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Semi-receding Horizon Algorithm for “Sufficiently Exciting” MPC with Adaptive Search Step

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Abstract— **In this paper, the task of finding an algorithm providing sufficiently excited data within the MPC framework is tackled. Such algorithm is expected to take action only when the re-identification is needed and it shall be used as the “least costly” closed loop identification experiment for MPC. The already existing approach based on maximization of the smallest eigenvalue of the information matrix increase is revised and an adaptation by introducing a *semi-receding horizon* principle is performed. Further, the optimization algorithm used for the maximization of the provided information is adapted such that the constraints on the maximal allowed control performance deterioration are handled more carefully and are incorporated directly into the process instead of using them just as a termination condition. The effect of the performed adaptations is inspected using a numerical example. The example shows that the employment of the semi-receding horizon brings major improvement of the identification properties of the obtained data and the proposed adaptive-search step algorithm used for the “informativeness” optimization brings further significant increase of the contained information while the aggravation of the economical and tracking aspects of the control are kept at acceptable level.**

I. Introduction

Over the few decades since its introduction, model predictive control (MPC)—being perhaps the most perspective member of the broad family of the advanced control approaches—has been cured of most of the “childhood” diseases, its theoretical properties have been well-proven and therefore, it has gained much popularity within both the theoretically- and practically-oriented branches of the control community. Thanks to numerous advantages, the areas in which MPC is employed have accrued rapidly and nowadays, its practical use is no more restricted to chemical engineering where it started [1]. The evolution in the field of numerical optimization [2] enabling implementation of MPC algorithms on low-demand industrial computers and PLCs and their use for control of the fast systems opens the door for more and more challenging industrial applications of MPC.

However, not all drawbacks related to the deployment of MPC have been eliminated satisfactorily. The crucial role played by the mathematical model of the controlled system still restricts its usage. Furthermore, it is usually the process of obtaining of the suitable model candidate which is the most time-consuming part of the whole procedure of bringing MPC to life and requires much more time than the design, implementation and tuning of the controller itself. Actually, some references indicate that the identification phase may take up to $90\,\%$ of the overall time [3]. Taking this enormous percentage into account, it is desirable to pay sufficiently much attention to the identification of the model for MPC.

The identification aspects of MPC use are very often omitted in the literature. Usually, it is assumed that certain identification experiment has been performed to obtain a suitable model. This, however, does not correspond to the real life situation – in industrial practice, it is generally impossible to execute such experiment due to numerous economical and operational reasons. In such case the only data which are available for identification purposes are those from ordinary closed-loop operation of the controlled process. Such data suffer from lack of contained information and from negative aspects related to input-to-noise correlation.

So far, several methods for closed-loop identification have been introduced [4], [5]. The main disadvantage of these approaches is that while they work well for simple controllers with properties which are favorable from the identification point of view (causality, linearity), they do not work properly in case of advanced optimization based controllers [6]. The most effective way how to tackle this task in case of MPC seems to be simultaneous control and excitation of the system. If performed carefully, such approach has the potential to provide well-excited data suitable for identification purposes while also satisfying the given performance criteria.

The first approaches mentioned in the literature come up with adding of an external so-called dithering signal [7] to the control input while the next group of approaches is based on use of sufficiently excited reference signal. Both these branches can lead to a situation that the resulting closed-loop behavior will be far away from the desired control performance. In several other works [8], [9], an alternative method has been presented. The requirement on informativeness of the data has been added to the MPC cost function as an additional constraint. This demands solving of a complicated nonconvex task which is solved by the authors using a suitable relaxation as the semi-definite programming task. This approach suffers from one serious drawback – the excitation in the output directions is usually omitted.

In [10], [11], the authors have utilized the receding horizon principle which has enabled them to split the process of solving of the originally non-convex problem into two steps by solving twice the quadratic programming task. However, the output excitation has been omitted as well.

