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Regression Models for Repairable Systems

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Abstract When operating a device which is a subject to degradation, we want to estimate the distribution of the time to failure for maintenance optimization. Our aim is to describe the dependency of the failure time distribution on applicable regression variables. Models commonly used in survival analysis, such as the Cox model or the Accelerated failure time model, need to be adjusted to accommodate repairs and maintenance. For instance, we may use the number of repairs or maintenance actions or their cost as time-varying covariates. In this work we describe such models and demonstrate their application on real data.

Keywords Reliability analysis · Repair models · Regression

Mathematics Subject Classification (2010) 62N05 · 90B25

1 Introduction

We study data describing a service record of one ore more devices which degrade over time. When a device breaks down, it is necessary to perform a repair. We want to avoid breakdowns by performing preventive maintenance, and to optimize the maintenance costs, it is desirable to estimate the time to failure distribution with the help of available information. In this work we focus on methods of modeling the life time of the device with available regression models of survival analysis with suitable covariates. The models need to be adjusted to accommodate recurring repairs and maintenance actions. One such approach was described by Percy and Kobbacy (1998) and Percy and Alkali (2005) for the Cox

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proportional hazards model with covariates multiplicatively influencing a parametric baseline hazard. In a similar way, we show the use the Accelerated failure time model with time-varying covariates (Lin and Ying 1995), which states that the covariates influence multiplicatively the flow of the internal time of the device. Further, we show methods of estimating the cumulative baseline hazard nonparametrically if we have data on more devices, which allows us to estimate the regression parameters without assumptions on the shape of the baseline. Finally, we show the application of all described methods on real data from oil industry.

2 Modeling the Life Time of One Device

Let $T_1, ..., T_n$ be random variables representing the ordered times of actions performed on the observed device, both repairs and preventive maintenances. Denote $\Delta_1, ..., \Delta_n$ the indicators whether in j-th time a repair ($\Delta_j = 1$) or a preventive maintenance ($\Delta_j = 0$) was performed and let $Z(t)$ be a vector of additional explanatory variables, possibly time-varying.

The data is available either in form of the ordered times of actions $(T_j, \Delta_j)_{j=1}^n$ or in the form of times elapsed between the actions $(T_j - T_{j-1}, \Delta_j)_{j=1}^n$ with $T_0 = 0$, along with the covariate values $Z(t)$. We assume that the time elapsed during an action does not contribute to the total time elapsed, as the device is considered not to be under workload when a repair or maintenance action is being done. The duration of an action may even not be available if the data is presented in the second form.

We work with counting processes denoting the number of repairs $N_\bullet(t)$ and maintenance actions $M_\bullet(t)$ up to time t:

$$N_\bullet(t) = \sum_{j=1}^{n} I(T_j \le t, \Delta_j = 1), \qquad M_\bullet(t) = \sum_{j=1}^{n} I(T_j \le t, \Delta_j = 0).$$

Our aim is to model and possibly predict the distribution of the time to failure, depending on the history of the device and available covariates. We work with hazard function denoting the limit probability of immediate breakdown of the device (an increase of $N_\bullet(t)$) given its history:

$$\lambda(t) = \lim_{h \to 0} P(N_\bullet(t+h) - N_\bullet(t) \ge 1 | \mathcal{H}(t))/h$$

where $\mathcal{H}(t)$ is the history of events up to time t. Further denote the cumulative hazard function $\Lambda(t) = \int_0^t \lambda(s)ds$ and $S(t) = \exp(-\Lambda(t))$ and $f(t) = -S'(t)$ corresponding survival function and density of the time to failure distribution. We assume that each repair returns the device to a working state as it was directly before the failure, and that each subsequent repair or maintenance action somehow affects the hazard function. The aim is to determine whether repairs and maintenance actions increase or decrease the hazard and by how much. We parametrize the hazard function and estimate the parameters using maximum likelihood method.

