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Mean-Risk Optimal Decision of a Steel Company under Emission Control

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Abstract We propose a mean-risk decision model for a steel company facing emission limits and trading with emission allowances. The model is calibrated using data of a real-life steel company and is subsequently solved for five different scenarios of demand and different levels of risk aversion. It is found that while the limits are never reached, permit trading influences the decision to a great extent, especially given extremely low or extremely high demand when large amounts of permits need to be traded. We demonstrate that the risk caused by emission trading may increase not only with an increasing demand but also when the demand is low and a great amount of allowances must be sold.

Keywords Stochastic programming · Risk management · Mean-risk modeling · Emissions trading · Emissions management

1 Introduction

In recent years, industrial companies in the European Union are facing new institutional constraints associated with policies aimed at reducing emissions of greenhouse gases. The goal of the present paper is to study the influence of these constraints on the production and profit of a steel company.

The regulations, imposed on companies emitting greenhouse gases within the European Union, include not only constraints limiting their emissions but

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also an obligation to cover their emissions of carbon dioxide by emission permits, a certain amount of which they obtain for free from the authorities while the remaining (excess) ones have to be bought (can be sold) on a market.

The scientific community has already paid a certain amount of attention to the influences of these constraints on decisions made by companies: Authors of paper [6] design a single-period deterministic optimization problem of a general industrial company in the situation at the very beginning of the EU emission trading system (EU ETS); according to the authors, their paper was the first basic attempt to assess the influence of European legislative environmental factors on companies. In [11], a multi-stage stochastic programming problem was formulated which maximized an energy-producing company's profit while taking its emission trading into account; however, the situation of such a company is different from a steel one in that it produces only a single product - the power - and its decision variable is not production but composition of fuels. In a more recent paper [12], environmental factors are incorporated into the production function of an industrial company but no optimal policy is considered. In [14], a single-stage news-vendor-like stochastic programming model with penalization is formulated; however, risk is not considered. Authors of paper [4] propose a stochastic dynamic model designed for companies producing a homogeneous good (e.g. pulp, paper or cement industry) by either "common" or green technology; however, the study is not focused on any specific emissions trading system. Most recently, [13] formulates and solves a decision-making problem of a steel company using real-life data but, again, without involving risk.

In the present paper, we propose a single-stage mean-risk optimal production model, calibrated to a situation of an anonymous real-life Czech steel company facing emission limits and trading with emissions. Unlike all the existing papers, we take both EUA and CER allowances into account (see Section 2 for details). The prices of the allowances and the demands are stochastic with parameters estimated from historical data.

In line with seminal paper [8], we use variance as the risk measure; even though there exist many, perhaps more sophisticated, risk measures (see, e.g., [1]), the advantage of the variance is its additivity given independent variables, allowing us to decompose the total risk faced by the company and mutually compare its components, which is actually the main goal of our present paper. Another advantage of variance in comparison with, e.g., VaR or CVaR, is its robustness with respect to small changes of distributions and the fact that the corresponding decision problem is always convex, hence easily solvable. Moreover, despite its alleged obsolescence, variance risk criterion was recently shown to be more or less compatible with recent approaches to risk management, namely with the second order stochastic dominance and prospect stochastic dominance (see [5], [7], respectively).

In the empirical part of our paper, we consider a continuum of risk aversion levels and five different scenarios of the mean demand. For each combination of a risk aversion and a scenario, we analyze the impact of emission-related issues on the decisions made by the company. Our results show that the dependence of the optimal behavior on the parameters is non-trivial: with increasing risk aversion, for instance, the optimal production of individual products may be either decreasing or increasing.

Further, we analyze the risk faced by the company, decomposing it into one part caused by demand and another caused by the emission trading. While the magnitude of the former is similar for all scenarios, the latter increases with deviation of the necessary amount of allowances from the number of freely obtained ones; therefore, the emission trading risk is the greatest for the most "pessimistic" and the most "optimistic" scenarios of the demand.

We find that the dependence of the risks on the risk aversion level is not trivial, either: while the behavior of the demand risk is such that we would expect – it is decreasing with the increasing risk aversion – the form of the dependence of the emission trading risk on the risk aversion level varies scenario by scenario.

This paper is organized as follows. After this Introduction, a brief description of emission constraints under consideration is given in Section 2. In Section 3, our optimization model is established. In Section 4, stochastic factors involved in the model are discussed and results of their estimation are presented. The actual application of our model and its results are presented in Section 5. In Section 6, the risk associated with the demand and the risk stemming from the emission trading are discussed and compared. In Section 7, the paper is concluded and suggestions for further research are given. In the Appendix, construction of the variance matrix of the demand for individual products, a Lemma deriving moments of a bivariate log-normal distribution, and a comparison between several models of stochastic factors are given.

2 Legislative Environmental Factors Influencing EU Steel Companies

The European Union has established many environmental legislative constraints in order to reduce emissions produced. In the present paper, several main mandatory regulations are taken into account, namely

- emission caps, i.e., limits setting maximum amounts of selected gas' emissions for one year
- emission limits, i.e., maximum amounts for half an hour (see [9] for details), and
- obligation to cover CO_2 emissions by allowances, i.e., permissions to emit a particular amount of carbon dioxide.

There are two types of allowances which can be applied and traded by European companies within the EU ETS system: the EUA (European Union Allowance), certain amount of which each European steel company obtains for free each year, and the CER (Certified Emission Reduction) allowances. Allowances of both types are traded at several particular markets, either within the secondary emission market or via emission auctions, see [3]. For both historical and institutional reasons, the CER permits are far less expensive than the EUA ones; however, the EU restricts the share of the CERs covering the emissions to be no greater than 10 percent of the total amount (see [2]).

3 The Decision Model

Before defining our model, let us make the following assumptions:

- there are n possible products, each of them potentially serving as material for the production of another one;
- there are m greenhouse gases limited by caps and limits;
- there is only a single decision period, lasting one year;
- the decision is made at the beginning of the period (no ex-post corrections are allowed);
- the production process is homogeneous during the entire period (the rates of both the production and the emissions are the same at any given moment);
- the company generates its profit only by selling its products or by selling unused emission allowances;
- the EUA and CER prices are stochastic;
- the company always exploits the possibility of using CER allowances to cover the 10% limit; in particular, having obtained an amount of r free EUA allowances and having emitted e tons of CO_2, the company always buys CER allowances to cover an amount of $0.1e$ of emissions (and sells excess EUAs, if there are any);[1]
- margins for each product are constant in the entire period (selling prices elastically respond to changes in unit variable costs);
- values of demands are stochastic;
- the production exceeding the demand cannot be sold (or, equivalently, may be sold only for zero price).

Under these assumptions, the profit, up to eventual fixed costs, from final production $\boldsymbol{y} \in \mathbb{R}^n$ is given by

$$V_p(\boldsymbol{y}) = \boldsymbol{c}^T \min(\boldsymbol{y}, \boldsymbol{D}) - \boldsymbol{d}^T \boldsymbol{y}$$

where $\boldsymbol{c} \in \mathbb{R}^n$ is the vector of selling prices, $\boldsymbol{d} \in \mathbb{R}^n$ is the vector of production costs, and $\boldsymbol{D} \in \mathbb{R}^n$ is the vector of (stochastic) demands.

