Multiobjective Stochastic Optimization Problems with Probability Constraints

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Abstract. Multiobjective optimization problems depending on a probability measure often correspond to situations in which an economic or financial process is simultaneously influenced by a random factor and a decision parameter. Moreover it is reasonable to evaluate the process by a few objective functions and it seems reasonable to determine the "decision" with respect to the mathematical expectation of objectives. Complete knowledge of the probability measure is a necessary condition to analyze the problem. However, in applications mostly the problem has to be solved on the data base. A relationship between "characteristics" obtained on the base of complete knowledge of the probability measure and those obtained on the above mentioned data base has been already investigated in the case when constraints set is not depending on the probability measure ([9], [10]). The aim of the work will be to try to relax this condition.

Keywords: Stochastic multiobjective optimization problems, (properly) efficient solution, (strongly) convex functions, empirical estimates, Lipschitz property, constraints depending on the probability measure.

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1 Introduction

To introduce a "rather general" multiobjective stochastic programming problem, let $(\Omega, \mathcal{S}, P)$ be a probability space; $\xi := \xi(\omega) = (\xi_1(\omega), \ldots, \xi_s(\omega))$ s–dimensional random vector defined on $(\Omega, \mathcal{S}, P)$; $F(:= F(z),\, z \in R^s)$, P_F and Z_F denote the distribution function, the probability measure and the support corresponding to ξ, respectively. Let, moreover, $g_i := g_i(x,\, z)$, $i = 1,\, \ldots,\, l$, $l \geq 1$ be real–valued (say, continuous) functions defined on $R^n \times R^s$; $X_F \subset X \subset R^n$ be a nonempty set generally depending on F, and $X \subset R^n$ be a nonempty "deterministic" convex set. If the symbol E_F denotes the operator of mathematical expectation corresponding to F and if for every $x \in X$ there exist finite $\mathsf{E}_F g_i(x, \xi)$, $i = 1,\, \ldots,\, l$, then a rather general "multiobjective" one–stage stochastic programming problem can be introduced in the form:

$$\text{Find} \quad \min \mathsf{E}_F g_i(x, \xi),\ i = 1,\, \ldots,\, l \quad \text{subject to} \quad x \in X_F. \tag{1}$$

The problem (1) depends on the measure P_F that usually (in applications) has to be estimated on the data base. Evidently, then an analysis have to be done with respect to an empirical problem:

$$\text{Find} \quad \min \mathsf{E}_{F^N} g_i(x, \xi),\ i = 1,\, \ldots,\, l \quad \text{subject to} \quad x \in X_{F^N}, \tag{2}$$

where F^N denotes an empirical distribution function determined by a random sample $\{\xi^i\}_{i=1}^N$ corresponding to the distribution F.

Of course by this approach we can obtain only estimates of the corresponding "theoretical" characteristics. A relationship between "theoretical" characteristics and those obtained on the data base has been already investigated in the case when the constraint set does not depend on the probability measure (see e.g. [9] and [10]). In this work we consider the case when there exist real valued convex functions

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$\bar{g}_i(:= \bar{g}_i(x),\, x \in R^n)$, $i = 1, \ldots, s$ and $\alpha_i \in (0, 1)$, $i = 1, \ldots, s$ such that

$$\text{either} \quad X_F(:= X_F(\alpha)) = \bigcap_{i=1}^{s} \{x \in X : P[\omega : \bar{g}_i(x) \le \xi_i] \ge \alpha_i\},\ \alpha = (\alpha_1, \ldots, \alpha_s), \tag{3}$$

the case (3) corresponds to a special form of individual probability constraints;

$$\begin{aligned} \text{or} \quad X_F(:= X_F(u_0, \alpha)) &= \bigcap_{i=1}^{s} \{x \in X : \min_{u^i}\{P[\omega : L_i(x, \xi) \le u^i] \ge \alpha_i\} \le u_0^i\}, \\ & \quad u_0^i > 0,\ u_0 = (u_0^1, \ldots, u_0^s),\ \alpha = (\alpha_1, \ldots, \alpha_s), \\ L_i(x, z) &= \bar{g}_i(x) - z_i,\ i = 1, \ldots, s,\ z = (z_1, \ldots, z_s), \end{aligned} \tag{4}$$

$L_i(x, z)$, $i = 1, \ldots, s$ can be considered as loss functions. This type of loss functions can appear, e.g., in connection with an inner problem of two stage stochastic (generally nonlinear) programming problems (for definition of two–stage problems see, e.g. [1]).