The times of actions are mutually dependent, however, if we take the distribution of each (T_j, Δ_j) conditional on the service history until T_{j-1}, we get independent parts. The joint distribution of the data can be written as

$$g((T_n, \Delta_n), ..., (T_1, \Delta_1)) = g((T_n, \Delta_n)|(T_{n-1}, \Delta_{n-1}), ..., (T_1, \Delta_1))$$

$$\cdot g((T_{n-1}, \Delta_{n-1})|(T_{n-2}, \Delta_{n-2}), ..., (T_1, \Delta_1)) \cdot ... \cdot g((T_1, \Delta_1))$$

with g standing for the respective distributions. The breakdowns of the device ($\Delta_j = 1$) can be seen as realizations of the conditional distribution of the time to failure, given that the device survived up to the last action, resulting in terms $\frac{f(T_j^-)}{S(T_{j-1})}$. The survival function in the denominator is included because the device is returned to a working state after a repair and the next action at T_j happens after the time T_{j-1}. The maintenance actions ($\Delta_j = 0$) can be interpreted as right censoring of that distribution, since we do not know for how long would the device operate until breakdown without the preventive maintenance. This contributes to the likelihood with terms $\frac{S(T_j)}{S(T_{j-1})}$. Therefore the likelihood can be written as

$$L = \prod_{j=1}^{n} \left(\frac{f(T_j^-)}{S(T_{j-1})} \right)^{\Delta_j} \left(\frac{S(T_j)}{S(T_{j-1})} \right)^{1-\Delta_j} = \prod_{j=1}^{n} \lambda(T_j^-)^{\Delta_j} \cdot S(T_n)$$

and the log-likelihood has the form

$$l = \sum_{j=1}^{n} \Delta_j \log \lambda(T_j^-) - \int_0^{T_n} \lambda(t) dt.$$

At this point, we want to incorporate the available history and covariates $N_\bullet(t)$, $M_\bullet(t)$ and $Z(t)$ into the likelihood through the hazard function, possibly in an easily interpretable way.

2.1 Cox Model

In the Cox model the covariates affect the hazard function multiplicatively. We assume that a baseline hazard function $\lambda_0(t)$ is multiplicatively increased or decreased by each repair, maintenance action and other explanatory variables. We work with the hazard function in the form (Percy and Alkali 2005)

$$\lambda(t) = \lambda_0(t) e^{M_\bullet(t)\rho + N_\bullet(t)\sigma + Z^T(t)\beta} = \lambda_0(t)(e^\rho)^{M_\bullet(t)}(e^\sigma)^{N_\bullet(t)}(e^{\beta^T})^{Z(t)}.$$

As the explanatory variable $Z(t)$ we can use for instance the cost of the last repair. If the covariate values change only in the times of observed events and the baseline hazard $\lambda_0(t)$ is parametric, it is possible to insert the hazard function into the log-likelihood and maximize. This approach was suggested by Percy and Alkali (2005) as the generalization of the proportional intensities model of Cox (1972). Given the conditional structure of the data explained above, under certain regularity assumptions and conditionally independent distribution of the maintenance times, the parameters estimated by maximizing the likelihood are consistent and their asymptotic distribution is normal.

2.2 Accelerated Failure Time Model

We can also assume that each repair or maintenance causes that the internal time of the device flows faster or slower (Accelerated Failure Time model, AFT). We use the time transformation (Lin and Ying 1995)

$$t \to \int_0^t e^{M_\bullet(s)\rho + N_\bullet(s)\sigma + Z^T(s)\beta} ds =: h(t, \boldsymbol{\beta}),$$

where $\boldsymbol{\beta} = (\rho, \sigma, \beta)^T$. In this framework t represents the observed age and $h(t, \boldsymbol{\beta})$ the internal age of the device. The hazard function has the form

$$\lambda(t) = \lambda_0(h(t, \boldsymbol{\beta}))e^{M_\bullet(t)\rho + N_\bullet(t)\sigma + Z^T(t)\beta}.$$

It is again possible to insert into the log-likelihood and obtain the parameter estimates by its maximization. The consistency and asymptotic normality follows in a similar way. If the baseline hazard function is constant (corresponding with the exponential distribution), both models coincide.

