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Estimation of Hopping Rates From Real Traffic Trajectories

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Abstract. Variety of hopping particle systems (ZRP, TASEP, MTP) have been investigated from the point of view of the traffic flow modelling. Such models are characterized by the local hopping rates determining the model dynamics. Several techniques of estimating appropriately the hopping rates from real traffic trajectories are introduced. Properties of corresponding models are discussed.

Key words: Hopping particle systems, estimation of hopping rates, Intelligent Driver Model.

1 Introduction

A number of hopping particle models are motivated to be models of traffic flow [1, 2, 5, 7, 8]. An important aspect of traffic flow are traffic jams. The mechanism of the traffic-jam formation is being explained by the effect of condensation in the associated hopping particle model, e.g. in [7, 4]. The condensation can be understood as a creation of one large cluster somewhere in the system in equilibrium, which blows up to proportions.

Our aim is to discuss, whether the condensation effect is really somehow connected to the formation of phantom traffic jam. The idea is straightforward: we tried to extract the corresponding hopping rates of considered model from real traffic trajectories, or rather from the simulation equivalents of real traffic.

This article provides an overview of methods we used for that purpose, discussing their advantages and disadvantages. The concept has been used for zero-range process using the simulation from the intelligent driver model of traffic flow. Although the overview contains several dead-ends and the results are not very satisfactory, we hope that it shows the way we should continue in the project.

2 Mass Transport Process

The investigated processes belong to the class of mass transport processes. Although we mainly focus on the zero-range process in this article, we will define the problem in more general way to open the platform for generalizations. Mass transport process (abbr. MTP) can be understood as a generalization of zero range process enabling more particles to hop from the block of n particles [12]. In [3] the MTP was treated for both discrete and continuous masses.

Let us define the process on the circular lattice, i.e., the lattice consists of L sites $\{0, 1, \ldots, L-1\}$. N particles are moving along the lattice by hopping from site x to the neighbouring site $x+1$. The hops are governed by the *chipping function* φ, where $\varphi(k|n)$ denotes the probability that exactly k particles from the block of the length n hop to the neighbouring site as depicted in Figure 1. It holds

$$\sum_{k=0}^{n} \varphi(k|n) = 1\,, \quad \varphi(k|n) \geq 0 \quad \text{for all} \quad n \in \mathbb{N}_0\,, \quad 0 \leq k \leq n\,. \tag{1}$$

There are two possible interpretations of the MTP as a traffic flow model. Both approaches are visualized in Figure 1.

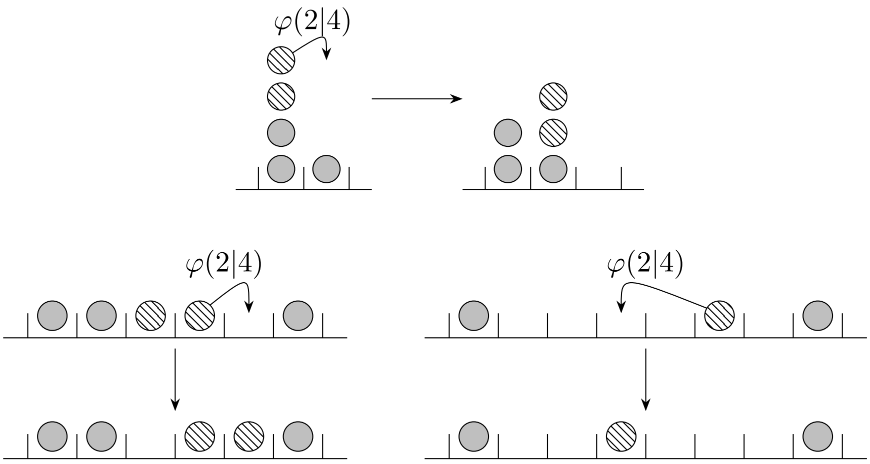

Figure 1: Mass transport model of traffic flow can be interpreted in the particle-based (left) and hole-based (right) mapping.

1. (particle-based) More natural way is to associate the pile of n particles in one site of the MTP with the block of consecutive particles in a linear lattice of the traffic flow model. The hop of k particles to the neighbouring site of the MTP then represents a situation in the traffic model, when k vehicles detach from the block and move one site forward, i.e., they may either be attached to an preceding block or form a new block of k vehicles.