To analyze the problem (1), the results of the multiobjective deterministic problems will be recalled. Since, it follows from them that the results of one–objective optimization theory can be useful to investigate the relationship between the results obtained under complete knowledge of P_F and them obtained on the data base, we recall also one–objective stochastic programming problems.

2 Some Definition and Auxiliary Assertion

2.1 Deterministic Case

To recall some results of the multiobjective deterministic optimization theory we consider a multiobjective deterministic optimization problem in the following form:

$$\text{Find} \quad \min f_i(x),\ i = 1, \ldots, l \quad \text{subject to} \quad x \in \mathcal{K}, \tag{5}$$

where $f_i(x)$, $i = 1, \ldots, l$ are real–valued functions defined on R^n, $\mathcal{K} \subset R^n$ is a nonempty set.

Definition 1. The vector x^* is an efficient solution of the problem (5) if and only if there exists no $x \in \mathcal{K}$ such that $f_i(x) \le f_i(x^*)$ for $i = 1, \ldots, l$ and such that for at least one i_0 one has $f_{i_0}(x) < f_{i_0}(x^*)$.

Definition 2. The vector x^* is properly efficient solution of the multiobjective optimization problem (5) if and only if it is efficient and if there exists a scalar $M > 0$ such that for each i and each $x \in \mathcal{K}$ satisfying $f_i(x) < f_i(x^*)$ there exists at least one j such that $f_j(x^*) < f_j(x)$ and

$$\frac{f_i(x^*) - f_i(x)}{f_j(x) - f_j(x^*)} \le M. \tag{6}$$

Proposition 1. ([4]) *Let $\mathcal{K} \subset R^n$ be a nonempty convex set and let $f_i(x)$, $i = 1, \ldots, l$ be convex functions on $\mathcal{K}$. Then x^0 is a properly efficient solution of the problem (5) if and only if x^0 is optimal in*

$$\min_{x \in \mathcal{K}} \sum_{i=1}^{l} \lambda_i f_i(x) \quad \text{for some} \quad \lambda_1, \ldots, \lambda_l > 0; \quad \sum_{i=1}^{l} \lambda_i = 1.$$

A relationship between efficient and properly efficient points is presented e.g in [3] or in [4].

Remark 1. Let $f(x) = (f_1(x), \ldots, f_l(x))$, $x \in \mathcal{K}$; $\mathcal{K}^{eff}$, $\mathcal{K}^{peff}$ be sets of efficient and properly efficient points of the problem (5). If $\mathcal{K}$ is a convex set, $f_i(x)$, $i = 1, \ldots, l$ are convex functions on $\mathcal{K}$, then

$$\mathcal{K}^{peff} \subset \mathcal{K}^{eff} \subset \bar{\mathcal{K}}^{peff} \quad \text{where} \quad \bar{\mathcal{K}}^{peff} \quad \text{denotes the closure set of} \quad \mathcal{K}^{peff}. \tag{7}$$

It follows from Proposition 1 that the properties of the multiobjective optimization can be (under some assumptions) investigated by one–objective theory. To recall suitable results, let $\|\cdot\| = \|\cdot\|_n$ denote the Euclidean norm in R^n and $\Delta_n[\cdot, \cdot]$ the Hausdorff distance of subsets of R^n (for the definition of the Hausdorff distance see e.g. [13]).