3 Inference When Observing More Devices

If we have data on m independent devices, we work with joint likelihood. On each device $i = 1, ..., m$, we have observed n_i events. Let us have $\lambda_i(t)$, T_{ij}, Δ_{ij}, $j = 1, ...n_i$ and $Z_i(t)$ the hazard function, times of events, repair indicators and covariate values for the i-th device respectively. The j-th event on the i-th device occured at the time T_{ij} and it was a repair if $\Delta_{ij} = 1$ and a preventive maintenance if $\Delta_{ij} = 0$. Denote

$$N_{ij}(t) = \Delta_{ij} I(T_{ij} \leq t),$$

$$M_{ij}(t) = (1 - \Delta_{ij}) I(T_{ij} \leq t),$$

$$Y_{ij}(t) = I(T_{i,j-1} < t \leq T_{ij})$$

the indicators, whether on the i-th device at time t the j-th action already occured and it was a repair ($N_{ij}(t)$) or a maintenance ($M_{ij}(t)$) or that the device is still at risk before the i-th action ($Y_{ij}(t)$). We get the log-likelihood in form

$$l = \sum_{j=1}^{m} \left(\sum_{j=1}^{n_i} \Delta_{ij} \log \lambda_i(T_{ij}^-) - \int_0^{T_{in_i}} \lambda_i(t) dt \right)$$

$$= \sum_{ij} \int_0^\infty \Big(\log \lambda_i(t^-) dN_{ij}(t) - Y_{ij}(t)\lambda_i(t^-) dt \Big).$$

The hazard function λ_i will contain the counts of repairs and maintenance actions $N_{i\bullet}(t)$ and $M_{i\bullet}(t)$, where $\bullet$ means the sum over corresponding index.

At this point, two options are available. We can either parametrize the baseline hazard and proceed as above, or it is possible to estimate the baseline hazard nonparametrically.

This may be desirable, since we then do not need to pose any assumptions on the form of the baseline and focus on the regression parameters.

3.1 Semiparametric Cox Model

Denote $\mathbf{X}_i^T(t) = (N_{i\bullet}(t), M_{i\bullet}(t), Z_i^T(t))$. Then the likelihood under the Cox model has the form

$$l = \sum_{ij} \int_0^\infty \Big((\log \lambda_0(t^-) + \mathbf{X}_i^T(t^-)\boldsymbol{\beta}) dN_{ij}(t) - Y_{ij}(t) e^{\mathbf{X}_i^T(t^-)\boldsymbol{\beta}} \lambda_0(t^-) dt \Big)$$

and we obtain the score function by taking derivative with respect to the regression parameters:

$$U(\boldsymbol{\beta}) = \sum_{ij} \int_0^\infty \Big(\mathbf{X}_i^T(t^-) dN_{ij}(t) - Y_{ij}(t) \mathbf{X}_i^T(t^-) e^{\mathbf{X}_i^T(t^-)\boldsymbol{\beta}} d\Lambda_0(t) \Big).$$

The score depends on an unknown cumulative baseline hazard $\Lambda_0(t)$. This can be replaced by the Nelson-Aalen type estimate

$$\hat{\Lambda}_0(t, \boldsymbol{\beta}) = \sum_{ij} \int_0^t \frac{dN_{ij}(s)}{\sum_{kl} e^{\mathbf{X}_k^T(s^-)\boldsymbol{\beta}} Y_{kl}(s)}.$$

Inserting the estimate we get the score function in form

$$U(\boldsymbol{\beta}) = \sum_{ij} \int_0^\infty \left(\mathbf{X}_i(t^-) - \frac{\sum_{kl} \mathbf{X}_k(t^-) e^{\mathbf{X}_k^T(t^-)\boldsymbol{\beta}} Y_{kl}(t)}{\sum_{kl} e^{\mathbf{X}_k^T(t^-)\boldsymbol{\beta}} Y_{kl}(t)} \right) dN_{ij}(t)$$

and we find the parameter estimates by solving the equations $U(\boldsymbol{\beta}) = \mathbf{0}$. The asymptotic properties are obtained in a different way. Firstly, we consider processes $\mathcal{M}_i(t)$ defined as

$$d\mathcal{M}_i(t) = dN_{i\bullet}(t) - Y_{i\bullet}(t) e^{\mathbf{X}_i^T(t^-)\boldsymbol{\beta}} d\Lambda_0(t).$$