The approach published recently in [12] offers another alternative. It works in two stages – in the first one, the original MPC task is solved and in the second step, the maximization of the information matrix is performed while the maximal allowed perturbation of the original MPC cost function is employed as the constraint. To simplify the second-stage optimization, elliptic approximation is exploited.

The authors themselves have also contributed to the lastly mentioned two-stage branch. Unlike [12], they performed no approximation and optimized directly the quantization of the

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provided information contained in data. In [13], two algorithms have been provided: the first of them considers only the first input sample for the optimization of informativeness while the second based on gradient optimization optimizes certain chosen subsequence of the whole optimal input sequence pre-calculated by MPC. According to comparison provided in [13], the two algorithms are equivalent for the class of the reference-tracking MPCs. In [14], the second (gradient-optimization-based) approach has been tested for the rapidly spreading class of zone MPC. Its versatility with respect to the used optimization criterion and considered constraints has been shown.

In the current paper, the authors provide several improvements of the existing methodology. First of all, in order to fully exploit the pre-calculated input samples ensuring sufficient excitation, *semi-receding horizon approach* is proposed. The authors provide also improvement of the optimization algorithm that is employed at the second stage in the view of constraints handling. Instead of keeping the optimization steps constant, the distance from the constraints on the allowed MPC performance deterioration is taken into account and the optimization steps are adapted accordingly. The semi-receding horizon approach together with the adaptive step provide major increase of the information gathered from the system.

The paper is organized as follows. In Section II, the problem to be solved is introduced including the model description, control requirement formulation and quantification of the amount of the information contained in the obtained data. Section III presents the proposed solution. Firstly, constraints-dependent step adaptation of the two-stage algorithm is provided and secondly, semi-receding horizon approach is introduced. Section IV presents the case study on which the performance of the proposed improvements is demonstrated. After the brief description of the considered system and the comparison quantifiers, the results are summarized and corresponding discussion is provided. The paper is concluded by Section V.

II. Problem formulation

In the following paragraph, the necessary background is provided. The descriptions of the model and the considered controller follow.

A. Model under investigation

In this paper, a simple linear time-invariant (LTI) model is considered. Such model can be described by the well-known ARX structure as

$$y_k = Z_k^T \theta + \varepsilon_k, \tag{1}$$

where y_k and u_k are the system output and input sequences and ε_k stands for zero-mean white noise. The vector of parameters θ is considered in the following form:

$$\theta = \left[b_{n_d} \dots b_{n_b} - a_1 \dots - a_{n_a}\right]^T \tag{2}$$

while $Z_k = \left[u_{k-n_d} \dots u_{k-n_b} y_{k-1} \dots y_{k-n_a}\right]^T$ is the regressor. Parameters of the structure n_a, n_b, n_d specify numbers of lagged inputs and outputs and a relative input-to-output delay ($n_d = 0$ means direct input-output connection).

B. Model predictive control

The objective of MPC is to find the optimal input sequence that minimizes the given performance criterion. The model of the system is used for predictions of the future behavior. Typical MPC formulation penalizes both the energy consumed for the control and the deviation of the outputs from the pre-defined reference trajectory. Such formulation can be mathematically expressed as:

$$J_{MPC,k} = \sum_{i=1}^{P} \left\| Q(y_{k+i} - y_{k+i}^{ref}) \right\|_2^2 + \left\| R u_{k+i} \right\|_2^2$$
$$\text{s.t.: linear dynamics (1)},$$
$$u_{k+i}^{min} \leq u_{k+i} \leq u_{k+i}^{max}, \tag{3}$$

with y_k^{ref} specifying the reference trajectory, Q and R being the control algorithm tuning matrices of the appropriate size and P being the prediction horizon. Formulation (3) can be solved by common solvers for quadratic programming.