Proposition 2. *Let X be a nonempty compact set. If*

1. *$\hat{g}_0 := \hat{g}_0(x)$, $x \in R^n$ is a Lipschitz function on X with the Lipschitz constant L,*
2. *$\bar{X}(v) \subset R^n$ are nonempty sets for $v \in Z_F$ and, moreover, there exists $\hat{C} > 0$ such that*

$$\Delta_n[\bar{X}(v(1)),\, \bar{X}(v(2))] \le \hat{C}\|v(1) - v(2)\| \quad \text{for} \quad v(1),\, v(2) \in Z_F,$$

then

$$|\inf_{x \in \bar{X}(v(1))} \hat{g}_0(x) - \inf_{x \in \bar{X}(v(2))} \hat{g}_0(x)| \le L\hat{C}\,\|v(1) - v(2)\| \quad \text{for} \quad v(1),\, v(2) \in Z_F.$$

Proof. Proposition 2 is a slightly modified version of Proposition 1 in [6]. □

Remark 2. An estimate of $\hat{C}$ can be found (for some special form of $\bar{g}_i$, $i = 1, \ldots, s$) in [6].

Definition 3. Let $h(x)$ be a real–valued function defined on a nonempty convex se $\mathcal{K} \subset R^n$. $h(x)$ is a strongly convex (with a parameter $\rho > 0$) function if

$$h(\lambda x^1 + (1-\lambda)x^2) \le \lambda h(x^1) + (1-\lambda)h(x^2) - \lambda(1-\lambda)\rho\|x^1 - x^2\|^2 \quad \text{for every} \quad x^1,\, x^2 \in \mathcal{K},\, \lambda \in \langle 0,\, 1\rangle.$$

Proposition 3. *[5] Let $\mathcal{K} \subset R^n$ be a nonempty, compact, convex set. Let, moreover, $h(x)$ be a continuous strongly convex (with a parameter $\rho > 0$) real–valued function defined on $\mathcal{K}$. If x^0 is defined by the relation $x^0 = \arg\min_{x \in \mathcal{K}} h(x)$, then*

$$\|x - x^0\|^2 \le \frac{2}{\rho}|h(x) - h(x^0)| \quad \text{for every} \quad x \in \mathcal{K}.$$

2.2 One–Objective Stochastic Programming Problems

To recall suitable for us assertions of one criteria stochastic optimization theory we start with the problem:

$$\text{Find} \quad \varphi(F,\, X_F) = \inf \mathsf{E}_F g_0(x,\, \xi) \quad \text{subject to} \quad x \in X_F, \tag{8}$$

where $g_0(x,\, z)$ is a real–valued function defined on $R^n \times R^s$.

First, if F and G are two s–dimensional distribution functions for which the Problem (8) is well defined and if $\mathcal{X}(F,\, X_F)$ denotes a solution set of the problem (8), then we can obtain that

$$|\varphi(F,\, X_F) - \varphi(G,\, X_G)| \le |\varphi(F,\, X_F) - \varphi(G,\, X_F)| + |\varphi(G,\, X_F) - \varphi(G,\, X_G)|. \tag{9}$$

If $g_0(x,\, z)$ is a strongly convex functions on X (with a parameter $\rho > 0$), then $\mathcal{X}(F,\, X_F)$ is singleton and consequently also

$$\|\mathcal{X}(F,\, X_F) - \mathcal{X}(G,\, X_G)\| \le \|\mathcal{X}(F,\, X_F) - \mathcal{X}(G,\, X_F)\| + \|\mathcal{X}(G,\, X_F) - \mathcal{X}(G,\, X_G)\|. \tag{10}$$

According to (9), (10) we can study separately stability of the problem (8) with respect to perturbation in the objective function and in the constraints set. To this end let F_i, $i = 1, \ldots, s$ denote one–dimensional marginal distribution functions corresponding to F and $k_{F_i}(\alpha_i)$ be defined by the relation:

$$k_{F_i}(\alpha_i) = \sup\{z_i :\, P_{F_i}\{\omega : z_i \le \xi_i(\omega)\} \ge \alpha_i\},\, \alpha_i \in (0,\, 1),\, i = 1, \ldots, s. \tag{11}$$