2. (hole-based) Another way is to associate the pile of particles in MTP to the empty sites in front of a vehicle in the traffic model. The hop of k particles in MTP than corresponds to the hop of one vehicle in the traffic model by k sites forward.

The most important property of the MTP is the fact that under certain conditions the process has a factorized steady state.

Theorem 1 *The mass transport process with parallel update and periodic boundaries with chipping function $\varphi(k|n)$ has a stationary distribution $F(\boldsymbol{\eta})$ decomposable to the factorized form*

$$F(\eta_0, \eta_1, \ldots, \eta_{L-1}) = \frac{1}{Z_{L,N}} \prod_{j=0}^{L-1} f(\eta_j) \tag{2}$$

if and only if the cross-ratio

$$R(k,n) = \frac{\varphi(k+1|n+2)\varphi(k|n)}{\varphi(k+1|n+1)\varphi(k|n+1)} \tag{3}$$

for $0 \le k \le n$ with $\varphi(0|0) = 1$ does not depend on k,i.e., $R(k,n) =: R(n)$. Then the one site marginals $f(n)$ are in the form

$$f(n) = f(0) \left(\frac{f(1)}{f(0)} \right)^n \cdot \prod_{j=0}^{n-2} R(j)^{-n+j+1}\,, \quad n \ge 2\,. \tag{4}$$

Proof. The proof is given in [12]. □

It is convenient to choose $f(0)$ and $f(1)$ to fulfil the normalization conditions

$$\sum_{n=0}^{+\infty} f(n) = 1\,, \quad \text{and} \quad \sum_{n=0}^{+\infty} n f(n) = \varrho := \frac{N}{L}\,. \tag{5}$$

Since the result of Thm 1 follows from the theory stated in [3] for both discrete and continuous time dynamical rules, one can consider also continuous time update and derive a similar result for chipping rates $\varphi(k|n)$ as well.

The investigated zero-range process (ZRP) is a special case of MTP, which allows the the hop of at most one particle, i.e.,

$$\varphi(1|n) = g(n)\,, \quad \varphi(0|n) = 1 - g(n)\,, \quad \varphi(k|n) = 0 \text{ for } k \ge 2\,. \tag{6}$$

Such chipping function fulfils the factorized steady state condition and the factors are

$$f(n) \propto \prod_{k=1}^{n} \frac{1}{g(k)} \tag{7}$$

for the continuous time update and

$$f(n) \propto \frac{1-g(1)}{1-g(n)} \prod_{k=1}^{n} \frac{1-g(k)}{g(k)} \tag{8}$$

for the parallel update.

For several of such processes including the ZRP it has been shown, that the large L limit (i.e., $L \to +\infty$ and $N = \lfloor \varrho L \rfloor$) of the factors $f(n)$ converge to the marginals of the product measure of the same process defined on an infinite line ([6] and references therein).

The effect of condensation means that there is a critical value ϱ_c such that for density $\varrho > \varrho_c$ there evolves a block of particles, which size grows above all limits. How is that captured by the stationary measure? Let the process has the one site marginal in the form $f(n) = z^n a_n$. The sum

$$\varrho(z) = \sum_{n=0}^{+\infty} n a_n z^n \tag{9}$$

converges for $z \in (-z_c, z_c)$ and diverges for $z \notin (-z_c, z_c)$. For $\rho > \rho_c := \rho(z_c)$ it means that in the large L limit $f(n;\rho) = f(n;\rho_c)$ almost sure. The remaining particles (mass $(\varrho - \varrho_c)L$) are stocked in one further unspecified site. This is the concept of condensation.

