



# Leverage effect in energy futures



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## ABSTRACT

We propose a comprehensive treatment of the leverage effect, i.e. the relationship between returns and volatility of a specific asset, focusing on energy commodities futures, namely Brent and WTI crude oils, natural gas and heating oil. After estimating the volatility process without assuming any specific form of its behavior, we find the volatility to be long-term dependent with the Hurst exponent on a verge of stationarity and non-stationarity. To overcome such complication, we utilize the detrended cross-correlation and the detrending moving-average cross-correlation coefficients and we find the standard leverage effect for both crude oils and heating oil. For natural gas, we find the inverse leverage effect. Additionally, we report that the strength of the leverage effects is scale-dependent. Finally, we also show that none of the effects between returns and volatility is detected as the long-term cross-correlated one. These findings can be further utilized to enhance forecasting models and mainly in the risk management and portfolio diversification.

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## 1. Introduction

The leverage effect is one of the well-established phenomena of the financial economics. Historically, Black (1976) discusses a possible relationship between returns and changes in volatility of stocks. The argumentation is based on changes in earnings, where decreasing expected earnings of the company push the price down and in turn it decreases the market value of the company which drives the leverage (ratio between debt and equity) up. Negative relationship between returns and volatility is thus referred to as ‘the leverage effect’. However, in the modern, high-speed, markets where the market prices of assets are driven by many more forces than simple expected earnings, such an explanation of the effect serves as just a little more than an anecdote. The leverage effect can be simply understood as a negative relationship between returns and volatility which are driven by opposite forces. When negative news reaches the market, volatility of the corresponding asset usually increases because of an uncertain future development. Contrarily, the negative news drives the prices down forming a negative return. The leverage effect thus seems a natural connection of the two characteristics (returns and volatility) of the traded assets.

The leverage effect is usually tightly connected, and sometimes even interchanged, with a notion of the asymmetric volatility. The standard asymmetric volatility is characterized by a lower volatility connected

to a bull (growing) market and a higher volatility connected to a bear (declining) market. The definition and interconnection between the two effects – the leverage effect and the asymmetric volatility – are thus very close and sometimes hard to distinguish between. Nonetheless, most authors agree on several characteristics of the relationship between returns and volatility – returns and volatility are negatively correlated, the correlation is quite weak yet still persists over quite long time (with slowly decaying cross-correlations), and the causality goes from returns to volatility and not vice versa (Bollerslev et al., 2006; Bouchaud and Potters, 2001; Bouchaud et al., 2001; Pagan, 1996).

Here we analyze the leverage effect in the future contracts of energy commodities, namely WTI and Brent crude oils, natural gas and heating oil. We try to provide a coherent treatment of the leverage effect starting from the long-term memory characteristics of volatility and its potential non-stationarity, then moving to the estimation of the correlation between returns and volatility under borderline (non-)stationary and a typical seasonality of futures contracts, and finally checking the slow decay of the cross-correlation function characteristic for long-range cross-correlated processes. We find that the leverage effect in its purest form (significant negative correlation between returns and volatility) is found for three out of four studied commodities. For the crude oil futures, the level of correlations is comparable to values found for other financial assets whereas the heating oil futures are characterized by a weaker effect. Interestingly, we find that the strength of the leverage effect is scale-dependent, i.e. the correlation coefficients vary across scales, which opens a potential new topic of research.

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Additionally, we show that the cross-correlations are not identified as hyperbolically decaying, i.e. there are no long-range cross-correlations between returns and volatility of the studied commodities. An important aspect of our analysis stems in not assuming anything about the relationship between returns and volatility which distinguishes our study from the other studies which are majorly built around assuming some kind of asymmetric volatility model (the leverage effect and asymmetric volatility are assumed *ex ante* to be frequently found *ex post* there).

The paper is structured as follows. In [Section 2](#), we provide a literature review of recent studies on the leverage effect and asymmetric volatility on energy markets. [Section 3](#) introduces the most important methodological aspects of our work – volatility estimation, long-term memory and its tests and estimators, estimation of correlations under borderline (non-)stationarity and seasonality, and long-range cross-correlations testing. [Section 4](#) presents the analyzed dataset and results. [Section 5](#) concludes.

## 2. Literature review

In this section, we review recent literature on the topic of leverage effect and asymmetric volatility in energy commodities in chronological order.

[Fan et al. \(2008\)](#) examine WTI and Brent crude oil prices with various specifications of the generalized autoregressive conditional heteroskedasticity (GARCH) models for purposes of risk management. They find significant two-way spillover effect between both crude oil markets as well as asymmetric leverage effect in the WTI returns but not in the Brent returns. Interestingly, the uncovered leverage effect implies that positive shocks have much higher impact on the future dynamics of the series than the negative ones which is opposite to the leverage effect found in stocks and it can be thus treated as an inverse leverage effect.

[Zhang et al. \(2008\)](#) study an interrelation between the US dollar exchange rates and crude oil prices with a special focus on spillover effects which they separate into three – mean spillover, volatility spillover and risk spillover. Apart from a significant long-term cointegration relationship, the authors find significant volatility asymmetry. In a similar way to the previous reference, they find the inverse leverage effect which they attribute mainly to the non-renewable property of oil and very different roles and behavior of suppliers and demanders of the commodity.

[Aloui and Jammazi \(2009\)](#) examine the relationship between crude oil and stock markets utilizing a two regime Markov switching exponential GARCH model. They show that the volatility clustering and the leverage effect can be significantly reduced by allowing for the regime switching. Transition between regimes is mainly connected to economic recessions together with stock market behavior. [Agnolucci \(2009\)](#) compares predictive powers of GARCH-type and implied volatility models on the WTI future contract. Apart from showing that the GARCH-type models outperform the implied volatility models, the author also finds no leverage effect for the WTI contract. [Cheong \(2009\)](#) then focuses on both WTI and Brent crude oil markets and applies GARCH specification. The author finds that the WTI volatility is more persistent than the one of the Brent crude oil. Even though the leverage effect is found for the Brent market and not for the WTI market, the out-of-sample forecasting exercise provides an evidence that a reduced GARCH model with no asymmetric volatility outperforms the others.

