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Research Note

Comments on "Weed recognition using image blur information by Peng, Z. & Jun, C., Biosystems Engineering 110 (2), p. 198–205"

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This research note is in reaction to a recent paper on weed recognition using image analysis (Peng & Jun, 2011). Here, the correct use of moment invariants in a weed recognition system is presented.

1. Introduction

This paper is a reaction to the paper (Peng & Jun, 2011) published recently in this journal where an automatic system for visual weed recognition was presented. Weeds were recognized on images acquired by a camera, moving quickly above the field. The authors correctly realised that due to the camera motion and possibly also due to incorrect focus, the images could be degraded by so called "blur" which makes object recognition more difficult. The blur can be (at least approximately for a flat scene and short acquisition time, which is the case here) modelled by two-dimensional convolution.

$$g(x, y) = (f * h)(x, y), \tag{1}$$

where $g(x,y)$ is the observed blurred image of a scene $f(x,y)$ and $h(x,y)$ is the *point-spread function* (PSF) of the system, which fully characterises the blur. The PSF is actually equivalent to the image of an ideal isolated bright point.

Since in Peng and Jun (2011) the source of blur is supposed to be known, the parametric form of the

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Nomenclature

PSF	Point Spread Function
$g(x,y)$	Blurred image
$f(x,y)$	Ideal image without blur
$h(x,y)$	PSF of the blurred image
*	Convolution
$\delta(x)$	Dirac function
$\mu_{pq}^{(g)}$	Central geometric moment of the image g
$B(p,q)$	Invariant to axial blur
$M(p,q)$	Invariant to linear motion blur
$C(p,q)$	Invariant to centrosymmetric blur

corresponding PSF is also known. In case of a linear horizontal motion the PSF has the following form:

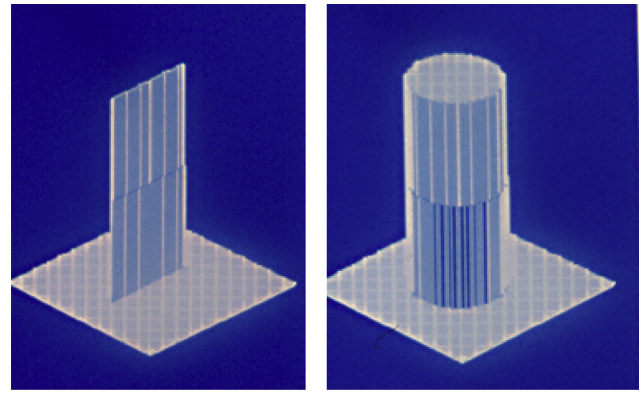
$$h(x,y) = \begin{cases} \frac{1}{vt}\delta(y) & \Leftrightarrow 0 \le x \le vt \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where v is the motion velocity, t is the exposure time and δ is a Dirac function (see Fig. 1a). If the motion vector has another direction, the PSF is just rotated accordingly. For an out-of-focus blur, the PSF is a cylinder[1] whose radius determines the size of the blur (see Fig. 1b). If these two (or more) factors act simultaneously, the composition PSF is a convolution of all particular PSF's.

The authors of Peng and Jun (2011) also correctly pointed out that, in order to beat the blur effect, the recognition should be based on image features which are not affected by blur. Such features are called *blur invariants* and were introduced by Flusser, Suk, and Saic (1996) and (1998). This blur-invariant solution is easier and faster to solve than the obvious approach, where the image is first de-blurred by means of image restoration techniques and then a standard recognition is applied.

However, in Peng and Jun (2011) a very important point was ignored: the blur invariance of these features is a direct consequence of the *symmetry* of the PSF. There are different invariants for different symmetries. Invariants for centrosymmetric PSF were published in Flusser and Suk (1998), for PSF symmetric with respect to both axes and diagonals in Flusser et al. (1996), for PSF with circular symmetry in Flusser and Zitová (2004), and for motion blur, Gaussian blur and PSF that have N-fold rotation symmetry in Flusser, Suk, and Zitová (2009), see Fig. 2 for symmetry examples. There exist no invariants with respect to a general non-symmetric blur. It is therefore necessary to use only invariants corresponding to the actual shape of the PSF, otherwise the invariance property is violated and the system performance decreases. However, in Peng and Jun (2011), the authors applied invariants designed for axial and diagonal symmetry taken from Flusser et al. (1996) to the recognition of images, blurred by the motion blur and combined motion-defocus blur. This is incorrect because neither motion nor motion-defocus blur have such a symmetry and this choice leads to a non-optimal recognition rate.

