Probabilistic Inspection of Multimodally Distributed Signals

Pavel Ettler and Ivan Puchr
COMPUREG Plzen, s.r.o.
306 34 Plzen, Czech Republic
ettler@compureg.cz, puchr@compureg.cz

Ladislav Jirsa and Lenka Pavelkova
Institute of Information Theory and Automation,
Czech Academy of Sciences,
182 08 Prague, Czech Republic
jirsa@utia.cas.cz, pavelkov@utia.cas.cz

Abstract

Condition monitoring of complex systems starts from an inspection of single measured signals. Results of health evaluation for each signal then enter the monitoring framework which uses a special type of probabilistic logic to assess condition of the whole inspected system while respecting its internal structure. A distribution of the involved signal can be multimodal in general. Therefore, a Gaussian mixture is more suitable for its approximation than a single theoretical distribution. The mixture estimated from a sufficiently long record of a particular signal can be used not only for its prediction but also for evaluation of the signal health. However, finding the values in a low probability region does not necessarily indicate bad condition of the signal – it can be interpreted as an increase in the information uncertainty instead. This approach is being tested on industrial data within the running project which aims to develop the novel probabilistic condition monitoring system.

1. Introduction

Condition monitoring, in particular its advanced or intelligent versions, play an important role in the nowadays industrial automation. This paper intends to contribute to hierarchical assessment of system conditions from the specific point of view: it focuses on inspection of the multimodally distributed signals and aims to evaluate their reliability by quantifying an uncertainty which is inherent in their measurement.
The contribution is a follow-up to the earlier paper(1) presented at the CM2014 conference a year ago. That is why both the introduction of the condition monitoring framework being built and the section about the subjective logic are rather brief and the bulk of the paper concerns modelling of the signals in question and evaluation of the uncertainty. Results illustrated on relatively big amount of real data are followed by concluding remarks.

2. Condition monitoring framework

The small international consortium working on the Eurostars project ProDisMon is building the industrial condition monitoring framework. Its main features are listed below. Issues of the standard specifications for moving information in a condition-based maintenance system belong to the scope of the other project partners and are not discussed here.

2.1 Hierarchical structure

The condition monitoring framework is based on the hierarchical, pyramid-like structure with the bottom-up evaluation. The lowermost blocks provide transformation from the measurement of particular signals to evaluation of their health/condition. Blocks in the upper rows of the pyramid combine information from the lower blocks to evaluate health of particular functional units or subsystems while output of the uppermost block indicates condition of the whole inspected system.

2.2 Nondeterministic evaluation

A straightforward evaluation of an overall condition can be performed by using logic rules. Nevertheless, the use of the binary logic throughout the system would be inadequate: if for example a slightly worn-away but functioning sensor is evaluated as faulty (*health = 0*) the whole system might be considered out of order; on the other hand, if the sensor evaluation says that *health = 1* its needed replacement might be neglected.
A good direction towards an adequate solution seems to be a utilization of the probabilistic logic where one considers probability of the unit being healthy, i.e. uses the measure from the interval [0, 1] and engages probabilistic logic operators for consequent compounding of information. Consideration of pervasive information uncertainty goes a step further and leads to the use of the subjective logic.

2.3 Considering uncertainty

Subjective logic is a type of probabilistic logic that allows probability values to be expressed with degrees of uncertainty[(2)]. Arguments in subjective logic are subjective opinions about propositions. A binomial opinion about the truth of state *h* (here *h* stands for *health*) is the ordered quadruple

$$\omega_h = (b, d, u, a), \tag{1}$$

where *b* is the belief that the specified proposition is true, *d* is disbelief, i.e. the belief that the specified proposition is false, *u* is uncertainty and *a* is the base rate, corresponding to prior probability. Details and properties of ω_h and its components and corresponding subjective logic operators can be found in [(2)] while basic remarks about the use of the calculus for the condition monitoring purposes are included in [(1)]. Here it is sufficient to note that it must hold

$$b + d + u = 1 \quad \text{and} \quad b, d, u, a \in [0,1]. \tag{2}$$

Evaluation of uncertainty *u* for multimodally distributed signals is the subject of further consideration.

