

Using Strain Analysis For Landslides



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Abstract The paper demonstrates why applying of the theory of strain analysis is useful to monitoring of landslides. The strain analysis going from mechanics of continuum may be applied as basis of communication during the multidisciplinary approach to geodesic, geological and geotechnical monitoring of landslides. It serves also as a technological and scientific communication basis between geodesists and specialists of other professions as geologists, geotechnics, geophysics, building engineers etc.

The independency of resulting deformation parameters to applied coordinate frame allows that the errors originated from erroneous pre-requisites about stability of some selected points that are taken during normal calculation as stable (in the stable part of location) will be completely eliminated. When compared to the sole listening of displacements this procedure enables to present deformation parameters in much more objective way and serves as a tool to demonstrate the relatively geodynamical trends of the territory in question.

Keywords strain analysis, deformation analysis, displacement vectors, mechanics of continuum, landslides

Introduction

More and more results of geodetic activity are used in other disciplines. Such cases are, e.g., geodynamic research, where geodesy offers objective and relatively sufficiently exact information about motions and shape and dimension changes of the terrestrial surface or building constructions, generally speaking of some monitored object or locality. Such information is used to further studies, physical interpretations and determining causative factors. That means there is a multidisciplinary lead approach to solving such problems. Such approach requires creating of

technological and scientific base for communication between specialists of different professions that will be suitable to all of them. In (Szostak-Chrzanowski et al. 2006) is proposed the mechanic of continuum to this purpose as support of deformation monitoring, their analysis and interpretation. Let us show other reasons and benefits leading to applying this theory in geodetic praxis.

Homogeneous territory or object in question is the required condition, of course. It means that if such condition is not fulfilled, then only division of the whole territory to several homogeneous parts must be tried and calculations has to be done for each one of them separately.

Analysis of deformations of the terrestrial surface or of building constructions does not belong to problems that many geodesists solve in their everyday praxis. The main goal of deformation analysis is determination of deformation mechanism if we look at the object as at the mechanical system under deformations according the laws of mechanics of continuum. After such geometric analysis physical interpretation of results may follow.

Geometrical analysis, describes the change in shape and dimensions of the monitored object, as well as its rigid body movements (translations and rotations). The goal of the geometrical analysis is to determine in the whole deformable object the displacement and strain fields in the space and time domains.

Physical interpretation is based on the relationship between the causative factors (loads) and deformations.

Geodetic monitoring of deformations should follow in two steps. The first one is determination of displacements of selected points on the object in question (standard task). After it determination of displacement field in continuous form by their interpolation (generalized task) can follow. The second step is determination of deformations parameters by geometric analysis of continuum mechanics (strain analysis). Determination of displacements in the first

step is usually done by repeated observation of geodetic network. But realisation of the second step is not so much usual among geodesists. Nevertheless strain analysis offers many conveniences.

Geometric analysis of deformations by continuum mechanics

As it was told earlier, the principle of geodetic methods applications is based on repeated measurement and comparison of results of individual stages of measurements. Obtained differences in positions of points represent their displacements. The vector of point displacement is

$$\mathbf{d}_i = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)_i^T = \mathbf{x}_i^o - \mathbf{x}_i^t \quad [1]$$

Where \mathbf{x}_i^o (resp. \mathbf{x}_i^t) is the vector of P_i point coordinates of fundamental (resp. actual in t-time) stage. This vector may be expressed as a function of coordinates:

$$\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)^T = \mathbf{u}(\mathbf{x}) = (\mathbf{u}_1(\mathbf{x}), \mathbf{u}_2(\mathbf{x}), \mathbf{u}_3(\mathbf{x}))^T = \mathbf{d} \quad [2]$$

Where $\mathbf{x} = (x, y, z)^T$. The strain tensor in point P_i is defined as a gradient of the function in this point:

$$\mathbf{E}_i = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}_i = \text{grad}(\mathbf{d}_i) = \begin{pmatrix} \frac{\partial u_1}{\partial x} & \frac{\partial u_1}{\partial y} & \frac{\partial u_1}{\partial z} \\ \frac{\partial u_2}{\partial x} & \frac{\partial u_2}{\partial y} & \frac{\partial u_2}{\partial z} \\ \frac{\partial u_3}{\partial x} & \frac{\partial u_3}{\partial y} & \frac{\partial u_3}{\partial z} \end{pmatrix}_i \quad [3]$$

In the displacement field is valid next relation, see (Welsch 1983):

$$\mathbf{d}_i = \mathbf{E}_i \mathbf{x}_i + \mathbf{t} \quad [4]$$

Where:

- \mathbf{d}_i is the displacement vector,
- \mathbf{E}_i is the displacement gradient,
- \mathbf{x}_i is the coordinate vector,
- \mathbf{t} is the vector of translation elements.

