilibrium in the credit market. In this position, it cannot make full use of, let alone abuse, its market power. An additional problem of interest in its own right would be that of choosing the optimal deposit rate for the merchant bank, which internalizes the impact of paid interest on the earnings of firms in its equity portfolio. I postpone this problem for future research but note that even the set of feasible deposit rates in such a problem would be relatively narrow. That is, the merchant bank is constrained in its ability to pay a low rate on its funds to such a degree that it turns out to be very moderately profitable and is forced to operate quite close to the default boundary. Its high default probability becomes a natural concern of macroprudential regulation.

Leverage stemming from collateral diversification will hardly be voluntarily reduced to zero by the non-financial private sector, since under scarce equity, its presence both provides better managerial incentives in firms and improves welfare. In certain cases, it can even be the only way to allow production financing, as standard

secondary stock market participation is limited by information barriers on the side of small shareholders.

However, what appears optimal from the micro perspective of a single enterprise can generate poorly sustainable leverage in aggregate. In principle, any amount of leverage reduces the distance to default as long as one counts on the possibility of sudden deleveraging based on a self-fulfilling collateral reappraisal. Such a reappraisal, in turn, entails a very probable solvency crisis in the merchant bank sector, since, as our examples have demonstrated, the default thresholds of the latter are much easier to attain than in a standard non-financial firm. The destiny of investment banks in the U.S. in 2007–2009 provides a good example of this.

The policy measures that are familiar to us from the latest crisis would, in our environment, roughly correspond to merchant bank bailouts by government funds in order to prevent collateral destruction. This policy entails considerable fiscal costs and soon reaches its limit, as the current sovereign solvency problem in Europe has clearly demonstrated. Accordingly, one should look for alternatives, preferably alternatives that, instead of making a futile attempt to transfer losses from sector to sector like a hot potato, would return them to their originators. This is the mechanism of collateral back-conversion into merchant bank equity with which we formally experiment in this paper. The results suggest that the formal effect of a simple legal status adjustment from plain deposits to CoCo deposits on aggregate economic indicators is likely to be of second order compared to the quantitative benefit of eliminating the contingent public sector exposure one creates by providing an across-the-board deposit guarantee.

Convertible bonds instead of government-insured deposits reduce fragility and public loss risk, but preserve both the welfare level and Townsend's (1979) CSV regularity. Quantitatively, in our model firms holding merchant bank CoCos invest and produce almost identically to the earlier government guarantee case (this is, of course, a huge simplification due to our manager risk-neutrality assumption and the primitive merchant bank balance sheet structure), but the expected fiscal costs are now zero as opposed to nearly half of GDP under guarantees.

In a small open economy, the adverse effect of international financial intermediary insolvency can be exacerbated if the real sector is the source of domestic GDP, whereas banks and their regulators are predominantly foreign, implying that they mostly care about gross investment and expected bank earnings on a consolidated basis. For this reason, macroprudential policies targeting a particular pattern of collateral diversification (in the notation of our model this is the ratio of ν to q and the structure of the portfolio in which ν is invested) can be important for systemic event propagation. In practice, explicit regulation of the balance sheet composition of global systemically important financial institutions (SIFIs) is extremely cumbersome and costly for everyone, if possible at all. Therefore, an arrangement based on conversion into common stock can simplify things enormously for small companies unable to bear legal representation costs in a multinational merchant bank resolution process. An international guarantee of their shareholder rights in the event of a SIFI insolvency is much easier. One possibility would be to delegate shareholder rights on a national principle to an official fiduciary agent. That is, instead of a long and uncertain search for a satisfactory international systemic risk containment mechanism, as one can currently observe, for example, on the G20 level, stepwise international harmonization based on support for shareholder rights seems a lot more feasible.

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