[Wei et al. \(2010\)](#) study both the WTI and Brent futures and compare a wide portfolio of GARCH-type models. Focusing on the performance of 1-day, 5-day and 20-day forecasting, they find that no single model is a clear winner in the horse race of testing. However, the authors favor the non-linear specifications of GARCH which can control for long-term memory as well as asymmetry. Similar to the previous studies, the results on asymmetry are mixed for the two markets. Even though

the asymmetry is found for a strong majority of specifications for the Brent market, the WTI shows mixed evidence.

[Chang and Su \(2010\)](#) focus on the relationship between crude oil and biofuels. Specifically, they are interested in the dynamics of volatility (using the exponential GARCH model) conditional on various phases of the market with respect to the crude oil prices. A significant asymmetric volatility reaction is found only for the soybean futures during the high oil prices. Other futures show no significant asymmetry. [Du et al. \(2011\)](#) examine the linkage between the crude oil volatility and agricultural commodity markets using the stochastic volatility approach in the Bayesian framework. The authors show that speculation, scalping and petroleum investors form important aspects of the volatility formation. In the model, they find a weak leverage effect between instantaneous volatility and prices.

[Reboredo \(2011\)](#) inspects the crude oil dependence structure with various copula functions. He shows that the correlation structure is similar during both bear and bull markets and further states that the crude oil market is strongly globalized. For the favored model of the marginals – exponential GARCH – the volatility asymmetry is found for all studied crude oil series. The same methodology is then applied in [Reboredo \(2012\)](#) where the relationship between oil price and exchange rates is examined. In general, the connection between the oil and exchange rate markets is reported to be very weak. The evidence of volatility asymmetry is mixed as well. [Wu et al. \(2012\)](#) propose a copula-based GARCH model and use it to model dependence between crude oil and the US dollar. In their specification, the leverage effect is not significant for either of the studied futures.

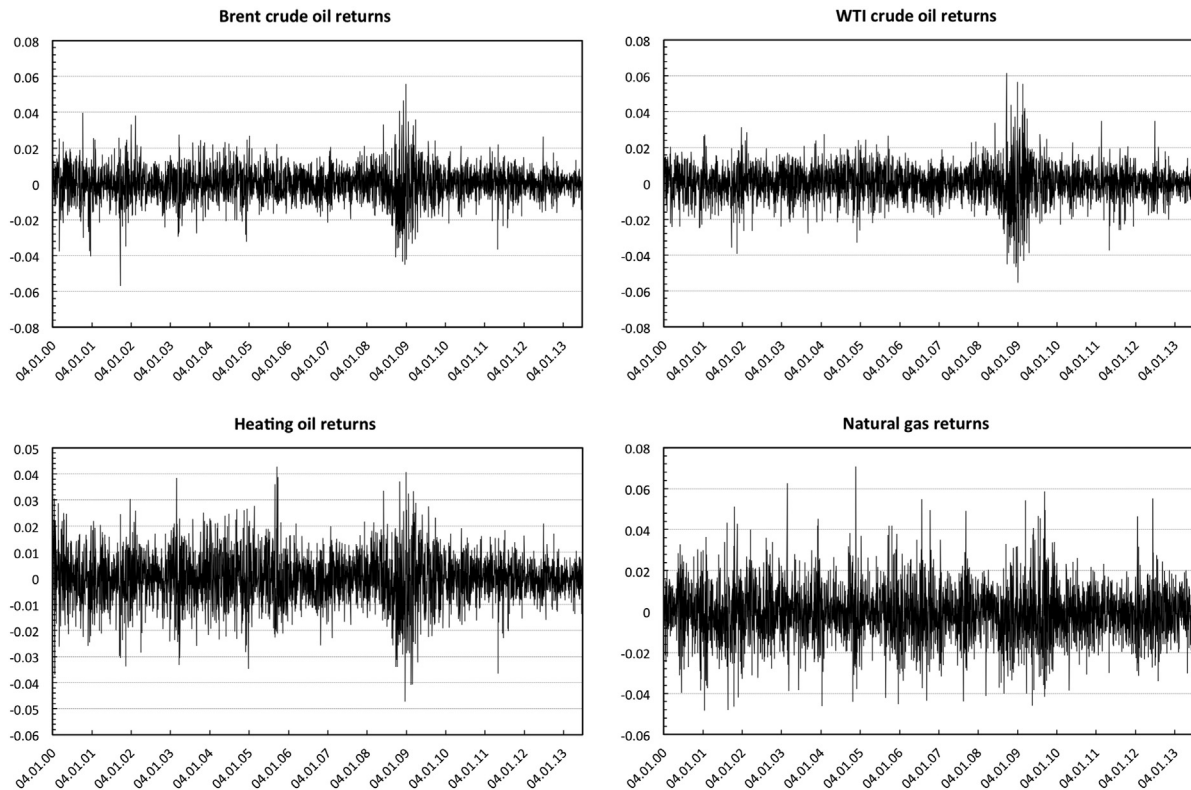
[Chang \(2012\)](#) employs a combined regime switching exponential GARCH model with Student-*t* distributed error terms to model crude oil futures returns. The model is able to capture the main stylized facts of the crude oil futures. Importantly, the model combines both the regime switching and asymmetric volatility to capture nonlinear dependencies between returns, volatility and higher moments. In accordance to other works, no leverage effect is found for the WTI futures.

[Ji and Fan \(2012\)](#) analyze the effect of crude oil volatility spillovers on non-energy commodities. After controlling for exchange rates, the authors utilize a bivariate exponential GARCH model with time-varying correlation structure. They show that the crude oil plays a core role in the commodities structure as its volatility spills over to other, non-energy, markets as well. The strength of these spillovers even increases after the 2008 financial crisis. Volatility asymmetry is studied as a difference in reaction to bad and good news. The authors find the effect to be significant for majority of the studied pairs.

[Nomikos and Adriosopoulos \(2012\)](#) investigate dynamics of eight energy spot markets on NYMEX. The authors combine a mean-reverting and a spike model with GARCH-type time-varying volatility focusing on risk management issues as well as their forecasting performance. The leverage effect is found for WTI, heating oil and heating oil-WTI crack spread, and the inverse leverage effect is uncovered for gasoline, natural gas, propane and gasoline-WTI crack spread.