It follows, that the expectation of $d\mathcal{M}_i(t)$ is zero for all t, motivating the Nelson-Aalen estimate. With some algebra, the score can be rewritten as

$$U(\boldsymbol{\beta}) = \sum_{i} \int_0^\infty \left(\mathbf{X}_i(t^-) - \frac{\sum_{kl} \mathbf{X}_k(t^-) e^{\mathbf{X}_k^T(t^-)\boldsymbol{\beta}} Y_{kl}(t)}{\sum_{kl} e^{\mathbf{X}_k^T(t^-)\boldsymbol{\beta}} Y_{kl}(t)} \right) d\mathcal{M}_i(t).$$

Under regularity assumptions similar to those of Lin et al. (2000) it can be then be shown with the help of the functional central limit theorem of Pollard (1990), that with $m \to \infty$ the score process $\frac{1}{\sqrt{m}} U(t, \beta_0)$ obtained by integrating the score up to t instead of infinity converges weakly to a zero-mean Gaussian process with a finite covariance function. Furthermore, using Taylor expansion we get that $\sqrt{m}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$ converges to a zero-mean normal distribution and $\sqrt{m}(\hat{\Lambda}_0(t, \hat{\boldsymbol{\beta}}) - \Lambda_0(t))$ converges also weakly to a zero-mean Gaussian process with a finite covariance. The idea or the proof is similar to the one for the recurrent event data studied by Lin et al. (2000), but had to be extended to accommodate $N_{i\bullet}(t)$ and $M_{i\bullet}(t)$ as covariates.

3.2 Semiparametric AFT Model

We assume that the internal time flows differently for each device and its rate changes at different moments. Therefore for each device we need its time transformation $t \to h_i(t, \boldsymbol{\beta})$.

Our aim is to do the inference with respect to the transformed, virtual time. We work with time-transformed processes

$$
\begin{aligned}
N_{ij}^*(t,\boldsymbol{\beta}) &= \Delta_{ij} I(h_i(T_{ij},\boldsymbol{\beta}) \le t),\\
M_{ij}^*(t,\boldsymbol{\beta}) &= (1-\Delta_{ij}) I(h_i(T_{ij},\boldsymbol{\beta}) \le t),\\
Y_{ij}^*(t,\boldsymbol{\beta}) &= I(h_i(T_{i,j-1},\boldsymbol{\beta}) < t \le h_i(T_{ij},\boldsymbol{\beta})),\\
\mathbf{X}_i^*(t,\boldsymbol{\beta}) &= \mathbf{X}_i(h_i^{-1}(t,\boldsymbol{\beta})).
\end{aligned}
$$

The score obtained by taking the derivative of the log-likelihood with respect to $\boldsymbol{\beta}$ has the form

$$
U(\boldsymbol{\beta}) = \sum_{ij} \int_0^\infty W_i(t^-,\boldsymbol{\beta}) \left(dN_{ij}^*(t,\boldsymbol{\beta}) - Y_{ij}^*(t,\boldsymbol{\beta}) d\Lambda_0(t) \right),
$$

where $W_i(t,\boldsymbol{\beta}) = \frac{\lambda_0'(t)}{\lambda_0(t)} \int_0^{h_i^{-1}(t,\boldsymbol{\beta})} \mathbf{X}_i^T(s) e^{\mathbf{X}_i^T(s)\boldsymbol{\beta}} ds + \mathbf{X}_i^*(t,\boldsymbol{\beta})$. This form is relatively complicated, with terms λ_0' and λ_0 not easy to estimate. The exact score can be replaced by the approximate score (Lin and Ying 1995)

$$
U(\boldsymbol{\beta}) = \sum_{ij} \int_0^\infty \mathbf{X}_i^*(t^-,\boldsymbol{\beta}) \left(dN_{ij}^*(t,\boldsymbol{\beta}) - Y_{ij}^*(t,\boldsymbol{\beta}) d\Lambda_0(t) \right).
$$