Proposition 4. *Let $\bar{g}_i$, $i = 1, \ldots, s$ be real-valued continuous function defined on R^n, $\alpha_i \in (0, 1)$, $i = 1, \ldots, s$, $\alpha = (\alpha_1, \ldots, \alpha_s)$. If P_{F_i}, $i = 1, \ldots, s$ are absolutely continuous with respect to the Lebesque measure on R^1, then*

$$X_F = \bar{X}(k_F(\alpha)), \quad k_F(\alpha) = (k_{F_1}(\alpha_1), \ldots, k_{F_s}(\alpha_s)),$$

where

$$\bar{X}(v) = \bigcap_{i=1}^{s} \{x \in X :\, \bar{g}_i(x) \le v_i\},\, v = (v_1, \ldots, v_s) \quad \text{in the case} \quad (3),$$
$$\bar{X}(v) = \bigcap_{i=1}^{s} \{x \in X :\, \bar{g}_i(x) - u_0^i \le v_i\},\, v = (v_1, \ldots, v_s) \quad \text{in the case} \quad (4).$$

Proof. The assertion of Proposition 4 has been proven in [6] (the case (3)) and in [8] (the case 4). $\square$

Proposition 5. *Let the assumptions of Proposition 2 and Proposition 4 be fulfilled. Then*

$$|\inf_{\bar{X}(k_F(\alpha))} \hat{g}_0(x) - \inf_{\bar{X}(k_G(\alpha))} \hat{g}_0(x)| \le L\hat{C}\|k_F(\alpha) - k_G(\alpha)\|.$$

If, moreover, $\hat{g}_0(x)$ is a strongly convex function on X, then there exists a constant $\bar{C}$ such that

$$\|\mathcal{X}(F, X_F) - \mathcal{X}(F, X_G)\|^2 \le \bar{C}\|k_F(\alpha) - k_G(\alpha)\|.$$

Proof. The first assertion of Proposition 5 follows from Proposition 2 and Proposition 4. To prove the second assertion we consider the distribution function $\bar{F}$ fulfilling the relations

$$k_{\bar{F}}(\alpha) = (k_{\bar{F}_1}(\alpha_1), \dots, k_{\bar{F}_s}(\alpha_s)), \quad k_{\bar{F}_i}(\alpha_i) = \max[k_{F_i}(\alpha_i), k_{G_i}(\alpha_i)], \, i = 1, \dots, s$$

and employ the relation (10), the first assertion and Proposition 3. $\square$

Furthermore, if we replace G by F^N we can employ the previous assertions to investigate the relationship between $\varphi(F, X_F)$, $\mathcal{X}(F, X_F)$ and $\varphi(F^N, X_{F^N})$, $\mathcal{X}(F^N, X_{F^N})$. To this end we assume:

A.2 $\{\xi^i\}_{i=1}^{\infty}$ is an independent random sequence corresponding to F; F^N is determined by $\{\xi^i\}_{i=1}^{N}$,

A.3 $P_{F_i}, i = 1, \dots, s$ are absolutely continuous w. r. t. the Lebesgue measure on R^1,

A.4 for $i \in \{1, \dots, s\}$ there exist $\delta > 0, \vartheta > 0$ such that $\bar{f}_i(z_i) > \vartheta$ for $z_i \in Z_{F_i}$, $|z_i - k_{F_i}(\alpha_i)| < 2\delta$, where $\bar{f}_i := \bar{f}_i(z_i)$, $i = 1, \dots, s$ denotes the probability density corresponding to F_i.