3. Multimodally distributed signals

Example of the multimodally distributed signal is depicted in **Fig. 1**. The left plot shows time progress of the main hydraulic pressure in the hydraulic screwdown of a cold rolling mill. The situation corresponds to processing of a metal sheet in several passes, prior to its rolling between coilers. Histogram on the right plot, which indicates distribution of the signal, clearly illustrates its several modes corresponding to single passes and low pressure for the empty rolling gap between passes.

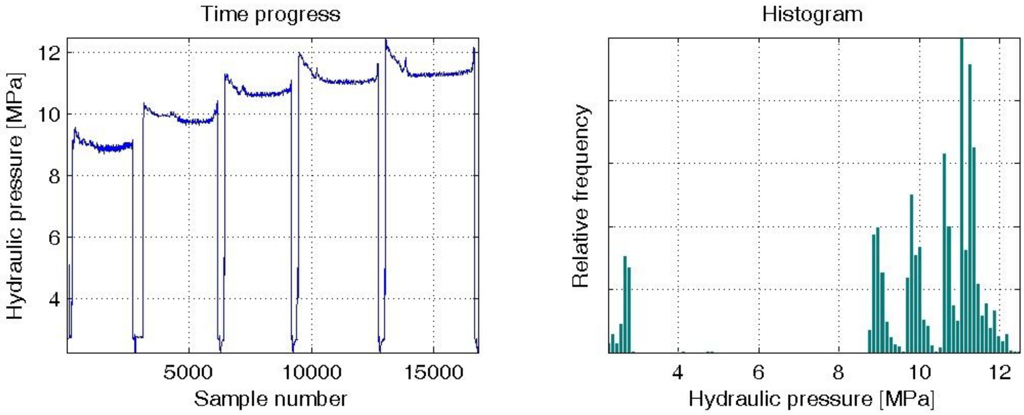

Fig. 1 Time progress and histogram of a multimodally distributed signal.

3.1 Modelling of multimodal signals

As dynamic properties of the signal are out of interest for now, course of study consists in static modelling of the signal, i.e. in smooth approximation of its empirical distribution depicted by the histogram.

3.2 Gaussian mixtures

Obviously, distribution of the signal in **Fig. 1** can hardly be approximated by a single/unimodal probability density function (pdf). Mixture of the unimodal distributions is suitable for this purpose, namely the Gaussian mixture. Then, conditioned probability density function approximating distribution of the signal is expressed as the weighted sum of Gaussian pdfs[3,4]

$$p(y \mid \Theta) = \sum_{c \in c_0} \alpha_c N_y(\theta_c, r_c), \qquad (3)$$

where y represents the signal in question, $\Theta = \{\alpha_c, \theta_c, r_c, c \in c_0\}$ is the vector of unknown parameters, α_c, θ_c, r_c are the weight, mean value and variance of the cth mixture component, c_0 is the set of all components comprising the mixture and $N_y(\theta_c, r_c)$ denotes pdf of the Gaussian (normal) distribution

$$N_y(\theta_c, r_c) \approx p(y \mid \theta_c, r_c) = \frac{1}{r_c\sqrt{2\pi}} e^{-\frac{(y-\theta_c)^2}{2r_c^2}}. \qquad (4)$$

3.3 Modelling example

Possible approximation of the empirical distribution of the above described signal is depicted in **Fig. 2**. In this case the mixture comprises 12 components.

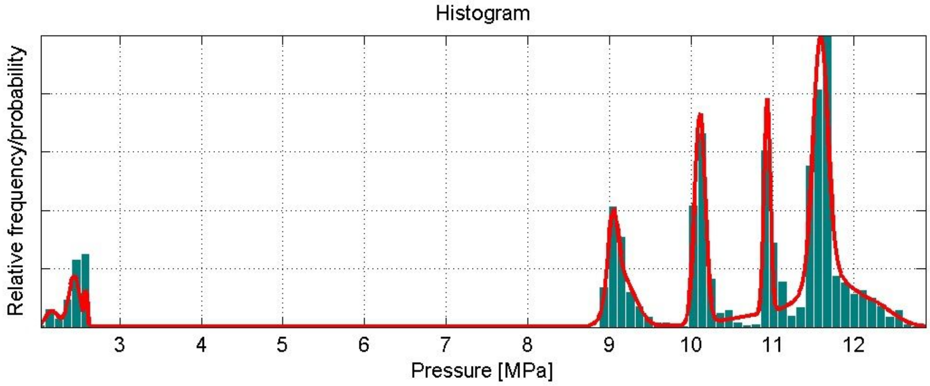

Fig. 2 Histogram of the pressure signal and its approximation by the Gaussian mixture.