Copulas are further utilized by [Tong et al. \(2013\)](#) who study tail dependence between crude oil and refined petroleum markets. Positive dependence is found in both tails so that the markets tend to move together in both bear and bull periods. Asymmetry in tail dependence is found between crude and heating oils, and between crude oil and jet fuel. Interestingly, the upper tail dependence is stronger than in the lower tail for the pre-crisis period. The authors report that the leverage effect, which is found in its standard form, is much stronger for the post-crisis period.

[Salisu and Fasanya \(2013\)](#) study the WTI and Brent crude oil with respect to the structural breaks while controlling for potential volatility asymmetry. Persistence as well as asymmetry of volatility is reported even after controlling for two structural breaks (Iraqi/Kuwait conflict



**Fig. 1.** Returns of energy futures. The dynamics of the analyzed futures follow standard patterns of financial returns, mainly non-Gaussian distribution, heavy tails and volatility clustering.

and the financial crisis of 2008) identified for both oil markets. The authors stress that neither of the effects should be studied separately and the constructed models should consider each of structural breaks, volatility persistence and asymmetry.

And Chkili et al. (2014) examine crude oil, natural gas, gold and silver markets using various linear and nonlinear GARCH-type specifications. The nonlinear specifications are found to fare better in a sense of in-sample and out-of-sample performance as well as risk management issues under the Basel II regulations. The direction and significance of the leverage effect are found to be strongly dependent on the model choice.

### 3. Methodology

Studying the leverage effect stems primarily in the analysis of the relationship between returns and volatility of the series. As such, this is connected with several issues. Firstly, the volatility itself needs to be extracted from the series. Secondly, the volatility is standardly considered as a long-term memory process. Thirdly, not only is the volatility process long-term dependent but also it is usually on the edge of stationarity, i.e. its fractional integration parameter  $d \approx 0.5$  and it is thus somewhere between a stationary short-term memory process with  $d = 0$  and a unit root process with  $d = 1$ . In this section, we introduce methodology and instruments that are used to deal with such specifics. Firstly, we describe the Garman–Klass estimator as an efficient estimator of the daily volatility which does not require availability of high-frequency data. Secondly, the concept of the long-term memory processes is discussed in some detail together with several of its tests and estimators to ensure that the results are robust. In the Data and results section, it is shown that the volatility processes are strongly persistent and on the edge of non-stationarity. For these purposes, we introduce methods that are able to efficiently estimate the correlation coefficient even for such series. The simplest leverage effect

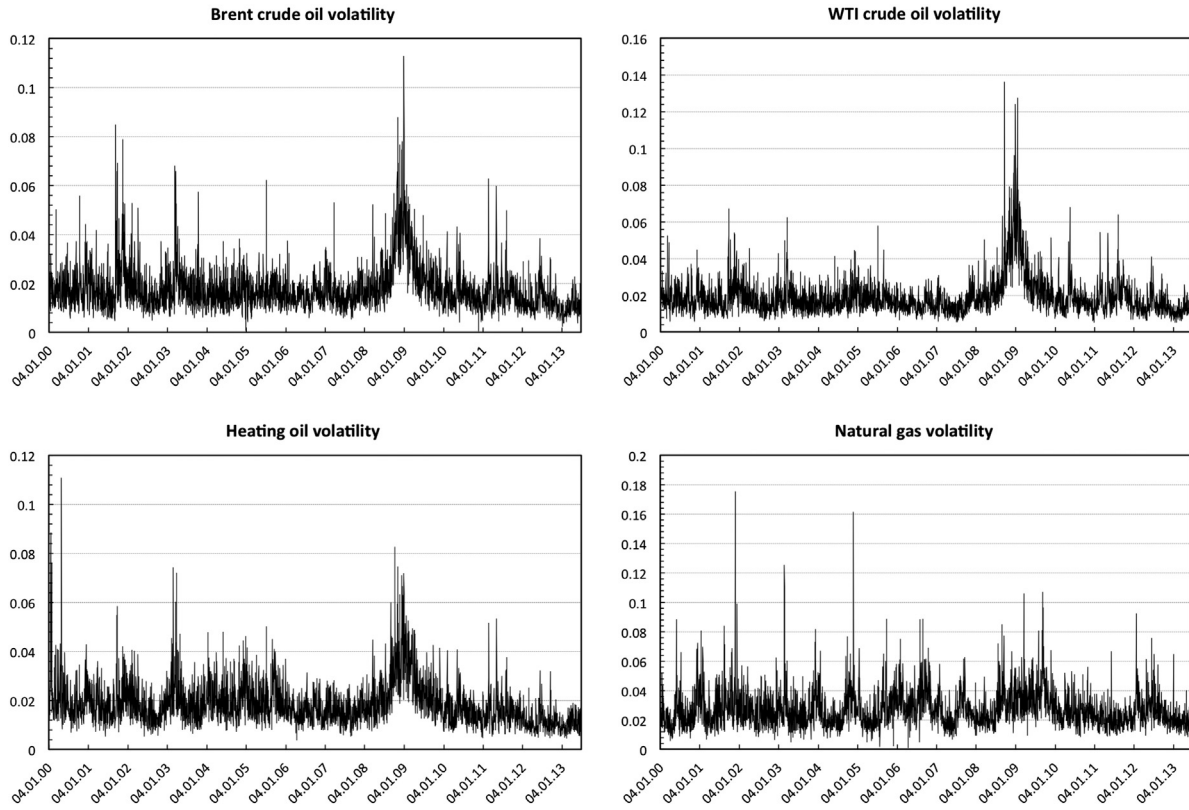
characteristic, i.e. the correlation between returns and volatility, is then estimated using these methods. And thirdly, we present the method for testing whether the leverage effect persists and decays slowly in time to give a more complete picture of the effect.

#### 3.1. Volatility estimation

In majority of the leverage effect and asymmetric volatility studies covered in the Literature review section, the volatility process has been estimated as a part of the complete model under various assumptions and restrictions. In turn, the volatility series and its characteristics are strongly dependent on the model choice and specifications. For our purposes, the leverage effect emerges from the model only if we assume correlation between the returns and volatility processes. However, if the effect is in reality not present, it can simply occur to be significant during the estimation procedure due to the model misspecification. In our study, we bypass this issue by estimating the volatility outside the returns model.