Proposition 6. *[7] Let $s = 1$, $\alpha \in (0, 1)$. If Assumption A.2, A.3 and A.4 are fulfilled, $0 < t' < \delta$, then*

$P\{\omega : |k_{F^N}(\alpha) - k_F(\alpha)| > t'\} \le 2\exp\{-2N(\vartheta t')^2\}, \quad N \in \mathcal{N}, \quad (\mathcal{N}$ denotes the set of natural numbers).

Proposition 7. *Let X be a convex, compact and nonempty set, $\alpha_i \in (0, 1)$, $i = 1, \dots, s$, $\alpha = (\alpha_1, \dots, \alpha_s)$, $N \in \mathcal{N}$. If assumptions A.2, A.3, A.4 and assumptions of Proposition 5 are fulfilled, then there exists a constant $C > 0$ such that*

$$P\{\omega : |\inf_{\bar{X}(k_F(\alpha))} \hat{g}_0(x) - \inf_{\bar{X}(k_{F^N}(\alpha))} \hat{g}_0(x)|| > t\} \le 2s \exp\{-2N(\vartheta t/LCs)^2\} \quad \text{for} \quad 0 < t/LCs < \delta.$$

If moreover $\hat{g}_0(x)$ is a strongly convex function on X, then there exists a constant $\bar{C}$ such that

$$P\{\omega : \|\mathcal{X}(F, X_F) - \mathcal{X}(F, X_{F^N})\|^2 > t\} \le 2s \exp\{-2N(\vartheta t/L\bar{C}s)^2\}.$$

Proof. The assertion of Proposition 7 follows from Proposition 5, Proposition 6 and the properties of the Euclidean norm. $\square$

Furthermore, we replace $g_0(x, z) := \hat{g}_0(x)$ independently of $z \in Z_F$ by more general case when

1a. $g_0(x, z)$ is a real–valued Lipschitz function on X with the Lipschitz constant L' not depending on $z \in Z_F$.

Then $\mathsf{E}_{F^N} g_0(x, \xi)$ is a Lipschitz function of $x \in X$ with the Lipschitz constant L' not depending on $\omega \in \Omega$. Consequently, according to Proposition 2 and Proposition 4 there exists C' such that

$$|\inf_{\bar{X}(k_F(\alpha))} \mathsf{E}_{F^N} g_0(x, \xi) - \inf_{\bar{X}(k_{F^N}(\alpha))} \mathsf{E}_{F^N} g_0(x, \xi)| \le L'C'\|k_F(\alpha) - k_{F^N}(\alpha)\| \quad \text{for} \quad \omega \in \Omega$$

and for $t > 0$

$$P\{\omega : |\inf_{\bar{X}(k_F(\alpha))} \mathsf{E}_{F^N} g_0(x, \xi) - \inf_{\bar{X}(k_{F^N}(\alpha))} \mathsf{E}_{F^N} g_0(x, \xi)| > t\} \le P\{\omega : L'C'\|k_{F^N}(\alpha) - k_F(\alpha)\| \ge t\}.$$

If, moreover, $g_0(x, z)$ is a strongly convex (with a parameter $\rho > 0$) function on X, then employing the proof technique from the proof of Proposition 5 we can obtan that

$$P\{\omega : \|\mathcal{X}(F, X_F) - \mathcal{X}(F, X_{F^N})\|^2 > t\} \le P\{\omega : 2L'C'\rho\|k_{F^N}(\alpha) - k_F(\alpha)\| \ge t\}$$

Furthermore, employing Proposition 7 and the relations (9), (10) we obtain

Proposition 8. *Let the assumptions A.2, A.3, A.4 and 1.a are fulfilled, $t > 0$. If there exists $\gamma \in (0,\, 1/2)$ such that*

$$P\{\omega : N^{\gamma} |\varphi(F,\, X_F) - \varphi(F^N,\, X_F)| > t\} \xrightarrow[N \to \infty]{} 0,$$

then

$$P\{\omega : N^{\gamma} |\inf_{\bar{X}(k_F(\alpha))} \mathsf{E}_F\, g_0(x,\, \xi) - \inf_{\bar{X}(k_{F^N}(\alpha))} \mathsf{E}_{F^N} g_0(x,\, \xi)| > t\} \xrightarrow[N \to \infty]{} 0.$$

