Day-ahead bidding on energy markets - a basic model and its extension to bidding curve

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Abstract

Wind resources energy production is highly influenced by uncertain weather conditions. We provide several simple models for bidding on day-ahead energy markets, which take into account the uncertainty. The obtained optimal bids and bidding curve are based not only on the point prediction, but also on the forecasted distribution of generated energy. We relate the resulting problems to two-stage stochastic programs with simple recourse.

Key words

Energy markets, wind energy, day-ahead bidding, uncertainty, two-stage stochastic programming

JEL Classification: D81, G11

1. Introduction

We deal with the problem of a wind energy producer. In the competition on a deregulated electricity day-ahead market under unknown (uncertain) market parameters, the energy producer has to provide its bids in advance, usually at the beginning of the trading day. However, the quantity offered by the producer need not to be available and the difference has to be bought on the intra-day market for a price which is not known at the time of bidding. On the other hand, when a producer is able to deliver more energy compared to its bid, then this surplus production can be sold on the intra-day market. In particular, this leads to asymmetrical uncertain penalties with respect to the surplus and shortage of production. The marked is then cleared by an independent system operator (ISO).

The optimal bidding problem is studied by several papers. Fleten and Kristoffersen (2008) employed a stochastic programming problem for short-term production planning of hydropower plants and applied it to a Norwegian hydropower producer trading on the Nordic power market. García-González et al. (2007) solved similar problem for optimal bidding for the next 24 hourly period. Moreover, they incorporated risk into their model in the form of Conditional Value at Risk (CVaR) constraint. Zhang (2012) studied a particular revenue function for a wind energy producer under fixed penalty costs. This assumption is relaxed in this paper and several extensions are discussed. Moreover, we propose relations to two-stage stochastic programming problems with simple recourse. For such problems many properties and solution techniques are known, see, e.g. Kall and Mayer (2011), Ruszczyński and Shapiro (2003), Shapiro et al. (2009).

The paper is organized as follows. In Section 2, we discuss deeply the revenue function and the resulting model for a single bid optimization. In Section 3, we extend the model for optimal bidding curve construction. Section 4 concludes the paper with an outline of future research.

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2. Model formulation - a single bid model

In this section, we assume that the producer sends a single bid to the day-ahead market. We use the following notation to formulate its revenue function:

- b - amount of energy nominated by the producer (shortly contract bid),
- P - uncertain (stochastic) spot market price cleared in day-ahead market by ISO (price),
- P_o - uncertain (stochastic) penalty price for exceeding the bid b (over-generation),
- P_u - uncertain (stochastic) penalty price for power generation lower than bid b (under-generation),
- G - uncertain (stochastic) amount of generated electrical power (production).

We assume that the distribution of the random variables is known or a good estimate is available.

The uncertain revenue function of a producer from the wind turbine can be written as follows

$$R(b) = b \cdot P + P_o \cdot [G - b]^+ \; - P_u \cdot [G - b]^-,$$

where the first summand corresponds to the pure revenue from the bid, which is then corrected by penalties for over- and under-generation represented by the remaining summands. Note that these penalties are usually asymmetric. Rayleigh, Gamma and Weibull distributions are often used for G, see, e.g. Zhang (2012).

The energy producer is looking for an optimal bidding strategy with respect to the random parameters, i.e. it maximizes the revenue

$$"max"_{b \geq 0} R(b).$$

However, in this form it is not clear how to solve the problem, since the objective function is random. The most natural way is to maximize the expected revenue function leading to

$$max_{b \geq 0} \mathrm{E}[R(b)].$$

We assume that the expected revenue function is well defined, i.e. the expectation is finite for all reasonable bids. If we put this problem into general framework of stochastic programming, it can be seen as a two-stage problem with recourse cf. Kall and Mayer (2011), Ruszczyński and Shapiro (2003). The contract bid serves as the first stage decision variable with fixed coefficient $\mathrm{E}[P]$. The second part

$$\varphi(\mathrm{b}) = \mathrm{E}[P_o \cdot [G - b]^+ - P_u \cdot [G - b]^-]$$

represents the second-stage recourse function. Its decomposition is highly dependent on random parts and their distributions. The simplest case is that when only generated amount G is random and penalty costs P_o, P_u are known and fixed. Then we obtain

$$\varphi(\mathrm{b}) = P_o \cdot \mathrm{E}[G - b]^+ - P_u \cdot \mathrm{E}[G - b]^-,$$

which corresponds to the simple recourse function, see, e.g., Klein Haneveld et al. (2006). Under fixed penalty prices and under additional distributional assumptions, Zhang (2012) derived explicit expression of the expected revenue function and its derivative. However, the assumption of fixed penalties needs not to be fulfilled in reality.