3 IDM as Reference Model

At the first phase we have used simulation data to develop a technique for hopping rates extraction. As a reference model has been used the Intelligent Driver Model [11, 10] (IDM). The IDM belongs to the family of car-following models and is based on a set of coupled ordinary differential equations

$$\begin{aligned} \dot{x}_i &= a\left[1 - \left(\frac{v_i}{v_0}\right)^{\delta} - \left(\frac{d_i^*}{d_i}\right)^2\right], \\ \dot{v}_i &= x_i\,, \end{aligned} \tag{10}$$

where $i \in \{1, \dots, N\}$ denotes the index of a vehicle on the road, $x_i(t)$ represents the position and $v_i(t)$ stands for the velocity of the vehicle i. Further, $d_i = x_{i+1} - x_i - l$ is the spatial distance between consecutive vehicles, $\Delta v_i = v_{i+1} - v_i$ the velocity difference. The term $d_i^* = d_0 + v_i T_i - (v_i \Delta v_i)/2\sqrt{ab}$ represents the comfortable distance of a vehicle to the preceding one. Remaining parameters with their interpretation are listed in Table 1

v_0	33 m/s	desired velocity
d_0	2.5 m	headway in jam
a	0.73 m/s^2	maximal acceleration
b	1.67 m/s^2	desired deceleration
T	2 s	safe time headway
l	5 m	car length

Table 1: Used parameter values of IDM model taken from [9].

The simulation has been performed for fixed number of vehicles $N = 1200$. The length of the road L has been calculated according to the required density ϱ as $L = N/\varrho$. The density varied from 30 veh/km to 90 veh/km.

To control the overall density, the simulations have been performed on a circular road without the possibility of overtaking. The simulation initiated in uniformly distributed vehicles along the road. The quantities started to be recorded after 0.1 h of the model time, i.e. 3600 steps of the Runge-Kutta solver with step $h = 0.1$ s. The quantities have been then recorded for the time corresponding to 3 h.

To simulate the variances between drivers, we have used randomized values of desired velocity v_0, acceleration a, and deceleration b. The parameters have been drawn from following distributions

$$v_0 \sim \mathrm{N}(33, 1)\,, \quad a \sim \mathrm{N}(0.73, 0.01)\,, \quad b \sim \mathrm{N}(1.67, 0.01) \tag{11}$$

For the purposes of the hopping rate estimation the data were aggregated over 100 simulations with different parameter sets for each density ϱ. In Figure 2 the cluster formation is illustrated.

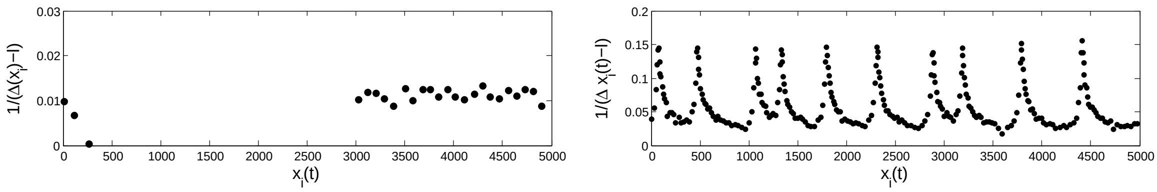

Figure 2: Snapshot from the IDM simulation for low (left) and high (right) density. The inverse value of the free space length in front of the vehicle is plotted against the vehicles position.

4 Rate Estimation

To use the ZRP for modelling a traffic flow it is necessary to consider a rougher scale than the microscopic motion of single vehicles. Our goal is to simulate the traffic flow on the level of motion of blocks composed of vehicles sufficiently close to each other. Separate vehicles are considered to be blocks of the length one and are referred to as singles. The idea of the project is to transform the traffic flow to the motion of such blocks. This motion should be then interpreted in the terms of the ZRP (or MTP), as suggested by the Figure 3.

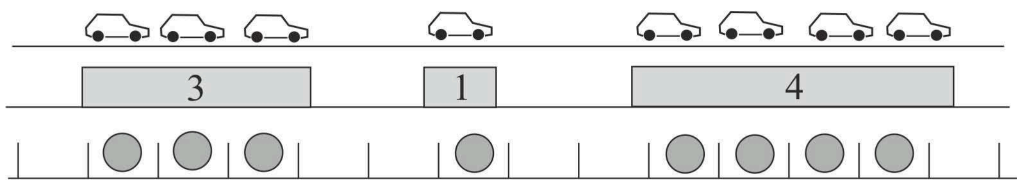

Figure 3: Mesoskopic view on traffic flow.

The first task is the identification of the blocks and their motion along the lattice. This section provides three possible techniques of the identification: Grid overlay, Envelope method, Threshold for inter-vehicle distance. The advantages and disadvantages of the techniques are listed below. Here we note that for the rate estimation, the Threshold for inter-vehicle distance seems to be most effective.