The profit from emission trading, namely from buying the maximal allowed amount of the CERs and subsequent buying/selling of missing/excess EUAs, given primary production $\boldsymbol{x} \in \mathbb{R}^n$, is

$$V_a(\boldsymbol{x}) = -0.1\boldsymbol{e}^T\boldsymbol{x}P^{CER} + [(r + 0.1\boldsymbol{e}^T\boldsymbol{x}) - \boldsymbol{e}^T\boldsymbol{x}]P^{EUA}$$

[1] As the prices of CERs were persistently about five times smaller than those of CERs during recent years, such a behavior would anyway emerge implicitly in our model with a very high probability.

where $\boldsymbol{e} \in \mathbb{R}^n$ is the vector of emissions amount per product, r is the amount of free-allocated emission permits and P^{EUA}, P^{CER} are (stochastic) emission permit prices (EUA type, CER type, respectively).

After a rearrangement, the total profit comes out as

$$V(\boldsymbol{x},\boldsymbol{y}) = V_p(\boldsymbol{y})+V_a(\boldsymbol{x}) = \boldsymbol{m}^T\boldsymbol{y}+\left(r - 0.9 \cdot \boldsymbol{e}^T\boldsymbol{x}\right)\cdot P^{EUA} - 0.1\cdot\boldsymbol{e}^T\boldsymbol{x}\cdot P^{CER} - \boldsymbol{c}^T\left[\boldsymbol{y} - \boldsymbol{D}\right]^+$$

where $\boldsymbol{m} = \boldsymbol{c} - \boldsymbol{d}$ is the margin and $[\cdot]^+$ denotes the positive part.

The (economic and environmental) constraints of the decision-making problem are

$$\begin{array}{lr} (E1) & \boldsymbol{y} = (\mathbf{I} - \mathbf{A})\,\boldsymbol{x}, \\ (E2) & \boldsymbol{x} \leq \boldsymbol{v}, \\ (L1) & \mathbf{H}\boldsymbol{x} \leq \boldsymbol{s}, \\ (L2) & \mathbf{H}\boldsymbol{x} \leq 17,520 \cdot \boldsymbol{l}, \end{array}$$

where $\mathbf{I} \in \mathbb{R}^{n\times n}$ is the unit matrix, $\mathbf{A} \in \mathbb{R}^{n\times n}$ is the matrix of technical coefficients of production, $\boldsymbol{v} \in \mathbb{R}^n$ is the production capacity, $\mathbf{H} \in \mathbb{R}^{m\times n}$ is the matrix of emission coefficients, $\boldsymbol{s} \in \mathbb{R}^m$ is a vector of emission caps, and $\boldsymbol{l} \in \mathbb{R}^m$ is the vector of emission limits per half-hour (there are 17,520 half-hours per year).

The mean-var optimization problem itself is then formulated as

$$\begin{array}{c} \max_{\boldsymbol{x}\geq 0,\boldsymbol{y}\geq 0} \mathbb{E}V(\boldsymbol{x},\boldsymbol{y}) - \lambda \mathrm{var} V(\boldsymbol{x},\boldsymbol{y}) \\ \text{given (E1), (E2), (L1), (L2)} \end{array} \tag{1}$$

where $\lambda \in \mathbb{R}^+$ is the degree of risk aversion.[2]

There are the $n = 5$ raw products manufactured – namely raw iron, brams, plates, profiles and cut shapes, all except the raw iron being sold – and there are $m = 3$ greenhouse gases, namely CO, NO_x, and airborne dust.

According to our agreement with the company, the particular values of the model's parameters are kept confidential.

As the most recent value of the dataset serving as a proxy for the demands (see Subsection 4.2 for details) comes from 10/2012, we take this date as the time of the decision.

4 Stochastic Factors

4.1 Prices of Allowances

Figure 1 shows the evolution of the EUA and CER prices between 1/2008 and 10/2012 (the allowance prices were obtained from the SendeCO2 stock exchange `[sendeco2.com]`). As the increments evidently scale with the absolute prices, we decided to analyze log-returns

$$R_t^{EUA} = \log P_t^{EUA} - \log P_{t-1}^{EUA}, \qquad R_t^{CER} = \log P_t^{CER} - \log P_{t-1}^{CER},$$

[2] It would be possible to formulate the problem by means of only one vector of variables instead of the pair $\boldsymbol{x},\boldsymbol{y}$ and substituting either sales ($\mathbf{y}$) or production ($\mathbf{x}$) according to (E1); however, we decided to leave both vectors in the model for the sake of readability.

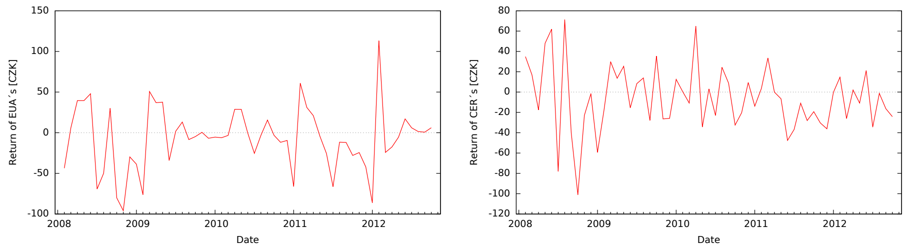
(a) P^{EUA} (b) P^{CER}

Fig. 1 Prices of allowances.

Variable	Mean	Median	Minimum	Maximum
R^{EUA}	-0.0150063	0.00706162	-0.294239	0.268549
R^{CER}	-0.0426674	-0.0419191	-0.590493	0.213908

Variable	Std. Dev.	Coeff.Var.	Skewness	Ex. kurtosis
R^{EUA}	0.130177	8.67480	-0.185831	-0.585533
R^{CER}	0.146702	3.43827	-1.03966	2.10217

Table 1 Summary statistics of monthly log-returns of the allowances



(a) R^{EUA} (b) R^{CER}

Fig. 2 Log-returns of allowances.

rather than absolute prices. The evolution of both returns series is shown in Figure 2, a basic summary statistics may be found in Table 1.

We applied two standard techniques – ARMA(2,2) and GARCH(1,1) – to the returns series. Even though the ARMA analysis of both series (with two past values of the opposite variable as independent variables) showed some of the coefficients as significant, a percentage of residual variance explained by these models, which may also be understood as a percentage improvement of the mean square error of predictions, came out very low (smaller than 6%); therefore, and also because the results of the GARCH analysis were similarly unconvincing, we decided to regard both R^{EUA} and R^{CER} as mutually independent i.i.d. series (for details, see Appendix C)).

The Doornik-Hansen test applied to R^{EUA} did not reject normality so we might regard the year-ahead log-return

$$\tilde{R}^{EUA}_{T^\star} = \sum_{i=1}^{12} R^{EUA}_{T+i}, \qquad T = 10/2012, \qquad T^\star = T + 12,$$

as normal.