Although MPC possesses many favorable properties, its potential and utilization crucially depend on the availability of a high accuracy mathematical model with good prediction behavior. In the real-life operation, it oftentimes happens that a model that used to work properly and reliably looses its accuracy and ability to provide good predictions and then, it is inevitable to obtain a new one. This illustrates the need for designing such controllers that are able to generate data which are sufficiently rich and contain enough information that can enable the occasional re-identification. Still, the overall control performance must not be significantly degraded and the resulting behavior should meet the requirements defined by the cost function (3). This might be a welcomed alternative to lengthy, complicated and (often also) cumbersome identification experiments which sometimes might not even be realizable due to either economical or operational reasons. The very first straightforward question before formulating the problem itself is how the "informativeness" of a set of data should be evaluated. One way is to quantify the information content of the data set based on the so-called information matrix [15] and the persistent excitation condition.

C. Persistent excitation condition

Let us consider ARX model structure (1). Then, the matrix ΔI_k^{k+M} defined as

$$\Delta I_k^{k+M} = \sum_{t=k+1}^{k+M} Z_t Z_t^{\mathrm{T}}. \tag{4}$$

represents the increment of the information matrix from the time k to the time $k+M$ and quantifies the amount of the gathered information. Knowing this matrix, the so-called persistent excitation condition can be formulated as follows

$$\Delta I_k^{k+M} \geq \gamma E > \mathbf{0}, \tag{5}$$

where γ is a scalar specifying the level of the required excitation and E is a unit matrix of corresponding dimension.

III. Persistent excitation within MPC

As already mentioned, the goal of this paper is to provide algorithm for the MPC which will be able to not only satisfy the control requirements but also to provide sufficiently excited data making the re-identification easier. Similarly to the recent work [12], we propose a two-stage algorithm based on the maximization of the information matrix. The procedure is as follows: firstly, the original MPC task (3) is solved and then the maximization of the information matrix is performed in the second step while the maximal allowed perturbation of the original MPC cost function is employed as the constraint:

$$
\begin{aligned}
U^* = \arg\max_U \gamma \\
\text{s.t.:} \sum_{t=k+1}^{k+M} Z_t Z_t^T \geq \gamma E, \\
J_{MPC,k}(U) \leq J^*_{MPC,k} + \Delta J, \\
u_{k+i}^{min} \leq u_{k+i} \leq u_{k+i}^{max}, i = 1, \dots, P
\end{aligned} \quad (6)
$$

Here, ΔJ specifies the maximum allowed increment of the original MPC cost function $J^*_{MPC,k}$. Note that in [12], the involved non-convex task which is to be solved in the second step is approximated using an elliptic approximation which works reliably only for simple low-order systems. On the other hand, we try to propose an algorithm that is able to solve the second-stage optimization task without any approximations with acceptable computational demands and favorable performance independent of the order of the system. The following subsection brings a more detailed description of the algorithm.

A. *Optimization with adaptive constraints-dependent step*

In the following text, the two-stage procedure that leads to the solution of the task of persistent excitation within the MPC is described. In the first stage, the original MPC problem is solved while in the second stage, numerical optimization algorithm with adaptive constraint-dependent search steps is employed to attack the optimization task (6).

First stage

The first step of the algorithm can be viewed as a kind of initialization for the second stage. The MPC task formulated by (3) and supplied by the corresponding constraints on the inputs is solved. As the output of the first stage, both the optimal input sequence $U^*_{MPC} = [u_{k+i}]$, $i = 1, 2, \dots, P$ and the corresponding cost function value $J_{MPC,k}(U^*_{MPC})$ are obtained. While the optimal input sequence U^*_{MPC} is used for the initialization of the numerical gradient search algorithm as the initial guess of the optimal input sequence $U^0 = U^*_{MPC}$, the optimal cost function value $J_{MPC,k}(U^*_{MPC})$ is used as a constraint.