The strain tensor may be divided into two parts:

$$\mathbf{E}_i = \mathbf{e}_i + \boldsymbol{\Omega}_i = (e_{jl})_i + (\omega_{jl})_i \quad j, l = 1, 2, 3 \quad [5]$$

Where:

- $\mathbf{e}_i = (e_{jl})_i$ is the symmetric tensor of deformation,
- $\boldsymbol{\Omega}_i = (\omega_{jl})_i$ is the asymmetric tensor of rotation,
- $e_{jl} = (\varepsilon_{jl} + \varepsilon_{lj}) / 2$,
- $\omega_{jl} = (\varepsilon_{jl} - \varepsilon_{lj}) / 2$.

$$\mathbf{e}_i = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{12} & e_{22} & e_{23} \\ e_{13} & e_{23} & e_{33} \end{pmatrix}_i$$

$$\boldsymbol{\Omega}_i = \begin{pmatrix} 0 & \omega_{12} & \omega_{13} \\ -\omega_{12} & 0 & \omega_{23} \\ -\omega_{13} & -\omega_{23} & 0 \end{pmatrix}_i$$

It could be written:

$$\mathbf{d}_i = (\mathbf{e}_i + \boldsymbol{\Omega}_i) \mathbf{x}_i + \mathbf{t} \quad [6]$$

We could determine the deformation parameters from \mathbf{e}_i and $\boldsymbol{\Omega}_i$. This may be done as a 3D solution as well as in plane. Such plane could be e.g., parallel with XY or XZ or YZ or in a more general way any plane of the local coordinate system to which space displacements will be projected.

It holds e.g., for displacements projected to XY plane of the local coordinate system:

$$\Delta = e_{11} + e_{22} \quad \text{- total dilatation}$$

$$\gamma_1 = e_{11} - e_{22} \quad \text{- shear strains}$$

$$\gamma_2 = 2e_{12} \quad \text{- shear strains}$$

$$\gamma = \sqrt{\gamma_1^2 + \gamma_2^2} \quad \text{- total shear}$$

$$\varepsilon_1 = \frac{1}{2}(\Delta + \gamma) \quad \text{- axis of maximum strain}$$

$$\varepsilon_2 = \frac{1}{2}(\Delta - \gamma) \quad \text{- axis of minimum strain}$$

$$\varphi = \frac{1}{2} \arctg \left(\frac{\gamma_2}{\gamma_1} \right) \quad \text{- direction of axis of maximum strain}$$

$$\psi = \varphi + \frac{1}{4}\pi \text{ for } \omega_{12} > 0 \quad \text{- direction of shear strain}$$

$$\psi = \varphi - \frac{1}{4}\pi \text{ for } \omega_{12} < 0 \quad \text{- direction of shear strain}$$

Deformation parameters characters and benefits resulting from them

It is worth noting that all displacements depend on selected coordinate frame. On contrary, all **deformation parameters** of last equations except the ψ and φ directions **are on used coordinate frame independent**, and insensitive to translation and rotation. And this is the reason why deformation

parameters and their applications are important in practice.

In the case of point displacements calculation the resulting character of displacements is quite unambiguously given by the applied coordinate frame or by conditions of geodetic network placing in the coordinate frame.

The first condition that has to be fulfilled is net adjustment as free network to prevent its scale changes or even its deformation. Usually selected points that are expected to be in the stable part of location are chosen like fiducial. More exactly: in the case of free networks, these points are chosen like points included at the condition of being placed in the coordinate frame (e.g. by selecting among identical points during the Helmert transformation). Selecting of such points is usually based on expected physical properties of the locality. Nevertheless we are never quite sure that our expectations about point stability are correct and well fulfilled. The situation will be shown on next practical examples of landslides.