We can again insert the estimate of the cumulative baseline hazard function

$$
\hat{\Lambda}_0(t,\boldsymbol{\beta}) = \sum_{ij} \int_0^t \frac{dN_{ij}^*(s,\boldsymbol{\beta})}{\sum_{kl} Y_{kl}^*(t,\boldsymbol{\beta})}.
$$

Note the analogy with the Nelson-Aalen estimate of the cumulative hazard for *iid* survival data. We get

$$
U(\boldsymbol{\beta}) = \sum_{ij} \int_0^\infty \left(\mathbf{X}_i^*(t^-,\boldsymbol{\beta}) - \frac{\sum_{kl} \mathbf{X}_k^*(t^-,\boldsymbol{\beta}) Y_{kl}^*(t,\boldsymbol{\beta})}{\sum_{kl} Y_{kl}^*(t,\boldsymbol{\beta})} \right) dN_{kl}^*(t,\boldsymbol{\beta}).
$$

Because the score is not continuous in $\boldsymbol{\beta}$, we obtain the parameter estimates by minimizing $\|U(\boldsymbol{\beta})\|$. The asymptotic properties are obtained in a similar manner, with

$$
d\mathcal{M}_i^*(t,\boldsymbol{\beta}) = dN_{i\bullet}^*(t,\boldsymbol{\beta}) - Y_{i\bullet}^*(t,\boldsymbol{\beta}) d\Lambda_0(t)
$$

having zero mean for all t and the score taking the form

$$
U(\boldsymbol{\beta}) = \sum_{i} \int_0^\infty \left(\mathbf{X}_i^*(t^-,\boldsymbol{\beta}) - \frac{\sum_{kl} \mathbf{X}_k^*(t^-,\boldsymbol{\beta}) Y_{kl}^*(t,\boldsymbol{\beta})}{\sum_{kl} Y_{kl}^*(t,\boldsymbol{\beta})} \right) d\mathcal{M}_i^*(t,\boldsymbol{\beta}).
$$

Again we obtain the asymptotic normality of $\sqrt{m}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$ and $\sqrt{m}(\hat{\Lambda}_0(t,\hat{\boldsymbol{\beta}}) - \Lambda_0(t))$ with the help of the functional central limit theorem. The covariance function of the latter depends on unknown functions λ_0 and λ_0' and therefore cannot be estimated easily.

4 Modeling Lifetime of Oil Pumps

We explore data on the service of oil pumps during several years, see Kobbacy et al. (1997) and Percy and Alkali (2007). For one device we have detailed data on $n_1 = 65$ times of repairs, maintenance actions and the cost of each action in man-hours. This data has been studied by Percy and Alkali (2005) using the parametric Cox model. We try to model the lifetime using both the Cox and the AFT model as shown above with various parametrized baseline hazard functions and compare the results. In the parametric case, it is possible to directly maximize the likelihood for all cases and see in which it was largest.

For four other pumps we have only the times of actions at disposal, with $(n_2, ..., n_5) = (51, 90, 30, 30)$. We use both the semiparametric methods and parametrized baseline hazards with the two described models to estimate the regression parameters utilizing data of all the five pumps. The likelihood in semiparametric methods depends on the unknown baseline hazard and therefore is not available for comparison of the used methods.

The data is given as the time elapsed between each of the successive actions. We assume that the duration of each action does not contribute to the total time elapsed, because the pumps are inoperable and not under workload at that time. It can still be argued that the devices do age even during a repair or maintenance action, but that could possibly lead to failures occuring at that time, necessitating further repairs, which would require more complex models.