For R^{CER}, the same test gave significant results; here, however, we relied on the Central Limit Theorem guaranteeing that standardised sums of independent variables are close to Gaussian, and took $\tilde{R}^{CER}_{T^\star} = \sum_{i=1}^{12} R^{CER}_{T+i}$ as normal, too.

Consequently, we took the conditional distributions of year-ahead prices

$$P^x_{T^\star} = P^x_T \exp\{\tilde{R}^x_{T+i}\}, \qquad x \in \{EUA, CER\},$$

given P^x_T as (approximately) log-normal.

Assuming zero means of R^x (which is justified by their insignificant estimates), this gives, by Lemma 1 (Appendix B),

$$\mathbb{E}(P^x_{T^\star}|\Omega_T) = P^x_T \exp\left\{6\sigma^2_x\right\}, \tag{2}$$

$$\operatorname{var}(P^x_{T^\star}|\Omega_T) = (P^x_T)^2(\exp\{12\sigma^2_x\} - 1)\exp\{12\sigma^2_x\}, \tag{3}$$

and

$$\begin{aligned}&\operatorname{cov}(P^{CER}_{T^\star}, P^{EUA}_{T^\star}|\Omega_T)\\ &\quad = P^{EUA}_T P^{CER}_T(\exp\{12\sigma_{EUA}\sigma_{CER}\operatorname{corr}(R^{CER}, R^{EUA}\} - 1)\\ &\qquad\qquad\qquad \times \exp\{6(\sigma^2_{EUA} + \sigma^2_{CER})\},\end{aligned} \tag{4}$$

where

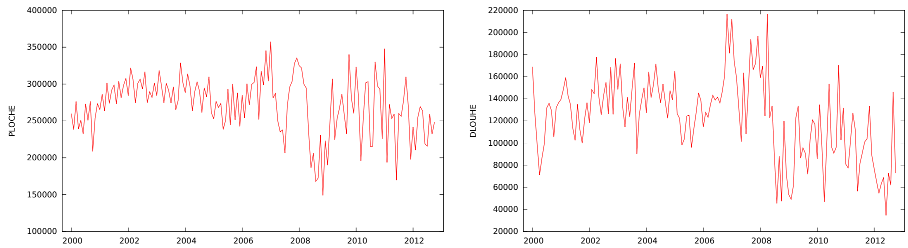
$$\sigma^2_x = \operatorname{var}(R^x), \qquad x \in \{EUA, CER\},$$

and where Ω_T stands for the history up to T.

By substituting the values from Tables 1 and 3 into (2)-(4) we estimated the mean and the variance matrix of $(P^{EUA}_{T^\star}, P^{CER}_{T^\star})^T$ as

$$\begin{bmatrix} 224.648 \\ 34.861 \end{bmatrix} \text{CZK}, \qquad \begin{bmatrix} 10,889.431 & 1,598.308 \\ 1,598.308 & 335.331 \end{bmatrix}, \tag{5}$$

respectively.



(a) S^F (b) S^L

Fig. 3 Nationwide sales [in tons]

Variable	Mean	Median	Minimum	Maximum
S^F	271,421.	274,609.	148,949.	357,424.
S^L	123,163.	126,904.	34,552.	216,539.

Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
S^F	38,660.7	0.142438	−0.611492	0.382079
S^L	36,266.3	0.294457	−0.0931467	−0.0989230

Table 2 Summary statistics of monthly nationwide sales

4.2 Overall Demands

Generally, steel products can be divided into two groups – flat ones (plates and products made of plates) and long ones (tubes, pipes, profiles, etc.). Out of the four final products of the modeled company, the plates and the cut shapes belong to the flat category, while the profiles and the brams are considered long ones.

As no historical data of demands for the individual products were available to us, we based our estimate of future individual demands on the history of aggregate nationwide sales of flat and long products, denoted by S^F, S^L (due to the recent decline in the metallurgical production, we might expect that the industry was always able to satisfy all the demand, so the demand was always equal to the sales).

Figure 3 shows the evolution of S^F and S^L between 1/2000-10/2012, the corresponding summary statistics are given in Table 2.

Similarly to the price returns, we tried to fit S^F and S^L by ARMA(2,2) and GARCH(1,1), both with two lags of the opposite series S as independent variables. Here, both the GARCH and ARMA models gave significant results; however, only ARMA models exhibited a considerable prediction power (the variance reduction for S^L was 27%, see Appendix C); therefore we decided to model the time series of sales equations resulted from re-estimating of the ARMA models after removing their insignificant coefficients:

$$S_t^F = 0.67 \cdot S_{t-1}^F + \eta_t, \qquad S_t^L = 0.86 \cdot S_{t-1}^F + 0.33 \cdot S_{t-1}^L + 0.12 \cdot S_{t-2}^L + \epsilon_t,$$

where η_t is an MA(2) process with coefficients 0.27 and -0.33 and ϵ_t is an MA(2) process with coefficients 0.00 and -0.59. Forecasts, given by this models, are

$$\mathbb{E}(S^F_{T^\star}|\Omega_T) = 268,439 \qquad \mathrm{var}(S^F_{T^\star}|\Omega_T) = 38,742^2$$
$$\mathbb{E}(S^L_{T^\star}|\Omega_T) = 106,900 \qquad \mathrm{var}(S^L_{T^\star}|\Omega_T) = 28,871^2.$$

As the Doornik-Hansen normality tests applied to the residuals of the models gave strongly insignificant results, we further presumed the forecast to be bivariate normal with $\mathrm{corr}(S^F_{T^\star}, S^L_{T^\star}|\Omega_T)$ estimated by a sample correlation between S^F and S^L being equal to 0.52.[3]

4.3 Individual Demands

As a natural prediction of the year-ahead demands for the individual products towards the examined company, denoted by

$$\boldsymbol{D}_{T^\star} = (D_1, D_2, D_3, D_4)^T.$$

we took the present sales of the company:

$$\boldsymbol{\mu}_D = (20, 510, 90, 28)^T. \tag{6}$$

Due to the absence of historical values of $\boldsymbol{D}$, however, we could not estimate its variance matrix directly; instead, we constructed it out of (estimated) moments of $(S^F_{T^\star}, S^L_{T^\star})$ under the simplifying assumption of an equal (unknown) correlations between the components

$$\mathrm{corr}(D_i, D_j) \equiv \rho, \qquad i \neq j \tag{7}$$

with the resulting vector of standard deviations and the correlation

$$\boldsymbol{\sigma}_D = (3.02, 53.27, 25.45, 7.97)^T, \qquad \rho = 0.051 \tag{8}$$

(from which the variance matrix of $\boldsymbol{D}$ might be constructed thanks to (7)). For details of the constructon of the variance matrix, see Appendix A.

Further, as the distribution of $\boldsymbol{D}$ is close to normal (see Appendix A), we further assumed $\boldsymbol{D}$ to be Gaussian with moments (6) and (8).