Second stage

In the second stage, the optimization task related directly to the maximization of the gathered information is solved. The performance criterion to be optimized is formulated as:

$$\mathcal{J}(U) = \max\left(\min \text{eig}(\Delta I_k^{k+M})\right) \quad (7)$$

where ΔI_k^{k+M} corresponds to (4). Here, let us mention that several other choices of the maximization criterion (e.g. determinant or the trace of the increment of the information matrix) can come to mind. The reason why the minimal eigenvalue has been chosen is that it corresponds to the direction in the gathered data which contains the least information. In other words, the criterion (7) reflects that the identifiability of the most difficultly identifiable parameter shall be improved.

The direct input constraints $u_{k+i}^{min} \leq u_{k+i} \leq u_{k+i}^{max}$ that are to be satisfied ensure that the calculated control action is practically realizable. Moreover, the optimizer in the second stage is allowed to perturb the original MPC criterion by at most ΔJ which is mathematically expressed as $J_{MPC,k}(U) \leq J^*_{MPC,k} + \Delta J,\ i = 1, 2, \dots, P$.

The optimal input sequence from the first stage U^*_{MPC} is then split into two parts –the first M samples are available for the optimization of the informativeness while the rest $P - M$ samples are kept fixed and with the first M samples, they are used to evaluate the original MPC cost function. The reason to optimize more than just 1 sample in the sense of data excitation is very pragmatical. Optimizing just 1 particular input sample, only a single direction corresponding to particular estimated parameter can be excited. The more parameters are to be identified, the more input samples should be taken into account. The numerical optimization of these samples is then performed utilizing a modified gradient search as follows.

The first M samples of the input sequence calculated by the MPC in the previous step are used as the initial guess U^0 of the profile which is optimized iteratively following the direction of the increase of the cost function (7),

$$U^{l+1} = U^l + \beta \star G^l, \quad (8)$$

where G^l is the search direction for the l-th iteration of the gradient search, β is the vector of lengths of the performed steps and $\star$ denotes element-wise multiplication of vectors. The gradient of the criterion (7) is calculated numerically: one by one, all M samples of U^l are gradually perturbed with chosen Δu. Performing this, a set of M perturbed input vectors is obtained,

$$
\begin{aligned}
\mathcal{U} = \{U_i = [u_1, u_2, \dots, u_i + \Delta u, u_{i+1}, \dots, u_M,], \\
i = 1, 2, \dots, M\}.
\end{aligned} \quad (9)
$$

Then, evaluating the change of the performance criterion for the second stage defined by (7) for each of the perturbed input profiles

$$\Delta\mathcal{J}_i = \mathcal{J}(U_i) - \mathcal{J}^c$$

with $\mathcal{J}^c$ denoting the current criterion value, the vector of numerical gradients G can be obtained as follows:

$$G = \left[\frac{\Delta\mathcal{J}_1}{\Delta u}, \frac{\Delta\mathcal{J}_2}{\Delta u}, \dots, \frac{\Delta\mathcal{J}_i}{\Delta u}, \dots, \frac{\Delta\mathcal{J}_M}{\Delta u}\right]. \quad (10)$$

Now, let us return to the search step vector β. While in the previous work, all samples of the vector β had the same magnitude (the movements in all M optimized dimensions was uniform), adaptive constraint-dependent search steps β are employed in the current paper. In the previous work, the maximal deterioration of the MPC performance was used as one of the terminating conditions – at each iteration, the actual deterioration was calculated and if it was higher than ΔJ, the gradient algorithm was terminated and the input subsequence from the previous iteration (which did not violate the MPC performance condition) was used as the output of the algorithm. In the current work, we combine the MPC-performance constraints with the search for the optimally excited inputs and the MPC performance criterion is directly incorporated into the optimization. In case that the perturbation of i-th input sample should cause deterioration close to the ΔJ, the gradient search step β_i in the corresponding i-th dimension is decreased and the movement in that corresponding direction is slowed down.