The aim of this note is to explain how proper invariants for motion and combined motion-defocus blur can be chosen



(a) (b)

Fig. 1 – The PSF of (a) an ideal linear motion blur and (b) an out-of-focus blur on a circular aperture.

and, consequently, how the performance of the recognition system can be increased. We believe this could be helpful for readers who want to use or re-implement the system proposed in Peng and Jun (2011).

2. Recalling blur invariants

Blur invariants are functions of the image moments. They can be defined for any kind of moments (Kautsky & Flusser, 2011) but for simplicity let us stay with *geometric moments* only. *Central* geometric moment of image f is defined as

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-x_c)^p (y-y_c)^q f(x,y)\,dx\,dy, \quad (3)$$

where x_c, y_c are the coordinates of the image centroid. Central moments are invariant to translation.

Under convolution, the central moments are transformed as

$$\mu_{pq}^{(g)} = \sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} \mu_{kj}^{(h)} \mu_{p-k,q-j}^{(f)}. \quad (4)$$

For each particular kind of symmetry, certain moments of the PSF are zero, which simplifies (4). For the horizontal motion blur PSF, $\mu_{pq}^{(h)} = 0$ for any odd $p+q$ and for any $q \neq 0$. Hence, Eq. (4) reduces to the form

$$\mu_{pq}^{(g)} = \sum_{k=0}^{[p/2]} \binom{p}{2k} \mu_{p-2k,q}^{(f)} \mu_{2k,0}^{(h)} \quad (5)$$

(symbol $[a]$ means the integer part of a). This allows to properly combine the moments of the blurred image and to eliminate all the non-zero moments of the PSF in order to obtain the desired invariance (see Flusser et al., 2009 for details).

3. Invariants to linear motion blur

In Peng and Jun (2011), it is proposed to use the following invariants of the 4th and 5th order (see Eqs. (4)–(5) in Peng & Jun, 2011) which were borrowed from Flusser et al. (1996).

[1] Provided that the camera aperture is circular and the light–lens interaction is governed by geometric optics rules.

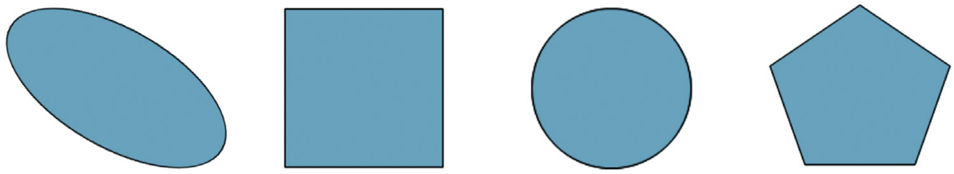


Fig. 2 – Examples of the PSF symmetries – central symmetry, symmetry with respect to both axes and diagonals, circular symmetry, 5-fold rotation symmetry. Specific blur invariants exist for each particular case.

- 4th order:

$$B(1,3) = \mu_{13} - \frac{3\mu_{02}\mu_{11}}{\mu_{00}},$$

$$B(3,1) = \mu_{31} - \frac{3\mu_{20}\mu_{11}}{\mu_{00}},$$

$$B(4,0) = \mu_{40} - \mu_{04} - \frac{6\mu_{20}(\mu_{20} - \mu_{02})}{\mu_{00}}.$$

- 5th order:

$$B(3,2) = \mu_{32} - \frac{3\mu_{12}\mu_{20} + \mu_{30}\mu_{02}}{\mu_{00}},$$

$$B(2,3) = \mu_{23} - \frac{3\mu_{21}\mu_{02} + \mu_{03}\mu_{20}}{\mu_{00}},$$

$$B(4,1) = \mu_{41} - \frac{6\mu_{21}\mu_{20}}{\mu_{00}},$$

$$B(1,4) = \mu_{14} - \frac{6\mu_{12}\mu_{02}}{\mu_{00}},$$

$$B(0,5) = \mu_{05} - \frac{10\mu_{03}\mu_{02}}{\mu_{00}},$$

$$B(5,0) = \mu_{50} - \frac{10\mu_{30}\mu_{20}}{\mu_{00}}.$$

As already pointed out, these invariants require a PSF with axial and diagonal symmetry, which is not the case of motion blur. Some of them (B(3,1), B(4,1) and B(5,0)) are still invariant thanks to the axial symmetry of the motion-blur PSF. Some others are invariants but uselessly complicated (B(1,3), B(3,2), B(2,3), B(1,4) and B(0,5)) and B(4,0) is not invariant at all.