4.4 Dependence between Prices and Demands

As it is evident from the estimated correlation matrix of $(S^F, S^L, R^{EUA}, R^{CER})$, shown in Table 3, no relation between (vectors of) prices and demands could be provd; therefore, we took (S^F, S^L) independent of (R^{EUA}, R^{CER}) and, consequently, $\boldsymbol{D}_{T^\star}$ independent of $(P^{EUA}_{T^\star}, P^{CER}_{T^\star})$.

[3] We could estimate the conditional correlation by using an unconditional one as the forecast value is 12 lags ahead, i.e., with a small influence of the historical values.

S^F	S^L	R^{EUA}	R^{CER}	
1.00	0.53***	0.22	0.12	S^F
	1.00	0.04	0.07	S^L
		1.00	0.67***	R^{EUA}
			1.00	R^{CER}

Table 3 Correlation matrix of prices and sales (*** denotes 1‰significance level)

4.5 The Value Function

Given models from Sections 4.1 - 4.5, the mean and the variance of the value function are

$$\mathbb{E}V(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{m}^T\boldsymbol{y} + \left(r - 0.9\cdot\boldsymbol{e}^T\boldsymbol{x}\right)\cdot\mathbb{E}P_{T^\star}^{EUA} - 0.1\cdot\boldsymbol{e}^T\boldsymbol{x}\cdot\mathbb{E}\left(P_{T^\star}^{CER}\right) - \boldsymbol{c}^T\mathbb{E}\left[\boldsymbol{y}-\boldsymbol{D}\right]^+, \tag{9}$$

$$\begin{aligned}\mathrm{var}(V(\boldsymbol{x},\boldsymbol{y})) = \Big[&\left(r - 0.9\cdot\boldsymbol{e}^T\boldsymbol{x}\right)^2\cdot\mathrm{var}\left[P_{T^\star}^{EUA}\right] + \left(0.1\cdot\boldsymbol{e}^T\boldsymbol{x}\right)^2\cdot\mathrm{var}\left(P_{T^\star}^{CER}\right)\\ &+\left(0.1\cdot\boldsymbol{e}^T\boldsymbol{x}\right)\left(r-0.9\cdot\boldsymbol{e}^T\boldsymbol{x}\right)\mathrm{cov}(P_{T^\star}^{CER},P_{T^\star}^{EUA}) + \boldsymbol{c}^T\mathrm{var}((\boldsymbol{y}-\boldsymbol{D})^+)\boldsymbol{c}\Big],\end{aligned} \tag{10}$$

respectively, with $\mathbb{E}P_{T^\star}^{EUA}$, $\mathbb{E}P_{T^\star}^{CER}$, $\mathrm{var}\left(P_{T^\star}^{EUA}\right)$, $\mathrm{var}\left(P_{T^\star}^{CER}\right)$ and $\mathrm{cov}(P_{T^\star}^{CER},P_{T^\star}^{EUA})$ taken from (5) and $\mathrm{E}((\boldsymbol{y}-\boldsymbol{D})^+)$ and $\mathrm{var}((\boldsymbol{y}-\boldsymbol{D})^+)$ evaluated by Monte Carlo estimates computed by means of 60,000 realizations of $\boldsymbol{D}$.

Remark. For the time series analysis, `Gretl` software package was used; to generate the Monte Carlo sample mentioned above, the `MASS` package of `R` was employed. The corresponding scripts, together with a spreadsheet for calculating moments of $\boldsymbol{D}$, may be downloaded from `https://github.com/cyberklezmer/modsc15`.

5 Solution of the Problem

In order to analyze the situation under various circumstances, we considered five possible mean vectors of the demand while keeping the remaining parameters of the stochastic factors unchanged; in particular, we considered scenarios of the form

$$\mathrm{S}_k : \mathbb{E}\boldsymbol{D}_{T^\star} = (k_F\cdot 20, k_F\cdot 510, k_L\cdot 90, k_L\cdot 28), \qquad 1\le k\le 5,$$

(compare with (6)) with the following coefficients of the individual scenarios:

$$\mathrm{S}_1: \ k_F = \frac{\min_t(S_t^F)}{\hat{S}^F} = 0.55, \quad k_L = \frac{\min_t(S_t^L)}{\hat{S}^L} = 0.32,$$

$$\mathrm{S}_2: \ k_F = \frac{1}{2}\left(1+\frac{\min_t(S_t^F)}{\hat{S}^F}\right) = 0.78, \quad k_L = \frac{1}{2}\left(1+\frac{\min_t(S_t^L)}{\hat{S}^L}\right) = 0.66,$$

$$\mathrm{S}_3: \ k_F = 1, \qquad k_L = 1, \quad \text{(the true state)}$$

$$\mathrm{S}_4: \; k_F = \frac{1}{2}\left(1 + \frac{\max_t(S_t^F)}{\hat{S}^F}\right) = 1.17, \quad k_L = \frac{1}{2}\left(1 + \frac{\max_t(S_t^L)}{\hat{S}^L}\right) = 1.51$$

$$\mathrm{S}_5: \; k_F = \frac{\max_t(S_t^F)}{\hat{S}^F} = 1.33, \quad k_L = \frac{\max_t(S_t^L)}{\hat{S}^L} = 2.03;$$

here, $\hat{S}^F = \mathbb{E}(S^F_{T^\star}|\Omega_T)$, $\hat{S}^L = \mathbb{E}(S^L_{T^\star}|\Omega_T)$, $\sigma_F = \mathrm{std}(S^F_{T^\star}|\Omega_T)$, $\sigma_L = \mathrm{std}(S^L_{T^\star}|\Omega_T)$, Further, we considered values from an interval, i.e.,

$$\lambda \in L, \qquad L = [5 \times 10^{-6}; 5 \times 10^{-4}],$$

which is a subset of interval $[1 \times 10^{-6}; 5 \times 10^{-3}]$ recommended by [10].[4]

In order to gain perspective, we solved the model for S_3 without involving the risk, i.e., with $\lambda = 0$, first. Consequently, the problem was evaluated for each scenario $\mathrm{S}_1, \ldots, \mathrm{S}_5$ and for each λ from L with step 5×10^{-6}.

In all these cases, the optimization problem was solved by `MS Excel 2013`, which we regarded as sufficient for dealing with problem (1) which is convex and hence solvable even by an average-quality solver. To automate the process of repeated computations, a Visual Basic macro has been created.

The results for S_3 and $\lambda = 0$ are shown in Table 4. Here, the company would be losing 110.4 million CZK (including costs of 3.67 million CZK spent for extra allowances), which is due to low values of demand.[5] It may be also observed that the influence of the emissions trading on the company is weak. The reason for the latter is that none of the limits is reached and little trading has to be done (the amount of CO_2 produced is close to the amount of freely allocated permits).

Table 5 compares results of all the considered scenarios with $\lambda = 0$.