Moreover, if $g_0(x,\, z)$ is for every $z \in Z_F$ a strongly convex function of $x \in X$ with a parameter $\rho >, 0$ then also

$$P\{\omega : N^{\gamma} \|\mathcal{X}(F,\, X_F) - \mathcal{X}(F^N,\, X_{F^N})\|^2 > t\} \xrightarrow[N \to \infty]{} 0.$$

3 Multiobjective Stochastic Case

To analyze stochastic multiobjective problem (1) we restrict our consideration to the case when

- B.1 $g_i(x,\, z)$, $i = 1, \dots, l$ are for every $z \in Z_F$ strongly (with a parameter $\rho > 0$) convex functions; moreover they are Lipschitz on X with the Lipschitz constant L_1 not depending on $z \in Z_F$.

Furthermore we introduce (for two s–dimensional distribution functions F, G) the sets $\mathcal{G}(F,\, X_F)$, $\bar{\mathcal{X}}(F,\, X_F)$, $\bar{\mathcal{G}}(F,\, X_F)$, Λ and the function $\bar{g}(x,\, z,\, \lambda)$ by the relations:

$$\begin{array}{lcl}
\mathcal{G}(F,\, X) & = & \{y \in R^l : y_j = \mathsf{E}_F g_j(x,\, \xi),\ j = 1, \dots, l \quad \text{for some} \quad x \in X;\ y = (y_1, \dots, y_l)\}, \\
\bar{\mathcal{X}}(F,\, X_F) & = & \{x \in X_F : x \quad \text{is a properly efficient point of the problem (1)}\}, \\
\bar{\mathcal{G}}^F(F,\, X_F) & = & \{y \in R^l :\ y_j = \mathsf{E}_F g_j(x,\, \xi),\ j = 1, \dots, l \quad \text{for some} \quad x \in \bar{\mathcal{X}}(F,\, X_F)\}, \\
\bar{\mathcal{G}}^F(G,\, X_G) & = & \{y \in R^l :\ y_j = \mathsf{E}_F g_j(x,\, \xi),\ j = 1, \dots, l \quad \text{for some} \quad x \in \bar{\mathcal{X}}(G,\, X_G)\}, \\
\Lambda & = & \{\lambda \in R^l :\ \lambda = (\lambda_1, \dots, \lambda_l),\ \lambda_i > 0,\ i = 1, \dots, l,\ \sum_{i=1}^{l} \lambda_i \ = 1\}, \\
\bar{g}(x,\, z,\, \lambda) & = & \sum_{i=1}^{l} \lambda_i g_i(x,\, z), \quad x \in R^n,\ z \in R^s,\ \lambda \in \Lambda.
\end{array} \tag{12}$$

Evidently, under the assumption B.1, the function $\bar{g}(x,\, z,\, \lambda)$ is (for every $z \in Z_F$, $\lambda \in \Lambda$) a (strongly with a parameter $\rho > 0$) convex and Lipschitz function on X with the Lipschitz constant L_1 not depending on $z \in Z_F$. Consequently, if we define the parametric optimization problem

$$\text{Find} \quad \varphi^{\lambda}(F,\, X_F) = \inf \mathsf{E}_F \bar{g}(x,\, \xi,\, \lambda)) \quad \text{subject to} \quad x \in X_F \quad \text{for} \quad \lambda \in \Lambda, \tag{13}$$

we can employ the auxiliary assertions of previous parts to analyze the Problem (1) and to see that the following Theorems are valid.