3.4 Data set for representative experiments

For the sake of clearness, **Fig. 1** and **Fig. 2** depict just about 16000 samples of the inspected signal. Much more comprehensive data file is necessary for representative experiments. To cover most of the possible working points of the machine, a number of data records were pre-processed, selected and merged to provide 4 channels of testing signals each consisting of more than 1.7 million samples. This data set was used for following experiments.

4. Uncertainty related to multimodal signals

Let us assume that the identified pdf in **Fig. 3** approximates the distribution of the whole signal data set and that it represents the expected distribution of future measurements.

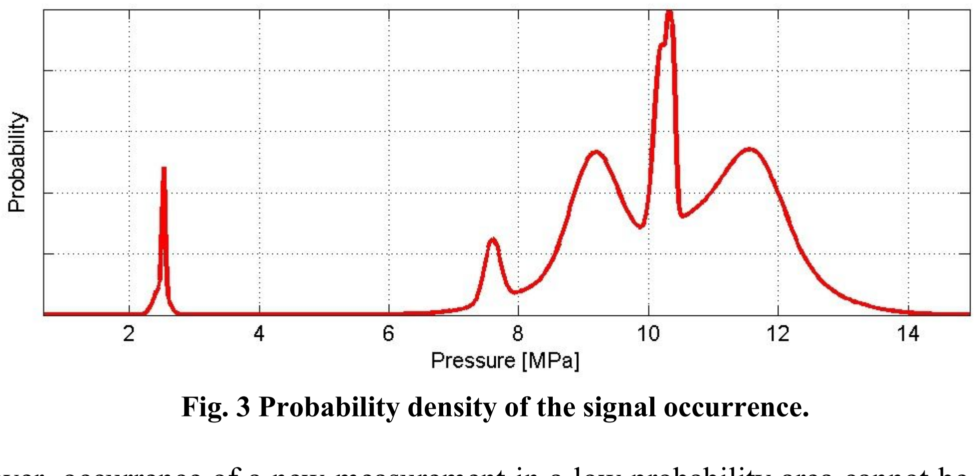

Fig. 3 Probability density of the signal occurrence.

However, occurrence of a new measurement in a low probability area cannot be simply interpreted as worsening of signal or process conditions. It may also indicate that the process has moved into a new, i.e. yet unrecorded working point. The only conclusion in that situation should be that information uncertainty u (1) related to the inspected unit has increased. However, the change of u also influences some of other elements of the opinion ω_h because they are bound together by the conditions (2). For the purpose of the project, we need to quantify this uncertainty. Two solutions to this problem are presented below.

4.1 Solution derived directly from the mixture pdf

In the simplest case denoted as A, the uncertainty is considered as the complement to the identified pdf (3)

$$u = 1 - p^*(y|\Theta), \tag{7}$$

Where symbol * denotes normalized function in the sense

$$x^* = \frac{x}{\max(x)}, \tag{8}$$

Nevertheless, evaluation (7) may be too strict: the uncertainty is positive even for minor peaks of the pdf, i.e. for "allowed" modes of the signal distribution. This discrepancy can be avoided by considering all the mixture components as equally important which results in the relation for the case B

$$u = 1 - \max_{c \in c_0}(N^*{}_y(\theta_c, r_c)). \tag{9}$$

Both cases are illustrated in **Fig. 4** for the pdf coming from **Fig. 3**. For the sake of simplicity, the uncertainty spans here the whole interval [0, 1]. In real situations the uncertainty is influenced by other factors as well (which are explained in [1] and [5]) and therefore the u-range coming from the inspection in question may be narrowed.