A practical example of monitoring network Jezerka

Deformation network Jezerka was taken to monitor landslides in foreland of the opencast CSA at Komorany by Most, Czech Republic, as a practical example of the application of continuum mechanics deformation analysis. The opencast with dimensions of about 4.3 x 2.6 km with a depth of 200 m is situated on at foothills of the "Ore Mountains" with slopes of about 600 m. There is therefore a legitimate concern of landslides. Fig. 1 shows the entire situation of the quarry monitoring networks and of the chain of the "Ore Mountains" situated in northwest of the quarry.



Fig. 1 The entire situation of the quarry monitoring networks and of the chain of the "Ore Mountains" situated in northwest of the quarry.

The purpose of the Jezerka monitoring network is to determine the magnitude and direction of landslides that occur. At the same time to analyze the relations between the geodynamic network points and thus

identify areas where the highest risks of landslides can be expected.

The network configuration is shown at Fig 2. There were a total of 77 repeated surveys of the network during a period of about 20 years. However, the network configuration of the network was changing. For the purposes of this article are taken only four stages 02.1992, 05.1992, 10.1992 and 01.1993 have been used on which the landslides and benefits of their analysis using continuum mechanics can be well demonstrated.



Fig 2 The network configuration.

Results of analysis of point displacements

To determine the displacement of the points all surveys of the monitoring network were adjusted as free positional network. In order to transform the adjusted network to the local coordinate system, two points, which could be considered as the most stable out of the all individual points were selected. They were the northernmost and westernmost point of the network. At the same time, the most northerly point and the direction to the westernmost point were introduced into the adjustment as fixed ones. At Figs. 3 to 5, the resulting shifts (displacements) of individual points are therefore related to a fixed point on the northern edge of the network.

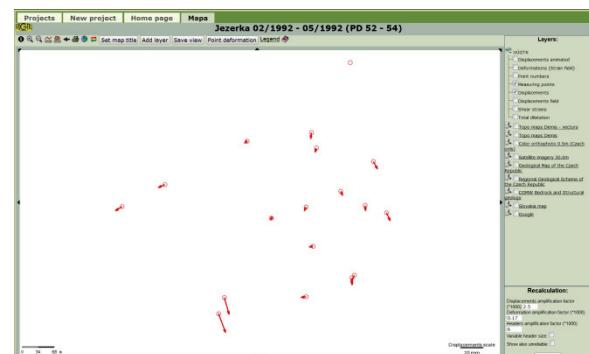


Fig. 3 Resulting shifts of points related to a fixed point on the northern edge of the network, stage 02. - 05.1992.

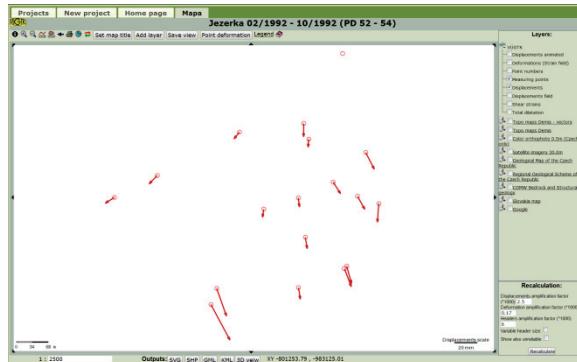


Fig. 4 Resulting shifts of points related to a fixed point on the northern edge of the network, stage 02. - 10.1992.

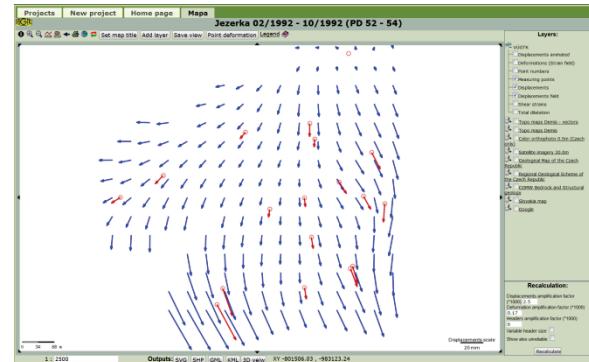


Fig. 7 Displacement field of points related to a fixed point on the northern edge of the network, stage 02. - 10.1992.