Theorem 9. *Let Assumptions B.1, A.2, A.3 and A.4 be fulfilled, X be a compact nonempty convex set, $g_i(x,\, z) := g_i^0(x)$, $x \in X$, $z \in Z_F$, $i = 1, \dots, l$, then*

$$\begin{array}{rcl}
P\{\omega : \Delta_l[\mathcal{G}(F,\, X_F),\, \mathcal{G}(F^N,\, X_{F^N})] \longrightarrow_{N \longrightarrow \infty} 0\} & = & 1, \\
P\{\omega : \Delta_n[\bar{\mathcal{X}}(F,\, X_F),\, \bar{\mathcal{X}}(F^N,\, X_{F^N})] \longrightarrow_{N \longrightarrow \infty} 0\} & = & 1, \\
P\{\omega : \Delta_l[\bar{\mathcal{G}}^F(F,\, X),\, \bar{\mathcal{G}}^F(F^N,\, X_{F^N})] \longrightarrow_{N \longrightarrow \infty} 0\} & = & 1.
\end{array}$$

Remark 3. It follows (under the assumptions of Theorems 9) that asymptotic properties, in the case of constraints sets fulfilling the relation (3), do not depend on the tails of distributions (including all stable case).

Theorem 10. *Let B.1, A.2, A.3 be fulfilled, X be a compact nonempty set, $t > 0$. If there exists $\gamma \in (0,\, 1/2)$ such that*

$$P\{\omega : N^{\gamma} |\inf_{X_F(\alpha)} \mathsf{E}_F g_i(x,\, \xi) - \inf_{X_F(\alpha)} \mathsf{E}_{F^N} g_i(x,\, \xi)| > t\} \longrightarrow_{N \longrightarrow \infty} = 0, \tag{14}$$

then

$$
\begin{aligned}
P\{\omega : N^{\gamma}\, \Delta_l[\mathcal{G}(F,\, X),\, \mathcal{G}(F^N,\, X_{F^N})] > t\} \quad & \longrightarrow_{N \longrightarrow \infty} \quad = \quad 0. \\
P\{\omega : N^{\gamma/2}\, \Delta_n[\bar{\mathcal{X}}(F,\, X),\, \bar{\mathcal{X}}(F^N,\, X_{F^N})] > t\} \quad & \longrightarrow_{N \longrightarrow \infty} \quad = \quad 0, \\
P\{\omega : N^{\gamma/2}\, \Delta_l[\bar{\mathcal{G}}^F(F,\, X),\, \bar{\mathcal{G}}^F(F^N,\, X_{F^N})] > t\} \quad & \longrightarrow_{N \longrightarrow \infty} \quad = \quad 0.
\end{aligned}
\tag{15}
$$

If, moreover, there exist functions $\hat{g}_i^0(x)$, $\imath = 1, \ldots, l$ defined on R^n such that $g_i(x,\, z) = \hat{g}_i^0(x)$, $i = 1, \ldots,$ then the relations (15) are valid with $\gamma \in (0,\, 1/2)$.

Remark 4. It follows from Theorems 9, 10 that asymptotic properties, in the case of constraints sets fulfilling the relation (3) or (4) and the condition $g_i(x,\, z) := g_0^i(x,\, z)$, $i = 1, \ldots, l$ (fulfilling the condition 1.a) do not depend on the tails of distributions (including stable distributions). To this end it is assumed that the one-dimensional marginals are absolutely continuous with respect to Lebesque measure in R^1. Practically this assertion follows from the old results of [2] or [14]. The distribution with heavy tails appear very often in economic and financial applications, see e.g. [11], [12].

4 Conclusion

The paper deals with multiobjective stochastic programming problems, especially with a relationship between characteristics of these problems corresponding to complete knowledge of the probability measure and those determined on the data base. However, in spite of the former papers there the assumption of "deterministic" constraints set is rather relaxed. On the other hand we still assume that the objective functions are strongly convex with respect to $x \in X$. Evidently linear functions do not fulfil this assumptions. It seems (under the corresponding analysis) that similar assertions are also valid in the linear case. However, a detailed investigation in this direction is beyond the scope of this paper.

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