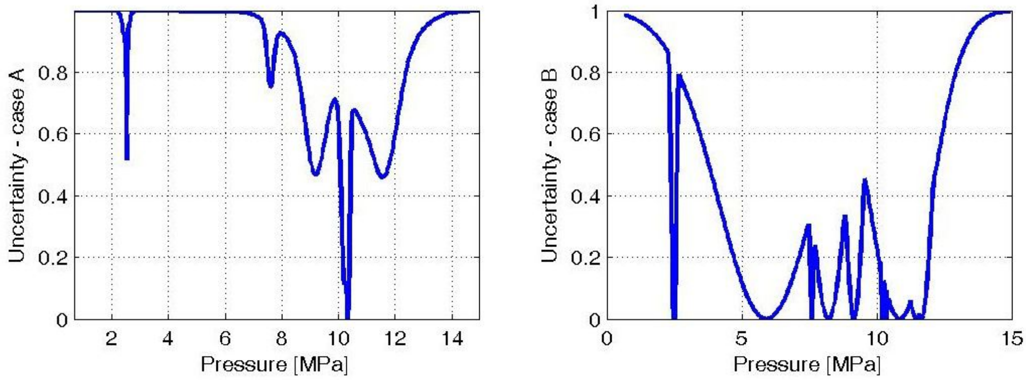

Fig. 4 Two basic cases of the quantified uncertainty.

It should be emphasised that according to (2) the non-zero uncertainty u influences elements b and d of the opinion ω_h. If there is for this particular case no subjective reason to increase the disbelief d, it must hold

$$d = 0 \quad \text{and} \quad b = 1 - u. \tag{10}$$

4.2 Solution using weights of the extended mixture

While the above mentioned method uses a current signal value and evaluates uncertainty (1) by the help of a shape of the reference mixture pdf, this section provides a solution based on continuous estimation of the signal in question and comparing the weight of the estimated pdf with weights of individual components of the reference mixture.

The reference mixture (3) is created from healthy data in an off-line phase. It has the form (4) and comprises n static components, i.e. $c = 1,2,...,n.$ In the on-line phase, a static model

$$p(y_t \mid \theta_{n+1}, r_{n+1}) = N(\theta_{n+1}, r_{n+1}) \tag{11}$$

is estimated from incoming data. This model represents a new component of the mixture. Thus, the obtained extended mixture comprises $n+1$ components

$$p(y \mid \Theta) = \sum_{c=1}^{n+1} \alpha_c N(\theta_c, r_c). \tag{12}$$

Here, $\Theta = \{\theta_{n+1}, r_{n+1}, \alpha_c, c = 1,...,n\}$. The parameters are estimated according to [4].

Let assume that (10) holds. Then, the uncertainty u (1) is evaluated using the estimates of weights α_c, i.e. probabilities of the particular components occurrence, as follows

$$u = \hat{\alpha}_{n+1}, \qquad b = \sum_{c=1}^{n} \hat{\alpha}_c. \tag{13}$$

As a result, the uncertainty in question corresponds to the weight of a new component and the belief b (1) corresponds to the sum of weights of reference components.

Assignment of uncertainty by considering a deterministic model of y is illustrated in **Fig. 5.** This diagram can be used directly for uncertainty assessment when data are assumed noiseless.

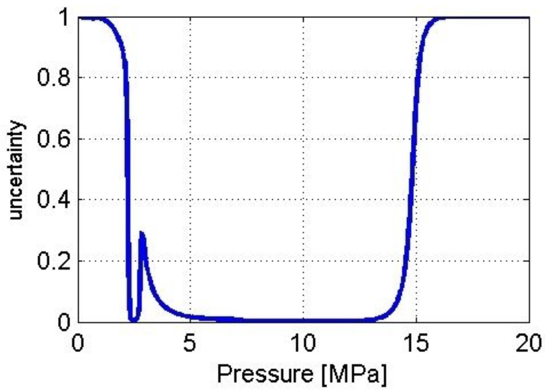

Fig. 5 Assignment of uncertainty by considering deterministic model of *y*.

Fig. 6 depicts a waveform of the healthy data and respective uncertainty. The uncertainty increases during switches between two modes.