Historically, the volatility and variance series were estimated simply as a squared or absolute returns of the series. In a sense, the GARCH-type models are built in the same logic. However, these simple measures turn out to be very poor estimators of the true volatility (Chou et al., 2010). Range-based estimators of volatility turn out to be much more efficient and precise than the absolute and squared returns and they stay close to the most efficient realized variance family measures.<sup>1</sup>

<sup>1</sup> We do not opt for the realized variance family measures due to their need of high-frequency data. Moreover, our study is a study of the relationship between returns and volatility, not of finding the best measure of volatility. The range-based estimators are in turn a very fitting compromise as these need only daily open, close, high and low prices which are freely available for practically all publicly traded assets.



**Fig. 2.** Volatility of energy futures. Volatility follows persistent behavior with strongly varying levels. Again, the dynamics is well in hand with the standard volatility dynamics of financial assets.

From several possibilities, we select the Garman–Klass (GK) estimator as a highly efficient estimator of daily variance. The estimator is defined as

$$\widehat{\sigma}_{GK,t}^2 = \frac{(\log(H_t/L_t))^2}{2} - (2\log 2 - 1)(\log(C_t/O_t))^2 \quad (1)$$

where  $H_t$  and  $L_t$  are daily highs and lows, respectively, and  $C_t$  and  $O_t$  are daily closing and opening prices, respectively (Garman and Klass, 1980). As the estimator does not take the overnight volatility into consideration, we further work with the open–close returns, i.e.  $r_t = \log(C_t) - \log(O_t)$ .

### 3.2. Long-term memory

Long-term memory (long memory, long-range dependence) is connected to specific features of the series in both time and frequency domains. In the time domain, the long-term memory process has asymptotically power-law decaying autocorrelation function  $\rho(k)$  with lag  $k$  such that  $\rho(k) \propto k^{2H-2}$  for  $k \rightarrow +\infty$ . In the frequency domain, the long-term memory process has divergent at origin spectrum  $f(\lambda)$  with frequency  $\lambda$  such that  $f(\lambda) \propto \lambda^{1-2H}$  for  $\lambda \rightarrow 0+$ . In both definitions, the Hurst exponent  $H$  plays a crucial role. For stationary series,  $H$  is standardly bounded between 0 and 1 so that  $0 \leq H < 1$ . No long-term memory is connected to  $H = 0.5$ , positive long-term autocorrelations are found for  $H > 0.5$  and negative ones for  $H < 0.5$ . The Hurst exponent is connected to the fractional differencing parameter  $d$  in a strict way –  $H = d + 0.5$  (Beran, 1994).

Hurst exponent is crucial for our further analysis. However, before estimating the exponent itself, we need to test the series for actually being long-range dependent. It has been shown that the estimators of Hurst exponent might report values different from 0.5, and thus hinting

long-term memory, even if the series are not long-range dependent (Barunik and Kristoufek, 2010; Couillard and Davison, 2005; Jiang et al., 2014; Kristoufek, 2010, 2012; Lennartz and Bunde, 2009; Taqqu and Teverovsky, 1996; Taqqu et al., 1995; Teverovsky et al., 1999; Weron, 2002; Zhou, 2012). To deal with this matter, we firstly test for the presence of long-range dependence in the series before estimating the Hurst exponent. We opt for two tests – modified rescaled range and rescaled variance.

The modified rescaled range test (Lo, 1991) is an adjusted version of the traditional rescaled range test (Hurst, 1951) controlling for short-term memory of the series. The testing statistic  $V$  is defined as

$$V_T = \frac{(R/S)_T}{\sqrt{T}} \quad (2)$$

where the range  $R$  is defined as a difference between the maximum and the minimum of the profile (cumulative demeaned original series),  $S$  is the standard deviation of the series and  $T$  is the time series length. Here  $(R/S)_T$  is the rescaled range of the series of length  $T$ . To control for the potential short-term memory bias (strong short-term memory might be mistaken for the long-term memory), the standard deviation  $S$  is used in its heteroskedasticity and autocorrelation consistent (HAC) version. For these purposes, we utilize the following specification which is later used in the bivariate setting and the rescaled covariance test as well:

$$\widehat{s}_{xy,q} = \sum_{k=-q}^q \left(1 - \frac{|k|}{q+1}\right) \widehat{\gamma}_{xy}(k) \quad (3)$$

where  $\widehat{\gamma}_{xy}(k)$  is a sample cross-covariance at lag  $k$ ,  $q$  is a number of lags taken into consideration and the cross-covariances are weighted with the Barlett-kernel weights. For the purposes of the modified rescaled



**Table 1**  
Descriptive statistics.

		Brent crude oil	WTI crude oil	Heating oil	Natural gas
Raw returns	Mean	0.0000	0.0001	0.0001	−0.0006
	SD	0.0089	0.0094	0.0092	0.0131
	Skewness	−0.2045	−0.1524	−0.0618	0.2035
	Ex. kurtosis	3.0454	3.5389	1.6150	1.6458
	Jarque–Bera	1358	1770	368	403
	p-Value	<0.01	<0.01	<0.01	<0.01
	Q(30)	56.5784	93.2582	48.8078	55.6547
	p-Value	<0.01	<0.01	0.0160	<0.01
	ADF	−9.3723	−8.9838	−9.0643	−12.7284
	p-Value	<0.01	<0.01	<0.01	<0.01
	KPSS	0.0729	0.2095	0.0936	0.4147
	p-Value	>0.1	>0.1	>0.1	0.0700
	Mean	0.0189	0.0313	0.0004	−0.0217
	SD	0.4530	0.4359	0.4569	0.4342
Standardized returns	Skewness	0.0051	0.0032	0.0046	0.0536
	Ex. kurtosis	−0.3861	−0.5599	−0.4995	−0.5268
	Jarque–Bera	21	44	35	41
	p-Value	<0.01	<0.01	<0.01	<0.01
	Q(30)	38.8799	49.0375	37.5385	38.4963
	p-Value	0.1280	0.0160	0.1620	0.1370
	ADF	−13.4725	−9.9805	−8.9793	−12.1317
	p-Value	<0.01	<0.01	<0.01	<0.01
	KPSS	0.1994	0.1133	0.0604	0.7664
	p-Value	>0.1	>0.1	>0.1	<0.01
	Mean	−4.1419	−4.0664	−4.0993	−3.7100
	SD	0.4612	0.4348	0.4422	0.4448
	Skewness	0.0974	0.5432	0.1736	−0.0990
	Ex. kurtosis	2.0910	0.9238	0.2861	1.4772
Logarithmic volatility	Jarque–Bera	634	285	28	311
	p-Value	<0.01	<0.01	<0.01	<0.01
	Q(30)	11,663	16,121	14,720	7948
	p-Value	<0.01	<0.01	<0.01	<0.01
	ADF	−4.1167	−4.1354	−3.77728	−5.2721
	p-Value	<0.01	<0.01	<0.01	<0.01
	KPSS	2.0720	1.0965	5.2119	0.7198
	p-Value	<0.01	<0.01	<0.01	0.013