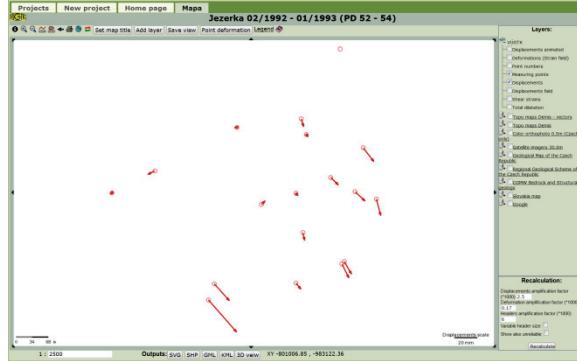


Fig. 5 Resulting shifts of points related to a fixed point on the northern edge of the network, stage 02.1992 - 01.1993.

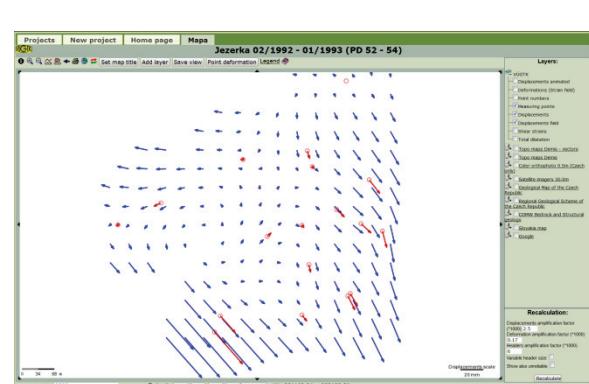


Fig. 8 Displacement field of points related to a fixed point on the northern edge of the network, stage 02.1992 - 01.1993.

For a better representation of the geodynamic situation Figs. 6 to 8 show the displacement fields determined by approximation from displacements of individual points of the network.

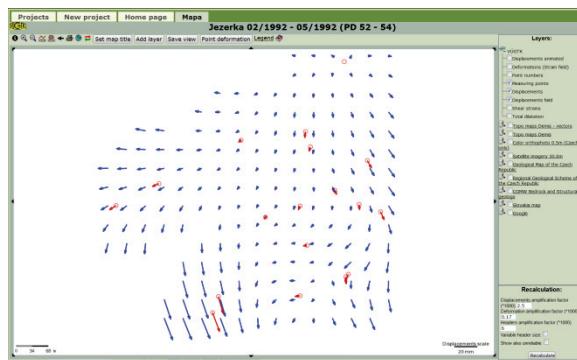


Fig. 6 Displacement field of points related to a fixed point on the northern edge of the network, stage 02. - 05.1992.

During the first two repeated stages (02-05.1992 and 02-10.1992) increased the size of shifts while maintaining their directions. The character of geodynamic activity is identical. The shifts are mostly in the direction down the slope. But at the last survey there was a total change. Points in the southern part of the network show similar character of displacements as in previous surveys, but at a lower speed. Some points in the middle of the network even show shifts up the hill, i.e. in the opposite direction than before. Therefore there was a strong suspicion about instability of the fixed point on the northern edge of the network. This suspicion was confirmed by repeated height determination of the fixed point by high precision leveling that confirmed a slope down of this point by about 3 mm comparing to previous survey. It means also its displacement down the slope. This has then influenced the amounts of displacements of other points. With respect to this fact it was found out that there is no point of the monitoring network stable enough that could be considered as a fixed one for the transformation of the adjusted monitoring network

into the local coordinate system and relate to it displacements of the other points of the network.

For these reasons the calculations of displacements were made by using Helmert similarity transformation as a condition for transformation of adjusted monitoring network into the local coordinate system. In this case the resulting displacements are related to the gravity centre point of the fiducial (supporting) points of the monitoring network, i.e. points entering transformation as identical. Resulting displacements are shown at Figs. 9 to 11. For a better demonstration of the geodynamic situation the fields of displacements determined by approximation from displacements of monitoring networks points are shown as well. The displacements are related to the center of gravity of the monitoring network.