	Iron	Brams	Plates 3	Profiles	Cut shapes
Production [tons]	328,712	439,481	519,375	82,641	25,470
Sales [tons]	0	11,027	481,323	82,641	25,470
Production capacity used [%]	48	49	69	69	85

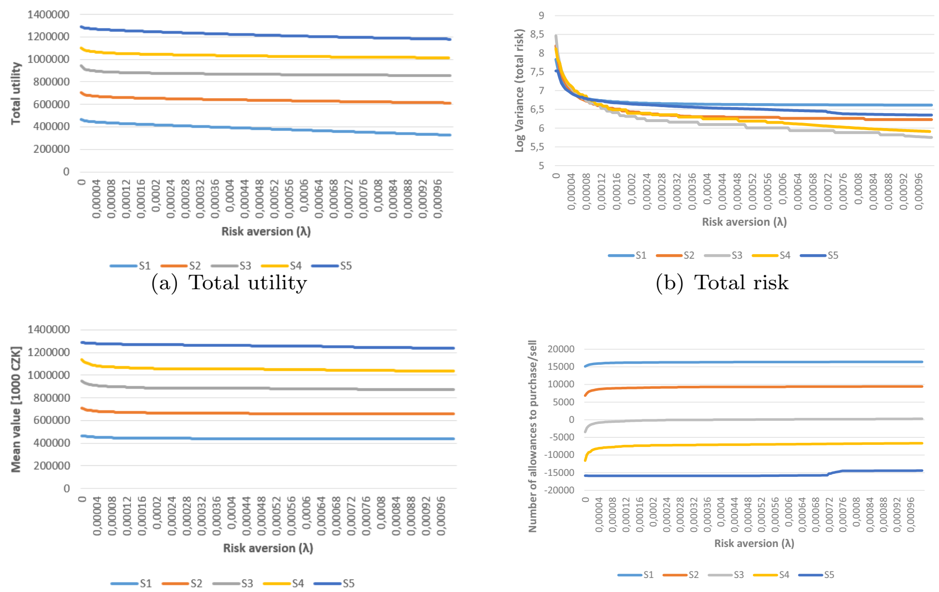
Table 4 Results of the model without risk in Scenario 3 - production portfolio

	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5
Total profit [1000 CZK]	-592,935	-350,866	-110,381	77,018	229,300
Total margin [1000 CZK]	465,951	708,020	948,505	1,135,904	1,388,186
Profit by permits [1000 CZK]	14,867	5,587	3,661	10,930	14,786
Slack/surplus of EUA's [pcs]	15,092	6,840	-3,469	-11,574	-15,860

Table 5 Results of the model without risk for all scenarios - financial measures

[4] We did not consider the whole recommended interval because the results, given the excluded values, differed from the rest only negligibly according to our preliminary computations.

[5] The present low demand is a persisting consequence of the economic crisis and company production would be unsustainable under these conditions over a longer term.



(a) Total utility (b) Total risk

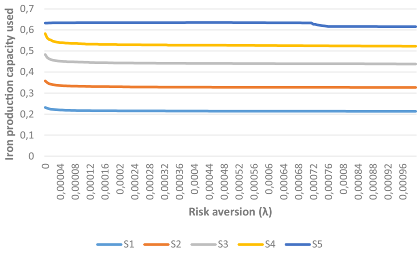
(c) Mean values (d) Allowances purchased/sold

Fig. 4 Results under changing λ ((a) Total utility, (b) Total risk, (c) Mean value of the objective function, (d) Amount of allowances purchased/sold)

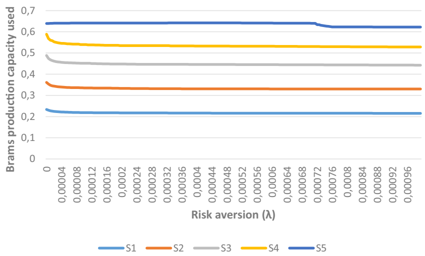
The results given a positive risk aversion are displayed graphically by Fig. 4 showing the overall utility, the risk, the mean value and total amount of allowances purchased given individual scenarios and values of λ. Fig. 5 displays the proportions of production capacity exploited given different scenarios and λ's and Fig. 6 shows the dependence of sales on λ. Mostly, production decreases with the increasing risk aversion, with the exception of brams given S_5 till $\lambda \doteq 0.0007$, when the graph is slightly increasing. As to the sales, an interesting situation happens given S_5: here, with an increasing risk aversion, the brams sales suddenly stop, which is because it simultaneously serves as raw material for making another product, which may be sold with a smaller risk.

In all the cases considered, neither emission limits nor emission caps were reached; even for Scenario 5, the space given by caps and limits was not used by more than 70%. So, the emissions trading is the only influential environmental factor for companies similar to this one, given the present low demand.

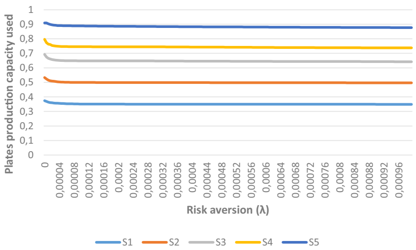
The production constraints were mostly not reached, either; the only exceptions are the profiles given S_5 (for all λ) and the cut shapes in the cases of S_4 and S_5 (for low values of λ).



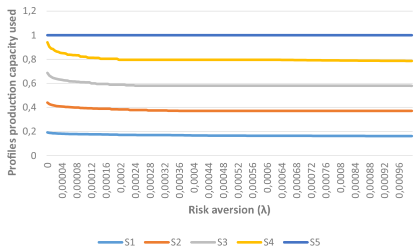
(a) Iron



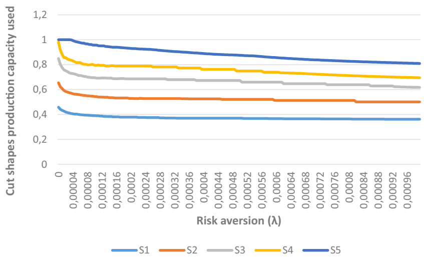
(b) Brams



(c) Plates



(d) Profiles



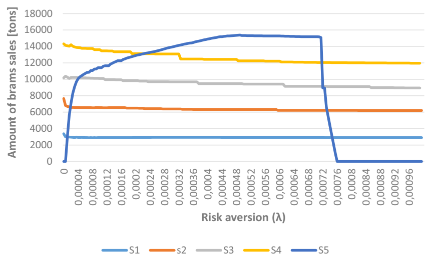
(e) Cut shapes

Fig. 5 Production capacity usage with changing λ

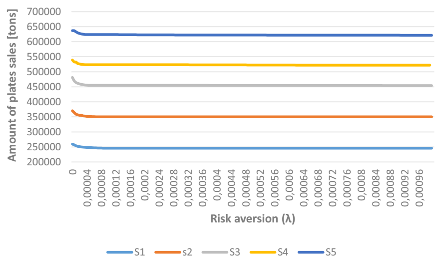
6 Analysis of Risks

The values of the total risk for all scenarios and λ's under consideration are shown in Fig. 4b, the components of risk – namely the risk associated with the demand and the risk originated from the emission trading – are depicted in Fig. 7 (the values of risk are shown in a log scale). Note that the risk might be unambiguously decomposed thanks to the independence of the demand and the prices of the allowances.