To accomplish that, the search steps are adapted using hyperbolic tangent function with the argument being the difference between the actual and maximal allowed degradation of the MPC cost function. In order to prevent the algorithm from "falling back" in case that the expected deterioration should be greater than ΔJ, the steps are restricted to be greater than or equal to 0. The resulting search steps β_i then correspond to

$$\beta_i = \max(0, \tanh(w(\Delta J - \Delta J_i))). \tag{11}$$

where ΔJ specifies the maximal allowed perturbation and ΔJ_i corresponds to the violation of the MPC cost function considering i-th perturbed input sequence U_i. The parameter w is used to shape the expression for the search step appropriately and is considered as the tuning parameter of the algorithm. With lower w, the algorithm is more "careful" and pays more attention to the distance from the maximal allowed perturbation. With $w \to \infty$, the expression (11) approaches $\max(0, \mathrm{sign}(\Delta J - \Delta J_i))$ and only the input perturbations causing unacceptable deteriorations $\Delta J_i \geq \Delta J$ are banned while the others are not handled at all. Let us note that at each iteration l of the gradient search algorithm, a new set of search steps β is obtained.

The box-constraints for the values of the particular input samples are satisfied performing a simple projection on the admissible input interval $\langle u^{min}, u^{max} \rangle$. The iterative search is terminated if the improvement of the criterion (7) is less than a chosen threshold.

B. Semi-receding horizon approach

In the following text, the adaptation of the usually followed methodology is proposed and explained.

Freely spoken, the main idea of the above mentioned approaches based on optimization can be summarized as follows: first, let us calculate the optimal input minimizing the MPC cost function. Then, let us consider that the first M samples of the optimal sequence are available for the optimization of the excitation and can be perturbed in order to maximize the obtained information. Meanwhile, the rest $(P - M)$ of the original input sequence calculated by MPC is considered fixed. The restrictions on the perturbation of these M samples are given by the original hard constraints on the applied inputs and by the maximal deterioration of the optimal value of the cost function calculated by MPC (here, the deterioration is obviously calculated for the whole prediction horizon of MPC). Following the well-known receding horizon control principle, once the second-stage optimization is accomplished, the first perturbed input sample is applied and the whole procedure is repeated.

Unfortunately, it can be expected that as long as the last M–1 samples optimized in the second stage are never applied to the system, the achieved excitation might not reach the calculated level. From this perspective, the difference between optimizing the whole subsequence of length M and optimizing just the first applied input might be negligible.

In the current work, we come up with a *semi-receding horizon approach* which decreases the gap between the expected and achieved excitation level and therefore obtains more informative data. The procedure is as follows:

1) calculate the input sequence optimizing the MPC cost function
2) optimize the first M samples of the P-sample sequence with respect to the provided excitation
3) apply the whole M-sample sequence, go to 1).

Obviously, this approach makes use of the receding horizon principle in order to ensure sufficient feedback which is necessary to satisfy the control/safety requirements while it also introduces certain type of relaxation which favors the data excitation effort. Moreover, for stable systems with $M \ll P$ such relaxation of the feedback does not bring observable control performance degradation compared to the receding horizon approach which is also demonstrated in the following Section.

IV. CASE STUDY

A. Description

To show the properties and demonstrate the performance of the proposed algorithm, we consider a SISO system with ARX structure with the parameters $n_a = 3$, $n_b = 3$, $n_d = 1$, and $\theta_0 = [0.01 \quad 0.0008 \quad 0.00087 \quad 0.996 \quad 0.36 \quad 0.376]^{\mathrm{T}}$ and with white noise with variance $\sigma_e = 0.05$.

In fact, this example has not been chosen arbitrarily – it mimics a simplified heat transfer model between the heating medium (heating circuits in concrete ceiling) and zone air in a building with the constant ambient temperature and sampling period $T_s = 15\,\mathrm{min}$. Let us mention that rather than providing a procedure to design a controller for the building control, the objective of this illustrative example is to demonstrate the properties of both the newly proposed methodology and the improved numerical optimization algorithm. Therefore, certain level of simplification of both the model and controller is adopted.