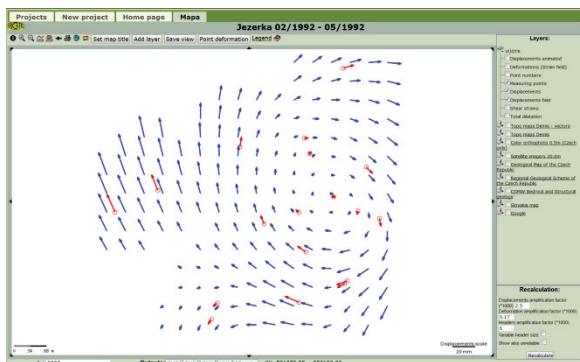


Fig. 9 Displacement field of points related to the center of gravity of the monitoring network, stage 02. - 05.1992.

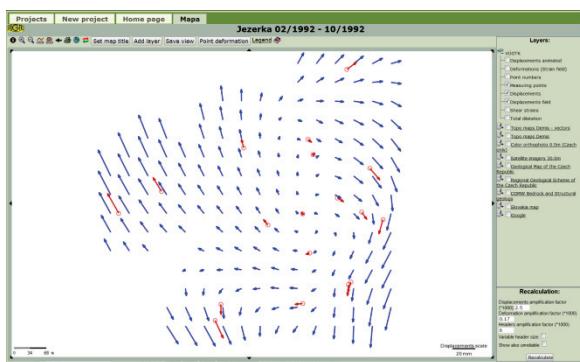


Fig. 10 Displacement field of points related to the center of gravity of the monitoring network, stage 02. - 10.1992.

These displacements are of a completely different character than those determined by previous method. Like this, we could create many other different variations resulting in displacements depending on the choice of conditions for the monitoring network location to a coordinate system. The question arises which of these displacements are correct and whether we are at all able to determine the "right"

displacements without having a stable point to which we could relate them. The answer is that we are not able to determine the absolute displacements in a local coordinate system without a stable point. We are only able to determine the relative relationships between monitoring network points without knowledge of the direction of the network movement.

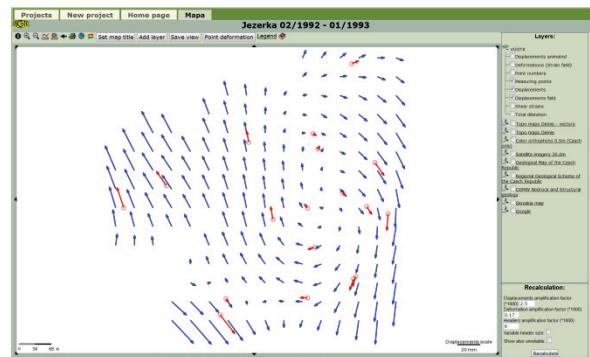


Fig. 11 Displacement field of points related to the center of gravity of the monitoring network, stage 02.1992 - 01.1993.

Analysis of deformation by continuum mechanics (strain analysis)

This creates a need for a mathematical tool that gives the identical results regardless of what method of location of the adjusted free monitoring network into a selected coordinate system. This tool is the continuum mechanics, as mentioned above. This method examines only the relative relationships between the monitoring network points and due to its insensitivity to translation and rotation gives always the same result. Various input values of points displacements, depending on the conditions selected for the location of the adjusted free network to a coordinate system vary only by translation and rotation.

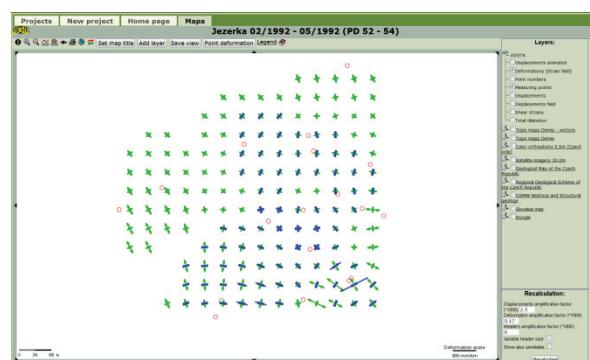


Fig. 12 The main axis of deformation tensors, stage 02. - 05.1992.