range, we set  $S \equiv \widehat{s_{xx,q}}$  as the autocovariance function is symmetric. We follow the suggestion of Lo (1991) and use lag  $q$  according to the following formula for the optimal lag:

$$q^* = \left(\frac{3T}{2}\right)^{\frac{1}{3}} \left(\frac{2|\widehat{\rho(1)}|}{1-\widehat{\rho(1)}^2}\right)^{\frac{2}{3}} \quad (4)$$

where  $\widehat{\rho(1)}$  is a sample first order autocorrelation and  $\cdot^{\frac{1}{3}}$  is the lower integer operator. Under the null hypothesis of no long-range dependence, the statistic is distributed as

$$F_V(x) = 1 + 2 \sum_{k=1}^{\infty} (1 - 4k^2 x^2) e^{-2(kx)^2}. \quad (5)$$

As an alternative to the modified rescaled range test, Giraitis et al. (2003) propose the rescaled variance test which simply substitutes the range in Eq. (2) by variance of the profile. The testing statistic  $M$  is then defined as

$$M_T = \frac{\text{var}(X)}{TS^2},$$

where  $X$  is the profile of the original series and the standard deviation  $S$  is defined in the same way as for the modified rescaled range test. Giraitis et al. (2003) show that the rescaled variance test has better properties than the modified rescaled range which is also supported by previous results of Lee and Schmidt (1996) and Lee and Amsler

(1997). Under the null hypothesis of no long-term memory, the statistic is distributed as

$$F_M(x) = 1 + 2 \sum_{k=1}^{\infty} (-1)^k e^{-2k^2 \pi^2 x}. \quad (6)$$

For the estimation of the Hurst exponent itself, we utilize two frequency domain estimators – the local Whittle estimator and the GPH estimator – and two time domain estimators – detrended fluctuation analysis (DFA) and detrending moving average (DMA) methods. The frequency domain estimators have well defined asymptotic properties and are well suited even for non-stationary or boundary series which turns out to be the case for the analysis we present. The time series estimators are used as a further robustness check as they do not assume any specific functional form of the analyzed process.

Robinson (1995) proposes the local Whittle estimator as a semi-parametric maximum likelihood estimator using the likelihood of Künsch (1987) while focusing only on a part of spectrum near the origin. As an estimator of the spectrum  $f(\lambda)$ , the periodogram  $I(\lambda)$  is utilized. For the time series of length  $T$ , and setting  $m \leq T/2$  and  $\lambda_j = 2\pi j/T$ , the Hurst exponent is estimated as

$$\hat{H} = \text{argmin}(R(H)) \quad (7)$$

where

$$R(H) = \log \left( \frac{1}{m} \sum_{j=1}^m \lambda_j^{2H-1} I(\lambda_j) \right) - \frac{2H-1}{m} \sum_{j=1}^m \log \lambda_j. \quad (8)$$

**Table 2**  
Long-term memory tests.

		Brent crude oil	WTI crude oil	Heating oil	Natural gas
Raw returns	$V_T$	1.4603	1.5970	0.7816	1.6995
	$p$ -Value	>0.1	>0.1	>0.1	0.0564
	$M_T$	0.0742	0.1141	0.0218	0.1809
	$p$ -Value	>0.1	>0.1	>0.1	0.0551
	$q^*$	2	1	0	4
Standardized returns	$V_T$	1.5398	1.5729	1.1521	2.0182
	$p$ -Value	>0.1	>0.1	>0.1	0.0171
	$M_T$	0.1058	0.0970	0.0663	0.2520
	$p$ -Value	>0.1	>0.1	>0.1	0.0281
	$q^*$	2	2	1	2
Logarithmic volatility	$V_T$	2.6776	2.8426	3.2812	2.6474
	$p$ -Value	<0.01	<0.01	<0.01	<0.01
	$M_T$	0.6971	0.5708	0.8271	0.4882
	$p$ -Value	<0.01	<0.01	<0.01	<0.01
	$q^*$	18	20	19	15

Geweke and Porter-Hudak (1983) introduce an estimator based on a full functional specification of the underlying process as the fractional Gaussian noise, which is labeled as the GPH estimator after the authors. The assumption of the underlying process is connected to a specific spectral density which is in turn utilized in the regression estimation of the following equation:

$$\log I(\lambda_j) \propto -(H-0.5) \log [4 \sin^2(\lambda_j/2)]. \quad (9)$$

Both estimators are consistent and asymptotically normal. To avoid bias due to short-term memory, we estimate both the local Whittle and GPH estimators only on parts of the estimated periodogram that are close to the origin (short-term memory is present at high frequencies and thus far from the origin). Specifically, we use  $m = T^{0.6}$ .

Detrended fluctuation analysis (Peng et al., 1993, 1994) is historically the most popular time domain estimator of Hurst exponent. The method is based on the power-law decaying autocorrelation of the long-term correlated series as the decay implies a power-law behavior of the detrended sums of the long-term correlated series. In the procedure, a profile of the series is constructed as a cumulative sum of a demeaned original series. The profile of length  $T$  is then split into  $n$  boxes of length  $s$ . In each box of length  $s$ , the profile is detrended and mean squared error is obtained. For each box size  $s$ , the average mean squared error  $F_{DFA}^2(s)$  is calculated and Hurst exponent is obtained from the regression on a scaling law  $F_{DFA}^2(s) \propto s^{2H}$ . There are various versions of the procedure using different types of detrending, minimum and maximum scales, and the overlapping windows are treated differently. In our specific case, we opt for  $s_{min} = 10$  and  $s_{max} = T/5$ , which are used for the final regression getting Hurst exponent  $H$ , and we use non-overlapping windows. If the time series length  $T$  is not divisible by  $s$ , we calculate averages based on boxes of length  $s$  taken both from the beginning and the end of the series. To obtain the standard errors of the estimate, we utilize the jackknife procedure so that we vary the minimal scale  $s_{min}$  from 10 to 100. The actual estimate is then taken as a simple average of the jackknife estimates and the standard error as their standard deviation. For more details about DFA, please refer to Kantelhardt et al. (2002) and Shao et al. (2012).