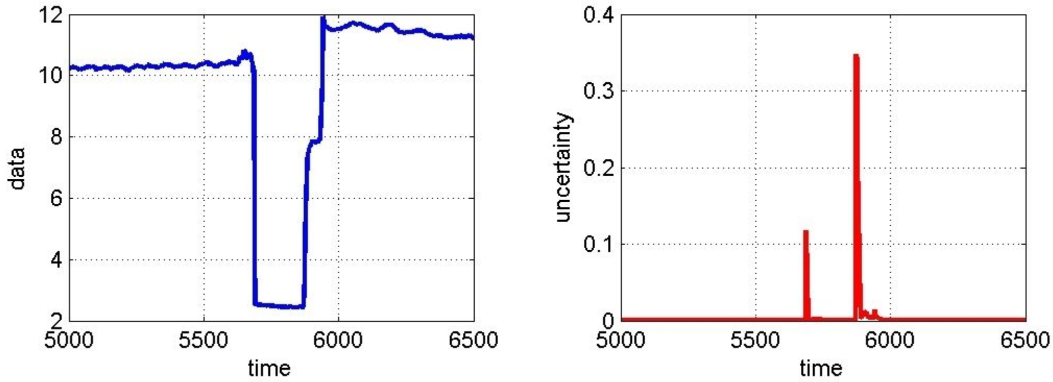

Fig. 6 A waveform of healthy data and respective uncertainty.

An example of the time behaviour of random data and respective uncertainty are shown in **Fig. 7**. Here the data do not match any reference component, i.e. they are outside the expected range given by the reference mixture.

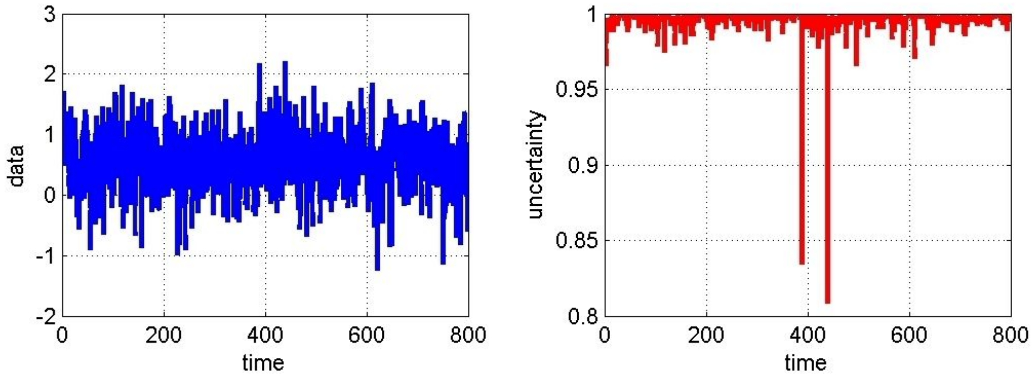

Fig. 7 Time behaviour of random data and respective uncertainty.

5. Conclusions

The paper deals with inspection of the multimodally distributed signals for the purpose of probabilistic reasoning about their and overall system health. Each selected signal is inspected independently while the information uncertainty is evaluated using the approximation of signal distribution by a Gaussian mixture. Two solutions of the problem are introduced. The developed methods enlarge the portfolio of algorithms being available for the pilot experimental application of the novel condition monitoring system in the industrial environment. Both solutions rely on the Gaussian mixtures identified off-line from the large set of representative data. Repetitive off-line updating of the mixtures is considered after collection of enough amount of new data.

Acknowledgements

The research is accomplished in the framework of the Eurostars project E!7262 ProDisMon and is supported by the grant MŠMT 7D12004.

References

1. P. Ettler and K. Dedecius, ‘Probabilistic reasoning in service of condition monitoring’, Proceedings of the 11th International Conference on Condition Monitoring and Machinery Failure Prevention Technologies (CM 2014/MFPT 2014), Manchester, UK, 2014.
2. A. Jøsang, ‘Conditional reasoning with subjective logic’, Journal of Multiple-Valued Logic and Soft Computing, 15(1), pp 5-38, 2008.
3. M. Kárný, J. Böhm, T. V. Guy and P. Nedoma, ‘Mixture-based adaptive probabilistic control’, International Journal of adaptive control and signal processing, John Wiley & Sons, Inc., 17 [2], pp. 119-132, 2003.
4. L. Jirsa and L. Pavelková, Testing of Sensor Condition Using Gaussian Mixture Model’, Proceedings of the 11th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2014), Vienna, Austria, 2014.
5. P. Ettler and K. Dedecius, ‘Probabilistic Condition Monitoring Counting with Information Uncertainty’, Proceedings of the 1st International Conference on Uncertainty Quantification in Computational Sciences and Engineering (UNCECOMP 2015), Crete Island, Greece, May 2015.