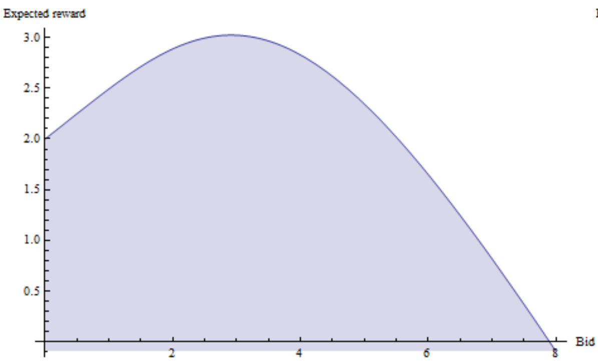
If the penalties are random and P_o, G and P_u, G are mutually independent, then we obtain

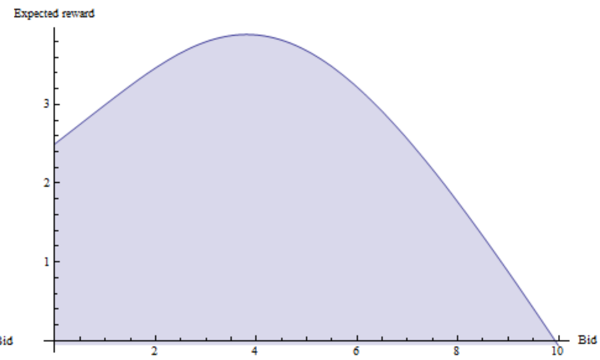
$$\varphi(\mathrm{b}) = \mathrm{E}[P_o] \cdot \mathrm{E}[G-b]^+ - \mathrm{E}[P_u] \cdot \mathrm{E}[G-b]^-,$$

which again represents a model with the simple recourse. However, also the penalties which are influenced by the price, which is simultaneously influenced by the energy production, can depend on the production *G*. Then no simplification of the recourse function is possible.

We can plot the expected revenue functions and find their maxima. Let $G \sim Gamma(4,1)$, $\mathrm{E}[G] = 4$, $P_o = 0.5$, $P_u = 2.0$, $\mathrm{E}[P] = 1$, and random penalties $P_o = 0.125 \cdot G$, $P_u = 0.5 \cdot G$, i.e. the expectations of the random penalties are equal to the deterministic values. Then we obtain expected revenue functions plotted in Figure 1. The optimal bids are 2.9132 and 3.8060 respectively, i.e. they are significantly different. Note that software *Mathematica* 9 was used for the computations.

Figure 1: Expected revenue functions – fixed penalties (left), random penalties (right)





When the penalties P_o, P_u are highly uncertain, then it can be favourable to solve the problem without including them. The formulation can be based on a chance constraint, where we prescribe a high probability $\alpha \in (0,1)$, usually 0.95 or 0.99, under which the production covers the bid. This leads to the chance constrained problem

$$max_{\mathrm{b}\geq 0} \mathrm{E}[P] \cdot b$$
$$s.t.\ P(b \leq G) \geq \alpha,$$

which is related to Value at Risk problem well-known from finance, cf. Rockafellar and Uryasev (2002). Let F^{-1} denote the inverse distribution function of random production G. Since the chance constraint has a deterministic reformulation $b \leq F^{-1}(1-\alpha)$, we can easily obtain the optimal solution

$$\tilde{b} = F^{-1}(1-\alpha).$$

Note that the problems with simple recourse and chance constraints are closely related, see Branda (2012, 2013) for a deep discussion.

3. Model formulation - an extension to bidding curve construction

Instead of a single bid, the producer can send whole bidding curve to the market. This curve is represented by couples of price and quantity, which is the energy producer willing to sell for the price. Let us denote these couples by $\{P_i, b_i\}, i = 1, \dots, I$. Simultaneous optimization over these couples can lead to a difficult nonconvex problem. Thus, usually the prices are selected from a discrete net and the corresponding bids are optimized. Such bid b_i is then valid for all prices $P \in [P_i, P_{i+1})$. In this case we are allowed to use the models from previous section with some minor modifications. The random price in substituted by the point P_i leading to the random revenue function

$$\hat{R}(b|P_i) = b \cdot P_i + P_o(P_i) \cdot [G - b]^+ - P_u(P_i) \cdot [G - b]^-,$$

where the penalties for under- and over-generation are dependent on the known price P_i in general. Then we solve the problem where we maximize the expected revenue, i.e. we can obtain the optimal bid as

$$\begin{aligned} b_i &= arg\,max_{\mathrm{b}\geq 0}\, \hat{R}(b|P_i) \\ &= arg\,max_{\mathrm{b}\geq 0}\, b \cdot P_i + P_o(P_i) \cdot \mathrm{E}[G - b]^+ - P_u(P_i) \cdot \mathrm{E}[G - b]^-. \end{aligned}$$

Note that we have obtained I models with the simple recourse structure.

4. Conclusion and future research

In this paper, we have formulated the revenue function for a wind energy producer who is bidding on the day-ahead energy market. The revenue function takes into account uncertainty represented by unknown energy production and random price which can influence the penalties for under- and over-generation. We have shown that under additional assumption the problem can be reformulated as the two-stage stochastic programming problem with simple recourse. Construction of the optimal bidding curve has been also discussed. Future research will be devoted to modelling of the competition on the energy markets and to its inclusion into individual revenue optimization, cf. Adam and Outrata (2014), Kozanidis et al. (2013). Another important task is the efficiency of the trading strategies, cf. Branda and Kopa (2014).

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