To see this, let us investigate how B(4,0) is transformed under horizontal motion blur of the image.

$$B(4,0)^{(g)} = \mu_{40}^{(g)} - \mu_{04}^{(g)} - \frac{6\mu_{20}^{(g)}\left(\mu_{20}^{(g)} - \mu_{02}^{(g)}\right)}{\mu_{00}^{(g)}}.$$

Since $\mu_{00}^{(h)} = 1$ we have $\mu_{00}^{(g)} = \mu_{00}^{(f)}$. Using the convolution property (Eq. (4)), the fact that $\mu_{10} = \mu_{01} = 0$ for any image, and calculating the moments of the motion PSF explicitly, we obtain for the other moments.

$$\mu_{20}^{(g)} = \mu_{20}^{(f)} + \mu_{20}^{(h)}\mu_{00}^{(f)} = \mu_{20}^{(f)} + \frac{s^2\mu_{00}^{(f)}}{12},$$

$$\mu_{02}^{(g)} = \mu_{02}^{(f)} + \mu_{02}^{(h)}\mu_{00}^{(f)} = \mu_{02}^{(f)},$$

$$\mu_{40}^{(g)} = \mu_{40}^{(f)} + 6\mu_{20}^{(h)}\mu_{20}^{(f)} + \mu_{40}^{(h)}\mu_{00}^{(f)} = \mu_{40}^{(f)} + \frac{s^2\mu_{20}^{(f)}}{2} + \frac{s^4\mu_{00}^{(f)}}{80},$$

$$\mu_{04}^{(g)} = \mu_{04}^{(f)} + 6\mu_{02}^{(h)}\mu_{02}^{(f)} + \mu_{04}^{(h)}\mu_{00}^{(f)} = \mu_{04}^{(f)},$$

where $s=vt$ is the length of the blurring pulse. Now we can substitute into $B(4,0)^{(g)}$. After some manipulations we get

$$B(4,0)^{(g)} = B(4,0)^{(f)} - \frac{s^2\left(\mu_{20}^{(f)} - \mu_{02}^{(f)}\right)}{2} - \frac{7s^4\mu_{00}^{(f)}}{240},$$

which proves that $B(4,0)^{(g)}$ depends on the PSF and hence B(4,0) is not invariant under motion blurring.

If we investigate for instance B(1,3), which consists of two terms, we realize that both terms are invariant with respect to motion blur, so it would be better to use them separately.

A correct general recurrent formula for motion blur invariants is

$$M(p,q) = \mu_{pq} - \sum_{k=1}^{\lfloor p/2 \rfloor} \binom{p}{2k} M(p-2k,q)\mu_{2k,0}. \quad (6)$$

This formula provides a complete system of motion-blur invariants[2] of arbitrary order. Here we present explicit forms up to the 5th order.

- 2nd order:

$$M(1,1) = \mu_{11},$$

$$M(0,2) = \mu_{02}.$$

- 3rd order:

$$M(3,0) = \mu_{30},$$

$$M(2,1) = \mu_{21},$$

$$M(1,2) = \mu_{12},$$

$$M(0,3) = \mu_{03}.$$

[2] To be precise, this formula is valid also for linear motion the velocity of which is variable during the exposure. The necessary assumption is that the velocity is symmetric with respect to the central point, which might be true for certain vibration blurs.

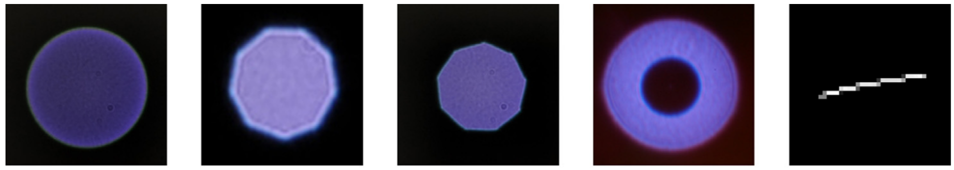


Fig. 3 – Real PSF's of an out-of-focus blur on (a) circular, (b, c) polygonal and (d) ring-shaped aperture, respectively. The PSF is given by the shape of the aperture. If the diaphragm is not fully open, we observe a polygonal shape due to the diaphragm blades. (e) Real motion-blur PSF. One can see that the theoretical assumptions may be violated in practice.