4.1 Parametric Modeling of the Service of One Pump

We have the times of repairs, maintenance actions and cost of each action for one pump. Using methods from section 2 we estimate the parameters ρ, σ and β in both the Cox model and the AFT model. We try to maximize the likelihood for exponential, Weibull $\lambda_0(t) = a\lambda^a t^{a-1}$, gamma $f(t) \propto t^{a-1}e^{-\lambda t}$, truncated Gumbel $\lambda_0(t) = \lambda a^t$ and log-normal baseline distributions.

Comparing the likelihood values in Table 1 we find that it is highest for both the Cox and AFT model with the truncated Gumbel distribution. Further we see that the more each action did cost, the more it increased the hazard function or accelerated the internal time, because $e^{\hat{\beta}} > 1$. Each man-hour of the action means an increase of hazard or acceleration of time by about $0.5 - 0.7\%$. A repair itself has a positive influence ($e^{\hat{\sigma}} < 1$), with the exception of the AFT model with log-normal baseline distribution, but that is the case with the lowest likelihood value. It is interesting that according to all cases except the Gumbel distribution in Cox model, the maintenance actions tend to have a negative influence ($e^{\hat{\rho}} > 1$). This

Table 1 The log-likelihood and parameter estimates from parametric models of the lifetime of one oil pump

Model	λ_0	log - lik	$e^{\hat{\rho}}$	$e^{\hat{\sigma}}$	$e^{\hat{\beta}}$	$\hat{\lambda}$	$\hat{a}$
	Exp.	-213.8	1.407	0.980	1.0066	0.0015	–
Cox	Weibull	-213.5	1.266	0.924	1.0064	0.0017	1.672
	Gamma	-213.8	1.405	0.918	1.0066	0.0016	1.027
	Gumbel	**-210.2**	0.701	0.745	1.0063	0.0006	1.010
	LN	-214.8	1.541	0.913	1.0069	$\hat{\mu}$=6.3	$\hat{\sigma}$=1.66
AFT	Weibull	-212.7	1.278	0.918	1.0061	0.0014	1.639
	Gamma	-213.8	1.418	0.916	1.0066	0.0014	0.918
	Gumbel	**-210.2**	1.318	0.877	1.0050	0.0005	1.001
	LN	-218.1	1.300	1.050	1.0070	$\hat{\mu}$=5.25	$\hat{\sigma}$=0.89

could be due to repairs often taking much more man-hours than maintenances (on average, a repair took 26.8 whereas a maintenance action took only 9.4 man-hours), resulting in negative influence of both.

4.2 Semiparametric Modeling of the Lifetime of Five Pumps

For five devices we have only the times of repairs and maintenances available. The data on the cost of the actions was not available for all pumps, therefore we estimate only the regression parameters ρ and σ. We tried the Cox and the AFT models, both parametric with the same baseline distributions as above and semiparametric. In the parametric cases we maximize the log-likelihood whereas in the semiparametric approach we insert the estimate of the cumulative baseline hazard into the score function and solve the equations $U(\boldsymbol{\beta}) = \mathbf{0}$ for the Cox model and minimize $\|U(\boldsymbol{\beta})\|$ for the AFT model.

In Table 2 we see that in all cases each repair increases the hazard or accelerates the internal time ($e^{\hat{\sigma}} > 1$). Among the parametric models, the Gumbel distribution with AFT model has the highest likelihood. In that case and also in the cases with log-normal baseline hazard and the semiparametric models, the maintenance actions have also a negative influence, whereas in the other cases it is positive. In Fig. 1 we see the estimates of the cumulative baseline hazard for both Cox and AFT models. The time in the AFT model is on the transformed scale $t \to h(t, \hat{\boldsymbol{\beta}})$.