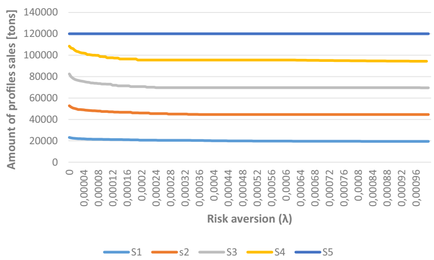
While the risk caused by the demand takes similar values for all the scenarios (which is mainly due to the fact that we assume the same variance given all the scenarios), the dependence of the emissions trading risk on scenarios and lambdas is more complex: in particular, there is no common pattern of



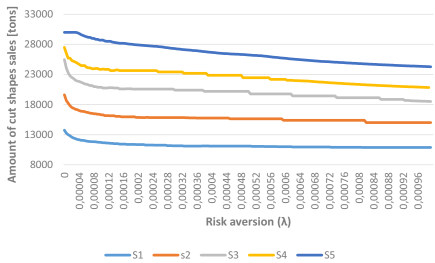
(a) Brams



(b) Plates

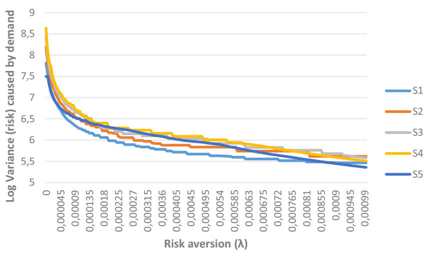


(c) Profiles

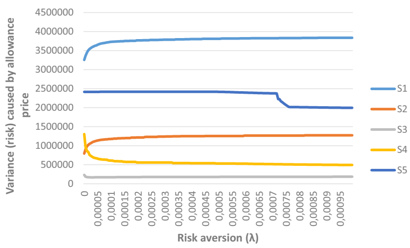


(d) Cut shapes

Fig. 6 Amount of sales with changing λ



(a) Risk (demand)



(b) Risk (allowances)

Fig. 7 Variance given by demand and emissions trading

the dependence. Quite naturally, the least trading risk is achieved given S_3, which is because the necessary amount of the permits is close to the amount allocated for free.

Given S_4 and S_5, the company does not have enough permits and must purchase the missing ones at the market, which subsequently increases the risk. However, with the increasing risk aversion coefficient, decreasing the amount of production causes a reduction of the amount of CO_2 released, which consequently decreases the need for additional allowances and, consequently, the corresponding risk.

Given S_1 and S_2, when the company does not use all of the permits it has obtained for free, the risk is increasing too but the increase is now caused by selling additional permits. If the company wanted to reduce the risk related to emissions trading here, it would have to reduce the unused amount of allowances by increasing production (which is exactly the opposite of the cases discussed above).

The shape of the graph describing the risk given S_3 is also worthy of interest: here, the value $\lambda \doteq 0.00003$ can be considered as a break point for the risk related to emissions trading. For lower risk aversions, the company has a lack of allowances, and the emission risk curve is decreasing and vice versa.

In Fig. 8, the risk caused by demand and that caused by trading are compared. It may be seen here that risk caused by emissions trading can exceed the risk caused by demand when λ is sufficiently high.

Fig. 9 shows efficient frontiers for all scenarios. The "shorter" shape given S_5, when the highest values of the demand are reached, may be explained by the (already mentioned) fact that, starting from a certain value of λ, one of the products (brams) is not sold at all (it is produced only as a raw material for other products) which leads to a substantial risk decrease.

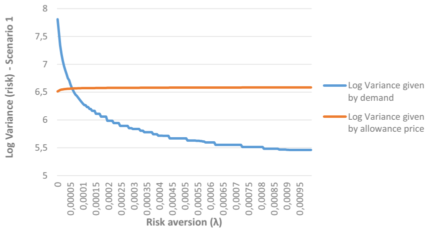
7 Conclusions

In this paper, a mean-variance profit-maximization model for companies under emission control was proposed. Even though the model was designed specifically for steel companies and calibrated to the situation of a particular one, it might be, after minor revision, applied to any, possibly non-steel, industrial company.

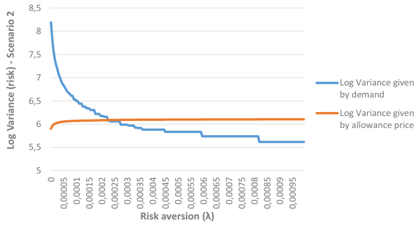
It was shown that the constraints stemming from the carbon regulation may have substantial effects on the decisions of such a company, so incorporating these constraints into decision-making models is highly beneficial.

In the particular situation of the modeled company, the emission limits have not been reached and the emissions trading did not have dramatic effects, which is because the Czech steel sector currently suffers from low demand. However, under different scenarios of demand, when the number of allowances needed differs positively or negatively from the amount allocated for free, the effects of emission trading increase, bringing a new source of risk.

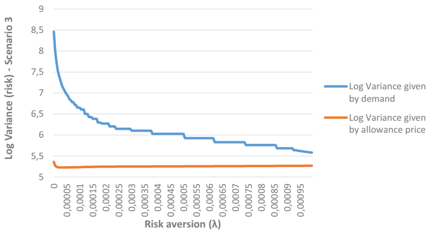
Of course, there are many ways in which to go further in this research: The first enhancement that suggests itself could be to model the decision-making by a two-stage stochastic programming incorporating a compensation for unsold production. A further generalization involving a multi-stage decision making would help to incorporate inter-temporal effects, which are currently neglected. It would also be worth using risk measures other than the variance; out of various options, a (nested) CVaR seems to be the most reasonable choice because it may be easily incorporated into the (convex) decision problem.



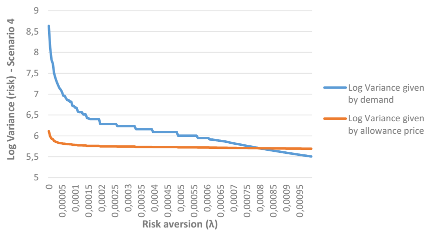
(a) Scenario 1



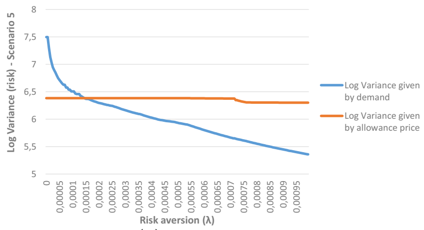
(b) Scenario 2



(c) Scenario 3



(d) Scenario 4



(e) Scenario 5

Fig. 8 Risk caused by demand and by emission trading with changing λ for particular scenarios