Let us assume we have at disposal the trajectories of individual vehicles $x_\alpha(t)$. Let vehicle $\alpha + 1$ directly precede the vehicle α. The inter-vehicle distance is $d_\alpha = x_{\alpha+1} - x_\alpha - l_{\alpha+1}$.

4.1 Grid Overlay

Such method is easy to implement, but is appropriate rather for the hole based mapping of the ZRP to traffic. The road is divided into regular grid with grid constant $mk \geq \bar{l}$, where $\bar{l}$ is the average vehicle length. The vehicle is assigned to such site of the grid, in which the coordinate $x(t)$ is located. See Figure 4. The hop of a particle is associated to the spacial change of the coordinate location on the grid.

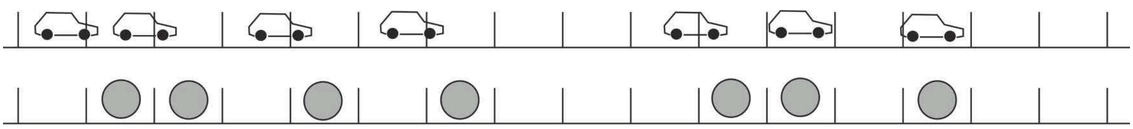

Figure 4: To the grid overlay.

Advantages

- Unambiguous interpretation of the location of particles and holes.
- Straightforward detection of the particle hops.

Disadvantages

- It is necessary to consider $l_\alpha \doteq \bar{l}$ for all vehicles α.
- If $mk \doteq \bar{l}$ there are not observed blocks longer then 2 particles even when evident traffic jam is observed.
- Using $mk > \bar{l}$ it is often observed that more vehicles occupy one site.

4.2 Envelope Method

This method seems to be dead-end. It was developed to enable the backwards construction of the system state in order to keep the ratio of occupied and empty sites. The idea consists in creation of an envelope around the position of a vehicle. The n vehicles are included in one block if the distance between the first and the last vehicle can accomodate all n envelopes, i.e. is less then $mk \cdot n$. See Figure 5. The algorithm is very computationally demanding and the blocks are often overlapping.

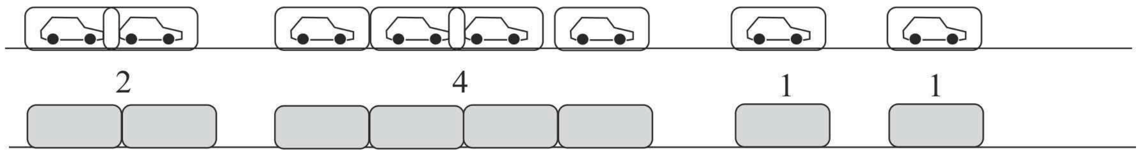

Figure 5: To the envelope method.

4.3 Threshold for inter-vehicle distance

Vehicles are assigned to one block if between each two consecutive vehicles the distance is less then the threshold c, $d_\alpha < c$, where c is fixed. Considering $l_\alpha = l$, the threshold c is in fact $c = mk - l$, see Figure 6.

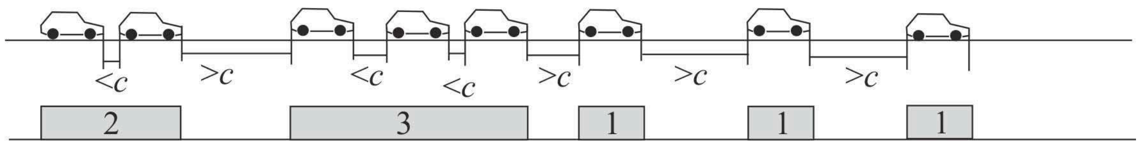

Figure 6: To the threshold for inter-vehicle distance.

If the motion is analyzed on the circular road, it is important not to artificially divide a block of vehicles which may be developed over the circle.

Here we notice the jump of the first particle of such block away from this block, i.e., we note a hop of a particle from a block of the length n if in time t there is a block of the length n and at $t + h$ there are two blocks of the length $n - 1$ and 1.

Advantages

- Unambiguous identification of blocks and their mutual positions.
- Unambiguous identification of the hop from a block of the length $n > 1$.