The system is controlled by the MPC corresponding to (3) minimizing the supplied energy (u corresponds to the temperature of the heating medium) and satisfying thermal comfort (to keep the output y as close to reference value as possible) with constraints $u^{max} = 50°C, u^{min} = 20°C$ while y^{ref} is generated according to the following 7 days schedule with night and weekend setbacks:

$$y^{ref} = \begin{cases} 22°\,\mathrm{C} & \text{from 8 a.m. to 6 p.m.,} \\ 20°\,\mathrm{C} & \text{otherwise.} \end{cases} \tag{12}$$

Weighting matrices are chosen as $Q = 10000$ and $R = 1$, the prediction horizon $P = 40$ steps (with the sampling period $T_s = 15\,\mathrm{min}$, it is equivalent to $10\,\mathrm{h}$) is assumed. In order to bring the example as close to reality as possible, the model which is used by the MPC for the predictions does not perfectly match the real system but its parameters are slightly shifted and are considered as $\hat{\theta} = [0.99 \quad 0.35 \quad 0.37 \quad 0.01 \quad 0.0007 \quad 0.0009]^{\mathrm{T}}$. The comparison is based on a numerical example with length $N = 10000$ samples (for the above mentioned sampling period, this corresponds to 3 months). The tuning parameters have been set as: input perturbation $\Delta u = 0.1$, search step shaping parameter $w = 0.02$, number of samples optimized in the second stage $M = 6$ and maximal allowed MPC function deterioration $\Delta J = 1500$. For a more detailed discussion of the tuning of the parameters, see [13].

B. Results

Let us remind the objective of the current work which is to develop an algorithm that is able to both satisfy the control performance defined by (3) and provide the data containing such amount of information that is sufficient for the re-identification. The evaluation of the performance can be found in the following table. In Table I, first the overall "informativeness" of data quantified by the normalized value of the smallest eigenvalue of the increment of the information matrix $\lambda_{min,n}$ is listed. The smallest eigenvalues for each particular algorithm are normalized with respect to the eigenvalue achieved by the original gradient algorithm making use of the ordinary receding horizon principle [13]. In the table, this algorithm is referred to as GA and the normalization basically means that the GA approach provides 1 "unit of information". The algorithms belonging into RH class make use of the classical receding horizon principle (contrary to GA, the second-stage optimization performed within the aGA algorithm employs adaptive search step) while algorithms listed in SRH class are those following the newly introduced semi-receding horizon approach (GA$_{SRH}$ algorithm employs constant search step, the aGA$_{SRH}$ employs the adaptive search steps proposed in the current paper). Regarding the last abbreviation, MPC refers to classical MPC control without sufficient excitation condition.

The tracking performance of the algorithms is evaluated using the absolute value of the overall tracking error,

$$e_y = \frac{1}{N} \sum_{k=1}^{N} \|y_k^{ref} - y_k\|,$$

while the consumed energy is evaluated using the quantifier:

$$I_E = \left(\frac{\sum_{k=1}^{N} u_k^2}{\sum_1^N u_{k,MPC}^2} - 1 \right) (\%).$$

Here, u_{MPC} represents the input applied by the classical MPC without the sufficient excitation condition. Here, the question of why a comparison with the original non-exciting MPC is provided could arise. The reason is that the proposed excitation-optimizing algorithm is supposed to restrict the deviation of the control performance from the ordinary operational conditions. Providing such comparison, it can be checked that our algorithm for the identification experiment does cause only negligible deviation from the ordinary regime, however, it is able to provide more excited data and therefore, it offers better conditions for the re-identification.

TABLE I

RESULTS COMPARISON.