8 Acknowledgement

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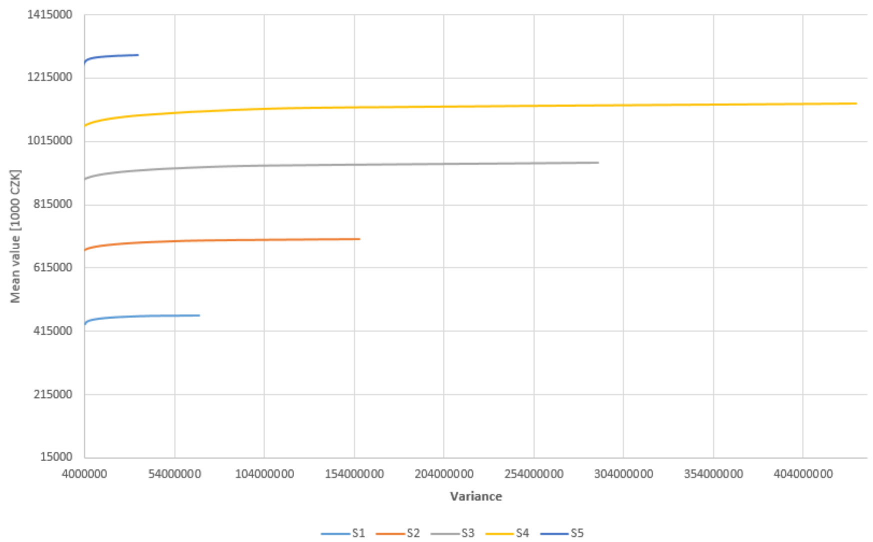

Fig. 9 Efficient frontier (dependence between risk and mean value)

References

1. Artzner, P., Delbaen, F., Eber, J.M., Heath, D.: Coherent measures of risk. Mathematical finance **9**(3), 203–228 (1999)
2. European Commission: Directive 2003/87/EC of the european parliament and of the council of 13 october 2003 establishing a scheme for greenhouse gas emission allowance trading within the community and amending council directive 96/61/ec. Union, European **46**, 32–46 (2003)
3. European Commission: Directive 2009/29/EC of the european parliament and of the council of 23 april 2009 amending directive 2003/87/EC so as to improve and extend the greenhouse gas emission allowance trading scheme of the community. Journal of the European Union **5**, 2009 (2009)
4. Gong, X., Zhou, S. X.: Optimal Production Planning with Emissions Trading. Operations Research **61**(4), 908–924, (2013)
5. Kopa, M.: Comparison of various approaches to portfolio efficiency. In: Proceedings of the 29th International Conference on Mathematical Methods in Economics, pp. 64–69 (2011)
6. Letmathe, P., Balakrishnan, N.: Environmental considerations on the optimal product mix. European Journal of Operational Research **167**(2), 398–412 (2005)
7. Levy, H., Levy, M.: Prospect theory and mean-variance analysis. Review of Financial Studies **17**(4), 1015–1041 (2004)
8. Markowitz, H.: Portfolio selection. The journal of finance **7**(1), 77–91 (1952)
9. Parliament of the Czech Republic, G.: Law No. 415/2012 on permitted level of pollution and its detection. (2012)
10. Raskin, R., Cochran, M.J.: Interpretations and transformations of scale for the pratt-arrow absolute risk aversion coefficient: Implications for generalized stochastic dominance. Western Journal of Agricultural Economics **11**(2), 204–210 (1986)
11. Rong, A., Lahdelma, R.: CO_2 emissions trading planning in combined heat and power production via multi-period stochastic optimization. European Journal of Operational Research **176**(3), 1874–1895 (2007)
12. Tang, H., Song, G.: Optimization of enterprise production based on carbon emissions credits trading. In: Proceedings of the 2nd International Conference on Green Communications and Networks 2012 (GCN 2012): Volume 5, pp. 325–330. Springer (2013)

13. Zapletal, F.: Mean-risk model optimizing the heavy industrial company's profit with respect to environmental aspects. Proceedings: 32nd International Conference on Mathematical Methods in Economics Conference in Olomouc (2014)
14. Zhang, B., Xu, L.: Multi-item production planning with carbon cap and trade mechanism. International Journal of Production Economics **144**(1), 118–127 (2013)

Appendix

A Model for Demands

The goal of the present Section is to deduce a variance matrix of demands (D_1, D_2, D_3, D_4) from their expectations and a variance matrix of (S^F, S^L).

To this end, assume that there are N potential customers where N is Poisson with parameter κ. Once during each time period (year in our case), each customer decides which amounts $\boldsymbol{Q} = (Q_1, Q_2, Q_3, Q_4) \in [0, \infty)^4$ of the products $1, 2, 3, 4$ he buys and, subsequently, which of these products he buys at the company of our interest.

As we do not have information about overall demand for individual products, we have to make simplifying assumptions, namely that the ratios of expectations and roots of second moments of Q's are the same within the same category, i.e.,

$$\mathbb{E}Q_1/\sqrt{\mathbb{E}Q_1^2} = \mathbb{E}Q_2/\sqrt{\mathbb{E}Q_2^2}, \qquad \mathbb{E}Q_3/\sqrt{\mathbb{E}Q_3^2} = \mathbb{E}Q_4/\sqrt{\mathbb{E}Q_4^2}. \tag{11}$$

Further, we assume that whenever a customer decides to buy a non-zero amount of two products from the same category, he will buy it at the same company; to formalize this, denote J^F and J^L zero-one variables indicating whether the customer decided to buy products from category F, L, respectively, at our company, and assume J^F and J^L to be mutually independent, independent of $\boldsymbol{Q}$.

Then, given that the demands of individual customers are i.i.d. with the same distribution as $\boldsymbol{Q}$, the distributions of S^F and S^L are Compound Poisson with embedded variables

$$K^F := (Q_1 + Q_2), \qquad K^L := (Q_3 + Q_4),$$

respectively, both with intensity κ; consequently, according to the well-known formulas for moments of the Compound Poisson distribution,

$$\mathbb{E}S^F = \kappa\mathbb{E}K^F, \qquad \mathbb{E}S^L = \kappa\mathbb{E}K^L,$$

$$\mathrm{var}S^F = \kappa\mathbb{E}(K^F)^2, \qquad \mathrm{var}S^L = \kappa\mathbb{E}(K^L)^2,$$

and, by the Law of Total Covariance,

$$\begin{aligned}
\mathrm{cov}(S^F, S^L) &= \mathbb{E}\mathrm{cov}(S^F, S^L|N) + \mathrm{cov}(\mathbb{E}(S^F|N), \mathbb{E}(S^L|N)) \\
&= \mathbb{E}\mathrm{cov}\left(\sum_i^N K_i^F, \sum_i^N K_i^L \middle| N\right) + \mathrm{cov}(N\mathbb{E}K^F, N\mathbb{E}K^L) \\
&= \mathbb{E}N\mathrm{cov}(K^F, K^L) + \mathrm{var}(N)\mathbb{E}(K^F)\mathbb{E}(K^L) \\
&= \kappa[\mathrm{cov}(K^F, K^L) + \mathbb{E}(K^F)\mathbb{E}(K^L)] = \kappa\mathbb{E}(K^F K^L).
\end{aligned} \tag{12}$$