4.3 Possible Model Selection and Validation Methods

When comparing two models, it is possible to perform a χ^2 test based on scaled deviance $D = 2 \cdot (l_1 - l_2)$. Based on the asymptotics explained in Section 2, we get $2\left(l(\hat{\boldsymbol{\beta}}) - l(0, \hat{\boldsymbol{\beta}}_{(2,\ldots p)})\right) \to \chi_1^2$ for testing $\beta_1 = 0$ to see whether one particular covariate significantly improves a model or not. For the one pump data using Cox model with Gumbel baseline, no significant improvement was found when adding the influence of the cost of the action (β) to a model containing only the regression parameters ρ and

Table 2 The log-likelihood and parameter estimates from modeling the lifetime of five pumps

Model	λ_0	log - lik	$e^{\hat{\rho}}$	$e^{\hat{\sigma}}$	$\hat{\lambda}$	$\hat{a}$
	Exp.	-880.3	0.985	1.016	0.016	–
Cox	Weibull	-880.2	0.976	1.016	0.014	1.063
	Gamma	-880.1	0.988	1.016	0.015	0.811
	Gumbel	-880.3	0.994	1.016	0.016	0.999
	LN	-894.4	1.090	1.016	$\hat{\mu}$=3.22	$\hat{\sigma}$=0.89
AFT	Weibull	-880.2	0.980	1.015	0.014	1.038
	Gamma	-880.1	0.988	1.016	0.015	0.812
	Gumbel	**-875.1**	1.022	1.036	0.013	0.999
	LN	-879.5	1.284	1.158	$\hat{\mu}$=2.67	$\hat{\sigma}$=1.56
Cox	nonparam.	–	1.043	1.020	–	–
AFT	nonparam.	–	1.028	1.084	–	–

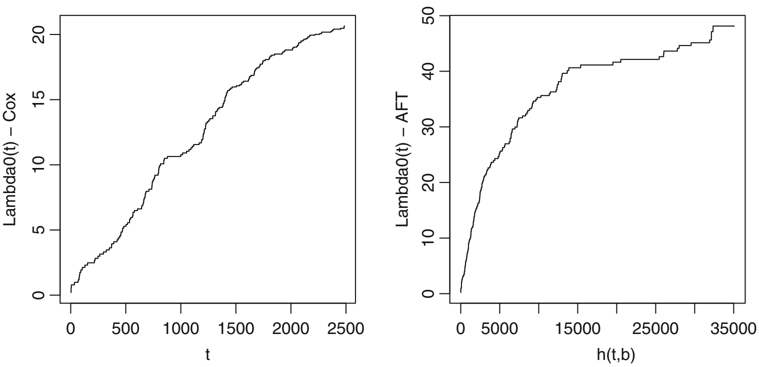

Fig. 1 Estimates of the cumulative baseline hazard in semiparametric Cox and AFT models

σ (Percy and Alkali 2005), while it can be argued that the covariate still adds some relevant information. It can be also tested, whether a Weibull or Gamma baseline hazard can be simplified to exponential with $a = 1$. This approach is, however, not usable when comparing models with different baseline hazards or the Cox model with the AFT model.

Aside from the likelihood value, we do not have direct means to determine which model fits the data best, especially when comparing the parametric and semiparametric approaches. For classic survival regression data, goodness-of-fit tests have been developed for parametric versions of both models by Lin and Spiekerman (1996), for the semiparametric Cox model by Lin et al. (1993) and for the semiparametric AFT model by Novák (2013). This methods could be adapted to accommodate repairs and maintenance actions. They are however, based on resampling approach and asymptotic convergence of certain martingale processes, and therefore it remains to be seen how well they would perform in such cases as with above data representing only a few independent devices.

5 Conclusion

We explored methods for modeling the influence of maintenance and repairs on the lifetime of the observed device. In the Cox model the covariates representing the count and size of repairs and maintenance actions influence the hazard function multiplicatively, whereas in the AFT model they accelerate or decelerate the flow of the internal time of the device. When we parametrize the baseline hazard function, the service record of one device is enough to obtain the estimates of the regression parameters. If we have data on more devices, it is possible to estimate the cumulative baseline hazard function nonparametrically. Further research could concern developing goodness-of-fit tests or testing whether a nonparametric estimate may be replaced by a suitable parametrized baseline hazard. It would be also possible to explore other transformations in the accelerated failure time model.

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