	RH		SRH		MPC
	GA	aGA	GA$_{SRH}$	aGA$_{SRH}$	
$I_E(\%)$	0.9	2.1	1.5	2.2	0
$e_y(°C)$	0.03	0.04	0.05	0.07	0.02
$\lambda_{min,n}$	1	1.9	2.4	4.2	0.2

As shown in Table I, the energy consumption increase for each of the tested algorithms compared to the original MPC never exceeds $2.2\,\%$. Here, it should be noted that this aggravation is expected only during the performance of the excitation algorithm until the sufficiently informative data appropriate for the model re-identification are gathered. This together with hardly observable violation of the average reference tracking guarantees very satisfactory control behavior even during the performance of the excitation experiment for which the algorithms have been developed. Even for the semi-receding horizon approaches, the deviation from the ordinary operational regime is insignificant which supports the claim that for $M \ll P$, the relaxation of the feedback does not cause serious degradation. The control performance is even more promising when realizing that nonzero noise and imperfect MPC model have been considered.

Regarding the "informativeness" of the provided data, it can be witnessed that the proposed improvements clearly fulfill their purposes – while the incorporation of the adaptive search step within the second-stage optimization improves the information quantifiers almost twice, the employment of the semi-receding horizon leads to even higher improvement ratio. As a result, when combined both adaptations, the results of the original GA algorithm have been improved by the aGA$_{SRH}$ algorithm by a factor greater than 4.

In order to visualize how the improvement of the smallest eigenvalue of the information matrix increase affects the identifiability, another comparison is provided. All 5 sets (one for each of the algorithms listed in Table I) of the obtained data containing $N = 10000$ samples have been used for identification. More than 100 models have been identified per each provided data set and their step responses have been confronted with the step response of the original system. The graphical comparison of all of them is shown in Fig. 1.

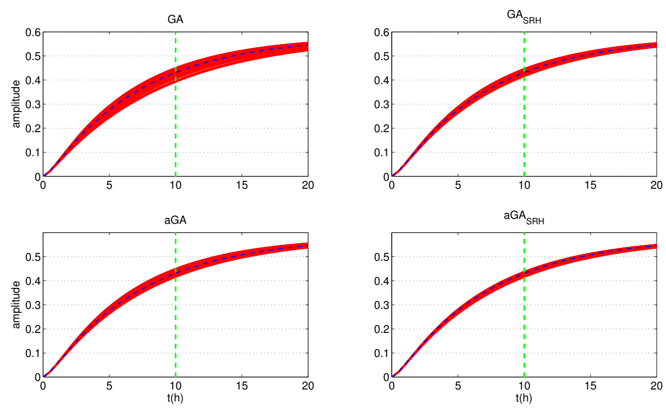

Fig. 1. Comparison of the step responses (red - identified models, blue dashed - real system).

Comparing the two step responses either in one row or one column, it can be observed that they get improved both when employing semi-receding or the adaptive search step proposal. In accordance with Table I, the biggest deviations from the real system step response occur for the GA algorithm while the models identified from the data provided by aGA$_{SRH}$ algorithm match the real step response almost perfectly. Here, the green dashed line marks the prediction horizon of the MPC being 10 h. Freely spoken, the prediction performance of the model on the horizons larger than P are of small interest as the behavior of the system for such horizons is not taken into account within the controller and neither the input is optimized for these horizons.

Fig. 2 compares the ultimate deviations from the real step response for the three most interesting candidates – MPC, GA and aGA$_{SRH}$. While in the previous work, the improvement which was provided by the GA algorithm cut the deviations from the real system response in half in average, very similar improvement has been obtained also in the current work. As can be seen, the deviations for the aGA$_{SRH}$ algorithm are condensed more tightly around zero and also the magnitude of the worst case deviation is at most

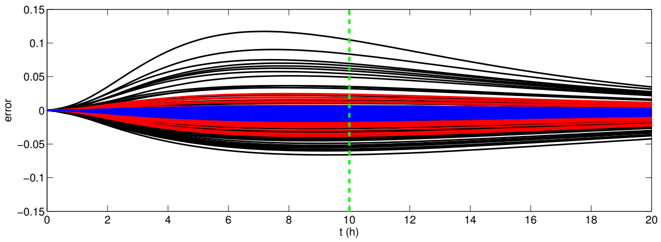

Fig. 2. Deviations from the real step response (black - MPC, red - GA, blue - aGA$_{SRH}$).

one half of the worst case deviation for the GA. Again, green dashed line marks the horizon of 10 h.