Similarly, D_1 and D_2 are Compound Poisson with embedded variables $J^F Q_1$, $J^F Q_2$, respectively, both with intensity κ , i.e.,

$$\mathbb{E}D_i = \kappa\mathbb{E}(J^F Q_i) = p_F\kappa\mathbb{E}Q_i, \qquad \mathrm{var}D_i = \kappa\mathbb{E}(J^F Q_i)^2 = \kappa\mathbb{E}(J^F)^2\mathbb{E}(Q_i)^2 = p_F\kappa\mathbb{E}Q_i^2, \quad i = 1, 2, \tag{13}$$

where $p_F = \mathbb{P}[J^F = 1]$. Further, as $D_1 + D_2$ is Compound Poisson with intensity κ and embedded variable $J^F K^F$, we have

$$\mathbb{E}(D_1 + D_2) = p_F \kappa \mathbb{E}(K^F) = p_F \mathbb{E} S^F \tag{14}$$

$$\begin{aligned} \mathrm{var}(D_1 + D_2) &= p_F \kappa \mathbb{E}(J^F K^F)^2 \\ &= \kappa \mathbb{E}(J^F)^2 \mathbb{E}(K^F)^2 = p_F \mathrm{var}(S^F) = \frac{\mathbb{E}(D_1) + \mathbb{E}(D_2)}{\mathbb{E} S^F} \mathrm{var}(S^F) \end{aligned} \tag{15}$$

(we have used (14) for the last equality) and, symmetrically,

$$\mathbb{E} D_i = p_L \kappa \mathbb{E} Q_i, \qquad \mathrm{var} D_i = p_L \kappa \mathbb{E} Q_i^2, \qquad i = 3, 4, \tag{16}$$

$$\mathbb{E}(D_3 + D_4) = p_L \mathbb{E} S^L, \qquad \mathrm{var}(D_3 + D_4) = \frac{\mathbb{E}(D_3) + \mathbb{E}(D_4)}{\mathbb{E} S^L} \mathrm{var}(S^L), \tag{17}$$

where $p_L = \mathbb{P}[J^L = 1]$. Moreover, similarly to (12),

$$\begin{aligned} &\mathrm{cov}(D_1 + D_2, D_3 + D_4) \\ &\quad = \kappa \mathbb{E}(J^F K^F J^L K^L) = \kappa \mathbb{E}(J^F) \mathbb{E}(J^L) \mathbb{E}(K^F K^L) = p_F p_L \mathrm{cov}(S^F, S^L). \end{aligned} \tag{18}$$

Now, to construct the variance matrix of $\boldsymbol{D}$ under (7), it suffices to compute ρ and $v_i := \mathrm{var}(D_i)$, $i = 1, \ldots, 4$, which we do in the following way: from (13) and (17), employing (11), we get that

$$v_1 / v_2 = (\mathbb{E} D_1 / \mathbb{E} D_2)^2, \tag{19}$$

$$v_3 / v_4 = (\mathbb{E} D_3 / \mathbb{E} D_4)^2, \tag{20}$$

respectively, while from (15) and (17) it follows that

$$v_1 + 2\rho\sqrt{v_1 v_2} + v_2 = \frac{\mathbb{E}(D_1) + \mathbb{E}(D_2)}{\mathbb{E} S^F} \mathrm{var}(S^F), \tag{21}$$

$$v_3 + 2\rho\sqrt{v_3 v_4} + v_4 = \frac{\mathbb{E}(D_3) + \mathbb{E}(D_4)}{\mathbb{E} S^L} \mathrm{var}(S^L); \tag{22}$$

finally, from (18), we get

$$\rho \sum_{i,j=1}^{2} \sqrt{v_i v_j} = \frac{\mathbb{E}(D_1) + \mathbb{E}(D_2)}{\mathbb{E} S^F} \frac{\mathbb{E}(D_3) + \mathbb{E}(D_4)}{\mathbb{E} S^L} \mathrm{cov}(S^F, S^L). \tag{23}$$

As (19)-(23) is in fact a system of five equations with five variables $\rho, v_1, \ldots, v_4$, we may easily get the values of $\rho, v_1, \ldots, v_4$ by a (numerical) solution.

Finally, as (Compound) Poisson variables may be approximated by normal ones provided that their intensities are large enough, our construction may be used in the case when S^F and S^L are normally distributed and, for the same reason, the resulting vector (Q_1, Q_2, Q_3, Q_4) may be regarded as normal.

B Bivariate Log-Normal Distribution

Lemma 1 *Let (X, Y) be a bivariate normal random vector with zero mean, variances σ_x^2, σ_y^2, respectively, and correlation ρ. Then*

(i) both $\exp\{X\}$ and $\exp\{Y\}$ are log-normal with means $\exp\left\{\frac{\sigma_x^2}{2}\right\}$, $\exp\left\{\frac{\sigma_y^2}{2}\right\}$, respectively and variances $\exp\{\sigma_x^2\}(\exp\{\sigma_x^2\} - 1)$, $\exp\{\sigma_y^2\}(\exp\{\sigma_y^2\} - 1)$ respectively.

(ii) $\mathrm{cov}(\exp\{X\}, \exp\{Y\}) = (\exp\{\sigma_x \sigma_y \rho\} - 1) \exp\left\{\frac{\sigma_x^2 + \sigma_y^2}{2}\right\}$

Proof While (i) might be calculated using well-known formulas for moments of a log-normal distribution, (ii) follows from the definition of covariance:

$$\begin{aligned}\operatorname{cov}(\exp\{X\},\exp\{Y\}) &= \mathbb{E}(\exp\{X\}\exp\{Y\}) - \mathbb{E}(\exp\{X\})\mathbb{E}(\exp\{Y\})\\ &= \mathbb{E}(\exp\{X+Y\}) - \mathbb{E}(\exp\{X\})\mathbb{E}(\exp\{Y\}) = \exp\left\{\frac{\sigma_{x+y}^2}{2}\right\} - \exp\left\{\frac{\sigma_x^2+\sigma_y^2}{2}\right\}\end{aligned}$$

where $\sigma_{x+y}^2 = \operatorname{var}(X+Y) = \sigma_x^2 + 2\sigma_x\sigma_y\rho + \sigma_y^2$.

C Time Series Model Selection

The Table below shows a comparison of three time series models for each series we examined. Here, "ARMA" means ARMAX(2,2,2) with two lags of the opposite variable as independent variables (for R^{EUA}, the opposite variable is R^{CER}, for S^L, the opposite variable is S^F, etc.). "GARCH" means GARCHX(1,1,2) with two lags of the opposite variable. "IID" means mutually independent, identically distributed series. Symbol R stands for a percentage improvement of the Mean Square Error of a particular model when compared with IID. Bold typeface denotes the best (most favourable) values. All the evaluations were done after removing insignificant coefficients from the model and a re-estimation.

	Akaiake criterion			MSE improvement	
	IID	ARMA	GARCH	ARMA	GARCH
R^{EUA}	−50	−56	**−57**	0.06	0.06
R^{CER}	59	52	**−22**	**0.05**	0.00
S^F	3,691	3,655	**3,631**	**0.13**	0.07
S^L	3,671	**3,534**	3,553	**0.27**	0.21