Disadvantages

- The state of the system cannot be transformed to the state of corresponding particle hopping system, since the method does not measure the length of the gap between two consecutive blocks. The gap can correspond to one, five, or only a half of a site.
- There is not direct way how to estimate the hopping rate of the single particle, such needs to be estimated from average velocity of singles.

5 Results from Simulations and Conclusions

The hopping rates $g(n)$, $n \geq 2$ have been estimated as

$$\widehat{g(n)} = \frac{1}{h} \cdot \frac{\text{number of jumps from block of the length } n}{\text{number of occurrence of the block of the length } n}, \tag{12}$$

where $h > 0$ is the time step of the Runge-Kutta method for the simulation of IDM. The hopping rate $g(1)$ has been estimated from the average velocity of single vehicles. The resulting estimation for variety of densities ϱ [veh/km] are depicted in Figure 7.

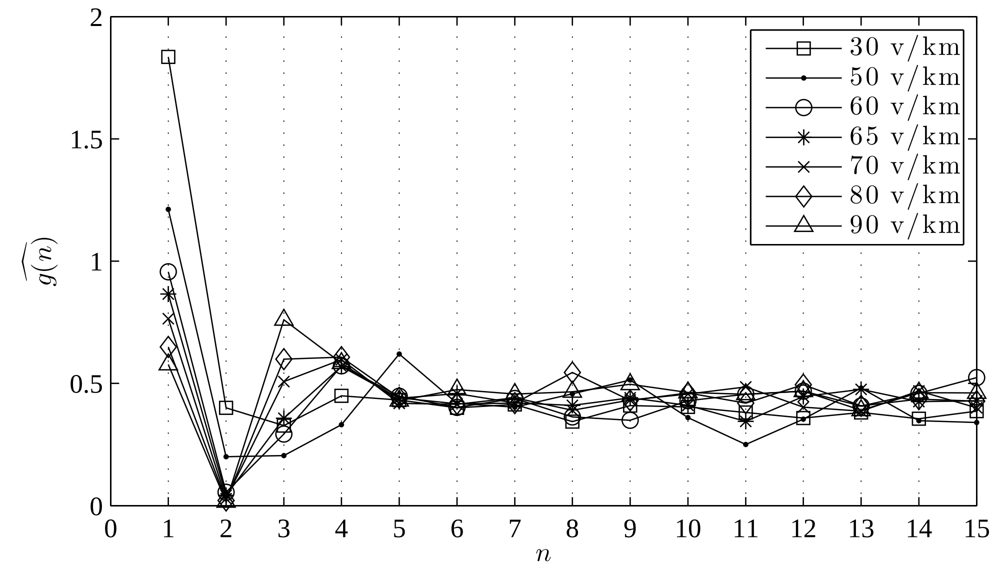

Figure 7: Estimation of hopping rates for various densities.

We can observe relatively low hopping rate for $n = 2$. This might be caused be the disproportion in the occurrence of the blocks of the length $n = 2$ and $n = 3$, as depicted in Figure 8.

The rather weird values of $\widehat{g(n)}$ can be explained as well by the fact that the estimation method supposes that the time intervals between hops from the block are distributed according to a memoryless distribution (exponential or geometric). The deterministic nature of the IDM model might cause that the distribution is far from memoryless and the estimation therefore collapses.

The next step of the research is to verify whether such unpleasant behaviour is caused only by the nature of the simulation model, or whether it is a generic property of traffic flow. In the future, we aim to try proposed methods on real traffic trajectories. Furthermore, it seems that the ZRP is too simple to capture the repulsive force between vehicles. A more complex model has been considered to address more complex interaction between particles. Nevertheless, the estimation methods has to be improved in order to work for this model.