As long as considerable part of the presented improvement can be owed to the incorporation of the adaptive search step exploited during the second stage, Fig. 3 illustrates its performance at the chosen sampling instance.

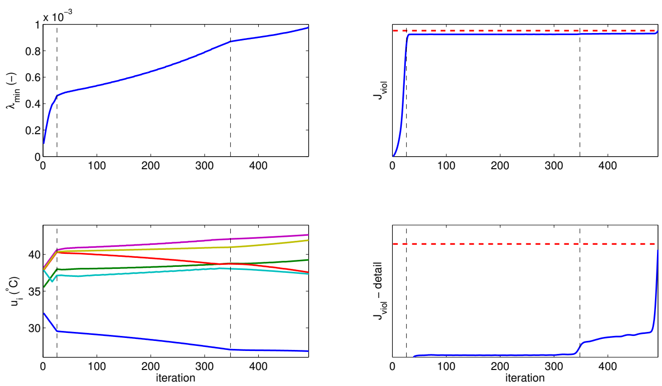

Fig. 3. Adaptive constraints-dependent step algorithm – illustration.

The subfigure in the left upper corner depicts the value of the smallest eigenvalue of the information matrix increase as the function of algorithm iterations. In each subfigure, two significant points (iterations) where its slope changes considerably are marked. Looking at the subfigures located in the right upper and right lower corner showing the actual violation of the MPC cost function ΔJ^l at l-th iteration, it is obvious that these two points are strongly related to the distance from the maximal allowed perturbation ΔJ (represented by the red dashed line). For the better clarity, the subfigure located in the right lower corner shows a detail of the subfigure located above it – here, the second point is clearly visible. In case that the algorithm approaches considerably the ΔJ threshold, the adaptive search step should "slow down" the movement in the most critical dimension. This can be witnessed inspecting the subfigure placed in the lower left corner which shows the evolution of the particular perturbed input samples u_i, $i = 1, 2, \ldots, M$. Here, it can be also seen that while at the first marked point, all optimized input samples "slow down", at the second marked point not all input samples are affected (apparently, the one plotted in dark green is not affected at all) which illustrates the performance of the incorporated adaptive step. Here, let us note that with the constant search step, the original algorithm would be terminated soon after the first marked point, the current improved algorithm continues in optimization and is able to improve the value of λ_{min} approximately twice compared to the value at the first marked point.

Last of all, let us note that the computational demands are kept admissibly low – in average, the calculations that need to be performed at particular sampling instance do not take more than 4 s.

V. Conclusion

Two improvements of the approach for sufficiently exciting MPC have been proposed. First of them modifies the utilization of the input samples optimized for the sufficient excitation. Instead of commonly considered receding horizon principle, its relaxed version called semi-receding horizon principle is employed. Second improvement introduces adaptive constraint-dependent search step for algorithm used in the second stage of the whole procedure. The adaptive step reflects the actual distance from the maximal allowed perturbation of the original MPC cost function. As long as both of them are able to provide approximately twofold increase of the information contained in the data, their combination improves the "informativeness" quantifier by a factor of more than 4 compared to the previously used algorithm. The control performance degradation has been also inspected – with the average energy consumption increase of no more than 2.2 % and negligible average tracking error, the solution presented in the current paper is highly suitable for utilization as a closed loop experiment for MPC.

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