Detrending moving average (Alessio et al., 2002; Vandewalle and Ausloos, 1998) approaches the detrending differently than DFA as it does not split the series into boxes but uses the scaling of the mean squared errors with the length of the moving average window. Specifically, the profile of the series is detrended with the moving average of length  $\lambda$  and mean squared error  $F_{DMA}^2(\lambda)$  is obtained for each  $\lambda$ . Hurst exponent is then obtained from the regression on  $F_{DMA}^2(\lambda) \propto \lambda^{2H}$ . There are again various specifications of the method mainly based on different types of moving averages. In our analysis, we opt for the centered moving average which has been shown to outperform the other options (Carbone and Castelli, 2003; Shao et al., 2012). Again,

the scaling range needs to be constrained and for that matter, we opt for  $\lambda_{min} = 11$  and as  $\lambda_{max}$ , we use the odd number closest to the fifth of the time series length. Odd numbers are needed for the centered moving average and the specific values are selected to be comparable with the DFA estimates. The jackknife procedure is utilized again and the minimum moving average window length is manipulated between 11 and 101.

### 3.3. Correlation coefficient for non-stationary series

As the leverage effect can be seen as a correlation between returns and volatility, a need for efficient estimators of correlation between potentially non-stationary series is high. Recently, two methods have been proposed in the literature – detrended cross-correlation coefficient (Zebende, 2011) and detrending moving-average cross-correlation coefficient (Kristoufek, 2014a).

Zebende (2011) proposes the detrended cross-correlation coefficient as a combination of the detrended cross-correlation analysis (DCCA) (Jiang and Zhou, 2011; Podobnik and Stanley, 2008; Zhou, 2008) and the detrended fluctuation analysis (DFA) (Kantelhardt et al., 2002; Peng et al., 1993, 1994). The detrended cross-correlation coefficient  $\rho_{DCCA}(s)$ , which measures the correlation even between non-stationary as well as seasonal series, is defined as

$$\rho_{DCCA}(s) = \frac{F_{DCCA}^2(s)}{F_{DFA,x}^2(s)F_{DFA,y}^2(s)}, \quad (10)$$

where  $F_{DCCA}^2(s)$  is a detrended covariance between profiles of the two series based on a window of size  $s$ , and  $F_{DFA,x}^2$  and  $F_{DFA,y}^2$  are detrended variances of profiles of the separate series, respectively, for a window size  $s$ . For more technical details about the methods, please refer to Kantelhardt et al. (2002), Podobnik and Stanley (2008) and Kristoufek (2014b). In words, the method is based on calculating the correlation coefficient between series detrended by a linear trend while the detrending is performed in each window of length  $s$ .

Kristoufek (2014a) introduces the detrending moving-average cross-correlation coefficient as an alternative to the above mentioned coefficient. The method connects the detrending moving average (DMA) procedure (Alessio et al., 2002; Vandewalle and Ausloos, 1998) and detrending moving-average cross-correlation analysis (DMCA) (Arianos and Carbone, 2009; He and Chen, 2011). The detrending moving-average cross-correlation coefficient  $\rho_{DMCA}(\lambda)$  is defined as

$$\rho_{DMCA}(\lambda) = \frac{F_{DMCA}^2(\lambda)}{F_{x,DMA}^2(\lambda)F_{y,DMA}^2(\lambda)}, \quad (11)$$

where  $F_{DMCA}^2(\lambda)$ ,  $F_{DMA,x}^2(\lambda)$  and  $F_{DMA,y}^2(\lambda)$  are, similar to the DCCA-based coefficient, detrended covariance between profiles of the two studied

**Table 3**  
Estimated Hurst exponents for logarithmic volatility.

	Brent crude oil	WTI crude oil	Heating oil	Natural gas
Local Whittle	1.0448	1.1008	1.0803	1.0659
St. error	0.0437	0.0440	0.0440	0.0440
GPH	1.0383	1.0987	1.1861	0.9838
St. error	0.0580	0.0612	0.0660	0.0640
DFA	1.1431	1.1980	1.1633	0.9255
St. error	0.0249	0.0257	0.0298	0.0276
DMA	1.1570	1.2223	1.1858	0.9190
St. error	0.0239	0.0273	0.0298	0.0328
Average	1.0958	1.1550	1.1289	0.9735

series and detrended variances of the separate series, respectively, with a moving average parameter  $\lambda$ . Contrary to the previous DCCA-based method, the DMCA variant does not require box-splitting but estimates the correlation from the profile series detrended simply by the moving average of length  $\lambda$ . [Carbone and Castelli \(2003\)](#) show that the centered moving average outperforms the backward and forward ones so that we apply the centered one in our analysis. For more detailed description of the procedures, please refer to [Alessio et al. \(2002\)](#), [Arianos and Carbone \(2009\)](#) and [Kristoufek \(2014a\)](#).

### 3.4. Rescaled covariance test

Motivated by the rescaled variance test for the univariate series, [Kristoufek \(2013\)](#) proposes the rescaled covariance test which is able to distinguish between long-term and short-term memory between two series. In a similar way as for the univariate series, the long-term memory can be generalized to the bivariate setting so that the long-range cross-correlated (cross-persistent) processes are characterized by asymptotically power-law decaying cross-correlation function and divergent at origin cross-spectrum. By applying the test to the relationship between returns and volatility, we can comment on possible power-law cross-correlated relationship between the two series which is usually connected to the leverage effect ([Cont, 2001](#)). Compared to the methods presented in the previous paragraph, which describe instantaneous relation between a pair of series, the rescaled covariance test provides more insight into temporal structure of the relationship.

The testing statistic for the rescaled covariance test is defined as

$$M_{xy,T}(q) = q^{\widehat{H}_x + \widehat{H}_y - 1} \frac{\widehat{\text{Cov}}(X_T, Y_T)}{T \widehat{s}_{xy,q}}, \quad (12)$$

where  $\widehat{s}_{xy,q}$  is the HAC-estimator of the covariance of the studied series defined in Eq. (3),  $\widehat{\text{Cov}}(X_T, Y_T)$  is the estimated covariance between profiles of the series, and  $\widehat{H}_x$  and  $\widehat{H}_y$  are estimated Hurst exponents of the separate processes. For the estimated Hurst exponents, we use the average of the local Whittle, GPH, DFA and DMA estimators if the process is found to be long-range dependent. Otherwise, we set the corresponding exponent equal to 0.5.