- 4th order:

$M(0,4) = \mu_{04},$

$M(1,3) = \mu_{13},$

$M(2,2) = \mu_{22} - \frac{\mu_{20}\mu_{02}}{\mu_{00}},$

$M(3,1) = \mu_{31} - \frac{3\mu_{20}\mu_{11}}{\mu_{00}}.$

- 5th order:

$M(1,4) = \mu_{14},$

$M(0,5) = \mu_{05},$

$M(3,2) = \mu_{32} - \frac{3\mu_{20}\mu_{12}}{\mu_{00}},$

$M(2,3) = \mu_{23} - \frac{\mu_{20}\mu_{03}}{\mu_{00}},$

$M(4,1) = \mu_{41} - \frac{6\mu_{20}\mu_{21}}{\mu_{00}},$

$M(5,0) = \mu_{50} - \frac{10\mu_{20}\mu_{30}}{\mu_{00}}.$

The first-order invariants and all invariants of the type $M(2k,0)$ formally also exist but they are identically zero. If the motion direction is not horizontal but known, the acquired image can be rotated properly to make the blur horizontal (note that convolution and rotation commute). Such approach was employed in Peng and Jun (2011) where the motion direction was estimated from the Fourier spectrum of the acquired image. Thanks to the invariance knowledge of the motion velocity is not required.

4. Invariants to composite motion-defocus blur

If out-of-focus blur of an unknown extent is also present in addition to the motion blur, then the composite PSF exhibits a central symmetry (provided that the aperture is circular) with respect to its centroid, i.e. $h(x - x_c, y - y_c) = h(-x + x_c, -y + y_c)$, regardless of the motion direction. Invariants to this kind of blur were introduced in Flusser and Suk (1998) and can be directly adopted. For $p + q$ odd, they are defined in a recursive form

$$C(p,q) = \mu_{pq} - \frac{1}{\mu_{00}} \sum_{n=0}^{p} \sum_{\substack{m=0 \\ 0<n+m<p+q}}^{q} \binom{p}{n} \binom{q}{m} C(p-n, q-m) \cdot \mu_{nm}. \quad (7)$$

so theoretically one may have as many invariants as needed for a sufficient discriminability. In the discrete domain, the meaningful number of invariants is of course limited. As explained in Flusser and Suk (1998), the invariants of even orders do not exist in this case.

5. Discussion

The applicability of the blur invariants in Eqs. (6) and (7) depends on the validity of the theoretical assumptions about the degradation model. These assumptions may be violated in practise. The real motion PSF may not be a straight line and the out-of-focus PSF also may differ from a cylinder (see Fig. 3 for some examples of real PSF's). Another important assumption which we incorporated is that the images have infinite support. In case of real digital images this is never true and we face so-called *boundary effect* when the convolution model (Eq. (1)) is violated in stripes along the image borders. As the PSF size increases relatively to the size of the image, the boundary effect becomes more significant. The boundary effect was responsible (along with the usage of incorrect invariants) for the drop of the recognition rate for fast motion, which was reported in Peng and Jun (2011).

Yet another aspect concerns the maximum order of the invariants we shall use. Low-order invariants usually do not provide enough discrimination power while the high-order ones are more vulnerable to numeric errors. Finding the proper number of the invariants is an important part of the recognition and should be based on a discrimination analysis of the given dataset.

6. Conclusion

In this note we corrected some misleading information in the recent paper (Peng & Jun, 2011) and showed how blur invariants may be correctly used in a weed recognition system.

The concluding recommendation for the users is as follows. For pure linear motion blur in a known direction the invariants (Eq. (6)) adjusted for direction should be used, while for any centro-symmetric blur the invariants (Eq. (7)), which cover linear motion blur in an arbitrary (unknown) direction and composite motion-defocus blur should be used. The potential influence of the boundary effect and possible violation of the assumptions must always be considered.

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