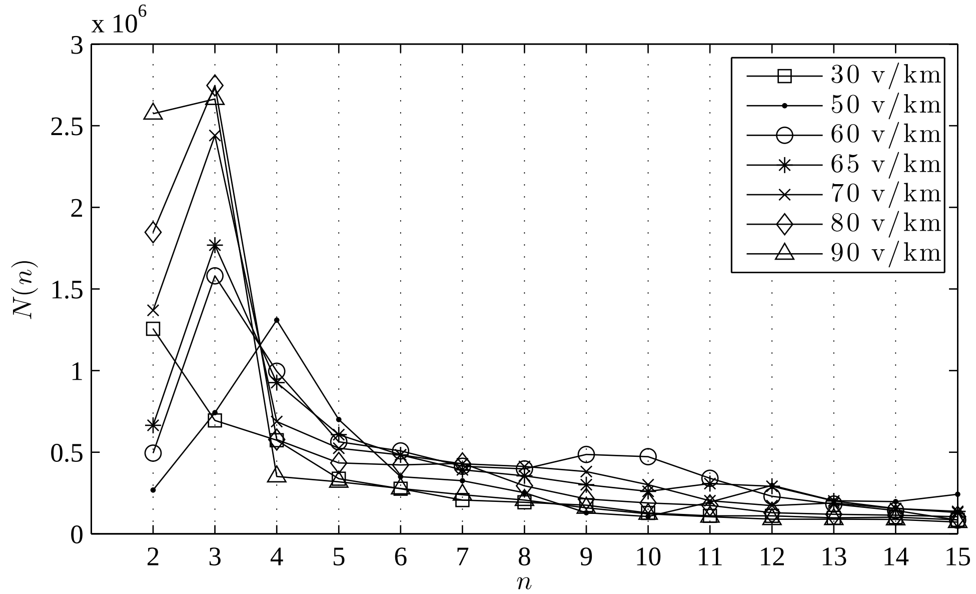

Figure 8: Occurrence of the blocks of the length n for various densities.

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References

[1] D. Chowdhury, K. Ghosh, A. Majumdar, S. Sinha, and R. Stinchcombe. Particle-hopping models of vehicular traffic: Distributions of distance headways and distance between jams. *Physica A: Statistical Mechanics and its Applications*, 246(3-4):471–486, 1997.

[2] A. E. Derbyshev, S. S. Poghosyan, A. M. Povolotsky, and V. B. Priezzhev. The totally asymmetric exclusion process with generalized update. *Journal of Statistical Mechanics: Theory and Experiment*, 2012(05):P05014, 2012.

[3] M. R. Evans, S. N. Majumdar, and R. K. P. Zia. Factorized steady state in mass transport models. *Journal of Physics A: Mathematical and Theoretical*, 37(25):L275–L280, 2004.

[4] M. R. Evans, S. N. Majumdar, R. K. P. Zia. Canonical analysis of condensation in factorised steady states. *J. Stat. Phys.* 123(2):357-390, 2006.

[5] C. Furtlehner and J.-M. Lasgouttes. A queueing theory approach for a multi-speed exclusion process. In C. Appert-Rolland, F. Chevoir, P. Gondret, S. Lassarre, J. P. Lebacque, and M. Schreckenberg, editors, *Traffic and Granular Flow 07*, pages 129–138. Springer Berlin Heidelberg, 2009.

[6] S. Grosskinsky, G. M. Schütz, and H. Spohn. Condensation in the zero range process: Stationary and dynamical properties. *Journal of Statistical Physics*, 113(3-4):389–410, 2003.

[7] J. Kaupuzs, R. Mahnke, and R. Harris. Zero-range model of traffic flow. *Physical Review E*, 72(5), 2005.

[8] E. Levine, G. Ziv, L. Gray, and D. Mukamel. Traffic jams and ordering far from thermal equilibrium. *Physica A: Statistical Mechanics and its Applications*, 340(4):636–646, 2004.

[9] A. Schadschneider, D. Chowdhury, and K. Nishinari. *Stochastic Transport in Complex Systems: From Molecules to Vehicles*. Elsevier Science B. V., Amsterdam, 2010.

[10] M. Treiber and D. Helbing. Macroscopic simulation of widely scattered synchronized traffic states. *J. Phys. A* **32**: L17, 1999.

[11] M. Treiber and A. Kesting. *Traffic Flow Dynamics, Data, Models and Simulation*. Springer, Berlin Heidelberg, 2013.

[12] R. K. P. Zia, M. R. Evans, and S. N. Majumdar. Construction of the factorized steady state distribution in models of mass transport. *Journal of Statistical Mechanics: Theory and Experiment*, 2004(10):L10001, 2004.