## 4. Data and results

We analyze front futures contracts of Brent crude oil, WTI (West Texas Intermediate) crude oil, heating oil and natural gas between 1.1.2000 and 30.6.2013.<sup>2</sup> As we are interested in the leverage effect, we focus on returns and volatility of the future prices. In [Figs. 1 and 2](#), we present returns and volatility based on the Garman–Klass estimator given in Eq. (1). From the returns charts, we observe that these behave very similarly to the standard financial returns with volatility clustering and non-Gaussian distribution. However, we also notice, mainly for the natural gas, that returns undergo certain seasonal pattern which is

connected to the rolling of the front and back futures contracts. This is dealt with by utilizing detrended cross-correlation and detrending moving-average cross-correlation coefficients which are constructed for such seasonalities. Volatility dynamics again reminds of standard volatility of other financial assets with evident persistence, which is dealt with later on. Again, the natural gas series stands out with more frequent volatility jumps and more erratic behavior.

In [Table 1](#), we summarize standard descriptive statistics and tests.<sup>3</sup> All returns series follow quite standard characteristics such as excess volatility, negative skewness (apart from natural gas in this case), non-Gaussian distribution and asymptotic stationarity. For each series, we also find significant autocorrelations. Later, we test whether these can be treated as the long-term ones or not. Apart from the returns and volatility, which we examine in its logarithmic form, we focus on the standardized returns as well. Note that the returns standardized by their volatility are usually close to being normally distributed and in general, they are more suitable for statistical analysis. From this point onward, we focus solely on the relationship between standardized returns and logarithmic volatility so that if returns and volatility are referred to, we work with the transformed series. Standardized returns are all approximately symmetric and do not exceed kurtosis of the normal distribution. Moreover, the autocorrelations have been filtered out by standardizing for three out of four series. For the volatility, we strongly reject normality of the distribution and we find very strong autocorrelations. Moreover, we reject both unit root and stationary behavior of the series. This leads us to an inspection of potential long-term memory in the analyzed series.

In [Table 2](#), we show the results for the modified rescaled range and the rescaled variance tests. Optimal lag has been chosen according to Eq. (4). We find that neither of the returns series are long-range autocorrelated, even though the testing statistics for natural gas are close to the critical levels. As expected, long-term memory is identified for all volatility series even after controlling for rather high number of lags (between 15 and 20). The results of the long-term memory tests thus give expected results – no long-term memory for the returns and statistically significant long-term memory for the volatility series.

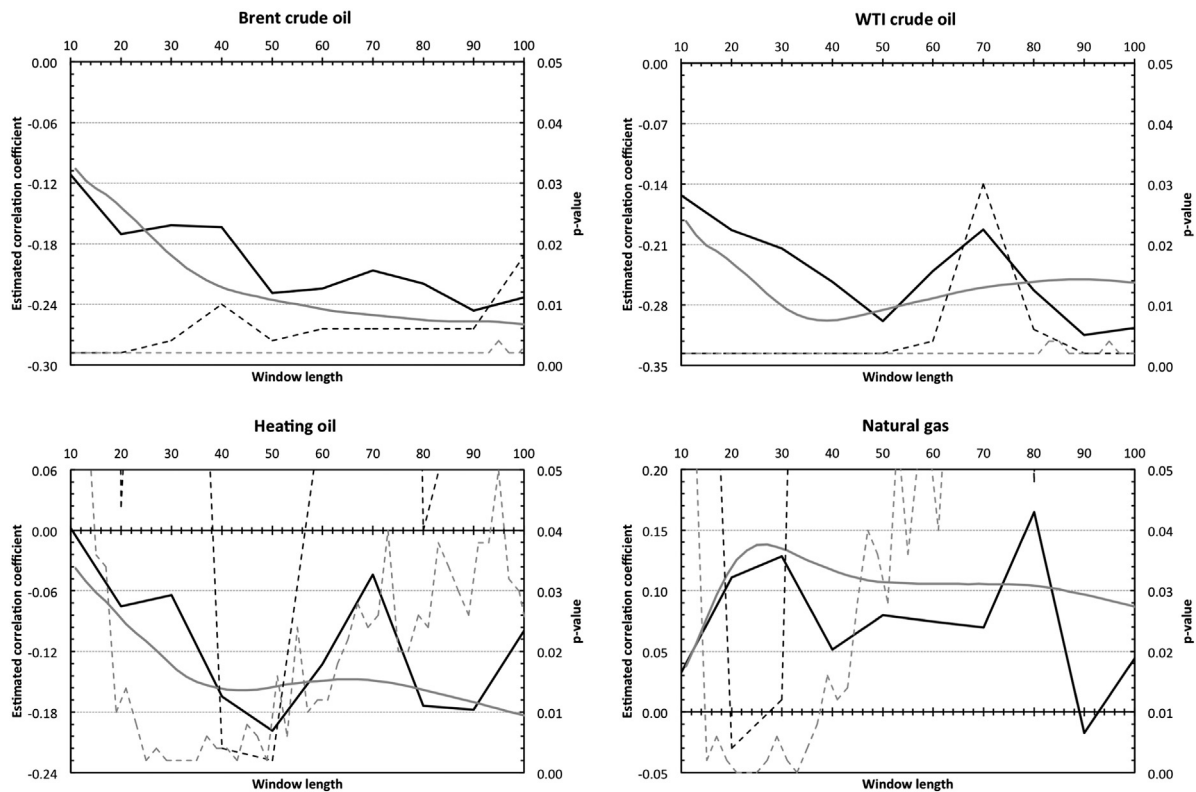
Based on the previous tests, we take that the returns series are not long-term dependent so that their Hurst exponent is equal to 0.5, which is later used in the rescaled covariance test. For the volatility series, we estimate the Hurst exponent  $H$  using the local Whittle, GPH, DFA and DMA estimators. The estimates are summarized in [Table 3](#). We observe that all the estimators give similar results – the Hurst exponent for volatility for all four studied series is estimated around  $H \approx 1$ . Based on the reported standard errors, we cannot distinguish whether the Hurst exponents are below or above the unity value with confidence. Such high values are rather non-standard for the volatility series of stocks, stock indices or FX rates. We attribute such a strong persistence to a specific structure of the futures contracts with well defined maturities and rolling of the front contracts which introduces stronger memory into the volatility process.

As the volatility series are long-term correlated, we need to apply correlation measures which are able to deal with such series. [Kristoufek \(2014b\)](#) shows that the standard correlation coefficient is not able to do so. We thus apply the detrended cross-correlation coefficient and detrending moving-average cross-correlation measures which are not only able to work under long-term memory and even non-stationarity but they can also filter out well-defined trends. To see whether the estimated correlation coefficient varies with a window length, i.e. with the time frame set for detrending, we present results for different  $s$  and  $\lambda$ . Specifically, we vary  $s$  between 10 and 100 with a step of 10, and  $\lambda$  varies between 11 and 101 with a step of 2 to get comparable results.<sup>4</sup> Results

<sup>3</sup> The descriptive statistics, stationarity tests, the local Whittle estimation and the GPH estimation were performed in Gretl 1.9.14.

<sup>4</sup>  $p$ -Values are constructed using 1000 series generated using Fourier randomization, which ensures that the autocorrelation structure remains untouched but the cross-correlations are shuffled away.

<sup>2</sup> The time series were obtained from <http://www.quandl.com> server on 23 July 2013.



**Fig. 3.** Correlation coefficients between returns and volatility. Correlation coefficients are based on DCCA (black lines) and DMCA (gray lines). Solid lines represent the estimated correlation coefficients (left y-axes) and the dashed lines show the  $p$ -values (right y-axes) for varying window lengths  $s$  and  $\lambda$  (x-axes). Both crude oils show the standard leverage effect with a quite stable behavior across scales while heating oil shows a weaker one. For natural gas, we report the inverse leverage effect.

are summarized in Fig. 3. For both crude oils, we find quite stable and statistically very significant leverage effect. The strength of the effect slightly varies with  $s$  and  $\lambda$  which suggests that the dynamics between returns and volatility is different for specific scales. Concretely, the leverage effect is stronger in longer time horizon. Both DCCA and DMCA based correlation coefficients bring very similar results which strengthens the findings. The level of the correlation between approximately  $-0.2$  and  $-0.3$  is in hand with results reported for the stock indices and stocks. The results are more mixed for the other two commodities. For heating oil, we again find the standard leverage effect, i.e. the negative correlation between returns and volatility. However, the statistical significance is much weaker than for the crude oils. These results also support the findings reported by Kristoufek (2014b) and Kristoufek, (2014a) who shows that the DCCA based coefficient is less efficient than the DMCA based one while using the same length of filtering windows. Even though the results of the DCCA estimator are erratic, we suggest that heating oil still follows the standard leverage effect albeit weaker than the crude oils do. Additionally, we again observe that the level of correlation depends on the scale, i.e. the leverage effect is again stronger at higher scales. Natural gas can be described as experiencing the inverse leverage effect, i.e. a positive relationship between returns and volatility. The estimated correlation coefficient is quite stable but weak at a level of approximately 0.1. The DCCA based coefficient again behaves quite erratically, yet the DMCA coefficient shows statistically significant correlations up to the scale of approximately 60 days, i.e. a trading quarter. Note that the inverse leverage effect is not completely unheard of even for stock markets (Qiu et al.,

2006; Shen and Zheng, 2009) but, as it is shown in the Literature review section, also for the energy markets (Fan et al., 2008; Zhang et al., 2008) and specifically for natural gas as well (Nomikos and Adriosopoulos, 2012).

Table 4 then summarizes the results of the rescaled covariance test which tests possible long-range cross-correlations. We use the same number of lags as for the univariate volatility tests in Table 3. Based on the reported  $p$ -values,<sup>5</sup> we find no signs of long-range dependence in the bivariate setting. This is tightly connected to quite weak correlations found above. Even though the series might be correlated, creating the leverage or the inverse leverage effects, the influence is not strong enough to translate into a long-term connection.

## 5. Conclusion

In this paper, we propose a comprehensive treatment of the leverage effect, focusing on energy commodity futures, namely Brent and WTI crude oils, natural gas and heating oil. After estimating the volatility process without assuming any specific form of its behavior, we find the volatility to be long-term dependent with the Hurst exponent on a verge of stationarity and non-stationarity. Bypassing this issue by using the detrended cross-correlation and the detrending moving-average cross-correlation coefficients, we find the standard leverage effect for both crude oils and heating oil, even though the latter one is weaker. For natural gas, we find the inverse leverage effect. This points out a need for initial testing for the presence of the leverage effect before constructing any specific models to avoid inefficient estimation or even biased results. Finally, we also show that none of the effects between returns and volatility is detected as the long-term cross-correlated one. The dynamics of the crude oil futures, as ones of the most traded

**Table 4**  
Rescaled covariance test for standardized returns and logarithmic volatility.

	Brent crude oil	WTI crude oil	Heating oil	Natural gas
$M_{xy,r}(q)$	-82.8163	80.8690	135.9459	-399.2733
$p$ -Value	$\gg 0.1$	$\gg 0.1$	$\gg 0.1$	$\gg 0.1$

<sup>5</sup> In the same way as for the DCCA and DMCA based correlation coefficients,  $p$ -values are constructed using 1000 series generated using Fourier randomization.



ones, is thus closer to the one of stocks and stock indices whereas the less popular natural gas somewhat deviates from the standard behavior. These findings can be further utilized to enhance forecasting models and mainly in the risk management and portfolio diversification.

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## Appendix A. Supplementary data

Scripts (written for R 3.0.0) for the rescaled variance, rescaled range and rescaled covariance tests as well as for the DCCA, DFA, DMCA and DMA based methods are appended to the manuscript. Data sets used in the analysis are publicly available at <http://www.quandl.com>. The specific analyzed series are appended to the manuscript as well. Supplementary data to this article can be found online at <http://dx.doi.org/10.1016/j.eneco.2014.06.009>.

## Appendix A. Supplementary data

Supplementary data to this article can be found online at <http://dx.doi.org/10.1016/j.eneco.2014.06.009>.

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