Both variants of shifts were used for determination of deformation tensors. The results are the same and are shown at Figs. 12 to 14 with the main axis of deformation tensors. Green shows the main axes of expansion and blue main axes of contraction.

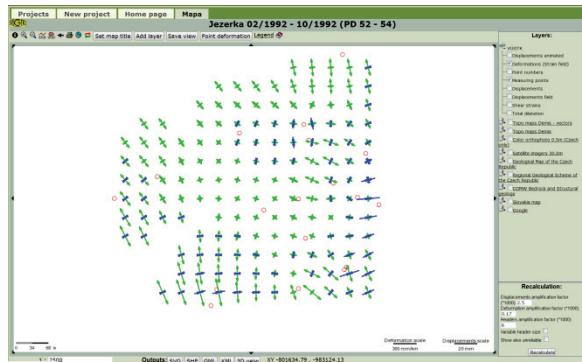


Fig. 13 The main axis of deformation tensors, stage 02. - 10.1992.

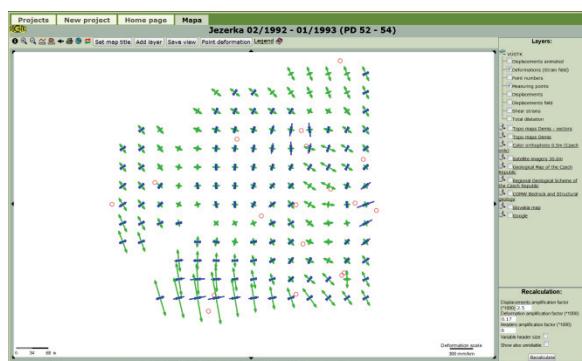


Fig. 14 The main axis of deformation tensors, stage 02.1993 - 01.1993

Evaluation of results and discussion

Each of the monitoring networks shows a large expansion area in the southwestern part of the network. The principal axes of expansion follow direction of the slope. Tab. 1 shows the maximum values of expansion for this area in individual stages of the monitoring network surveys and recalculated values to their velocity per month.

Table 1 Maximum values of expansion for southwestern part of the network.

Period	Value of expansion [mm/m]	Velocity of expansion [mm/m/month]
02/1992 - 05/1992	0.073	0,024
02/1992 - 10/1992	0.173	0,022
02/1992 - 01/1993	0.328	0,030

While the first two surveys confirmed approximately identical values of expansion, the last survey showed an increased value of expansion by nearly 50%. This confirms a growing geodynamic activity that may lead to a landslide in this area.

Another important area of the monitoring network is its northeastern part under the point that was initially considered to be stable. Again, this is an area with stronger expansion. As in the previous area also here the principal axes of the expansion maintain direction of the slope gradient. Tab. 2 shows again maximum values of expansion for this area for individual monitoring network surveys and further also recalculated values on their velocity by a month.

Table 2 Maximum values of expansion for northeastern part of the network.

Period	Value of expansion [mm/m]	Velocity of expansion [mm/m/month]
02/1992 - 05/1992	0.051	0,017
02/1992 - 10/1992	0.107	0,013
02/1992 - 01/1993	0.095	0,009

Reducing the velocity of expansion in this area can be most easily explained by the displacement down the slope of the fiducial point located in the area. This displacement has been verified by repeated high precision leveling and also by an analysis of displacements, as mentioned above.

This enabled to detect two distinct areas of geodynamic activity. Fig. 15 shows the resulting values of the principal axes of deformation tensors on background of an aerial image acquired 11 years later than these surveys and their evaluation. Here is clearly seen that the directions of the principal axes for expansion follow the direction downwards the slopes. Here can be also clearly seen that in the detected areas, there were a smaller landslides and that the direction of a slide always corresponded to the directions of the axes of principal expansion. Then it was in practice confirmed that the original calculations and analyses were giving correct results.

Conclusion

The article points out the distinct advantages of strain analysis tools, which can detect e.g. dangerous surges voltage surges of tension in monitored slopes, even when we do not have the time evolution of displacements of points related to a fixed coordinate frame. Thus the continuum mechanics tools enable to obtain more objective information about the geodynamic activity at monitored sites than a mere calculation and interpretation of point displacements in individual surveys.

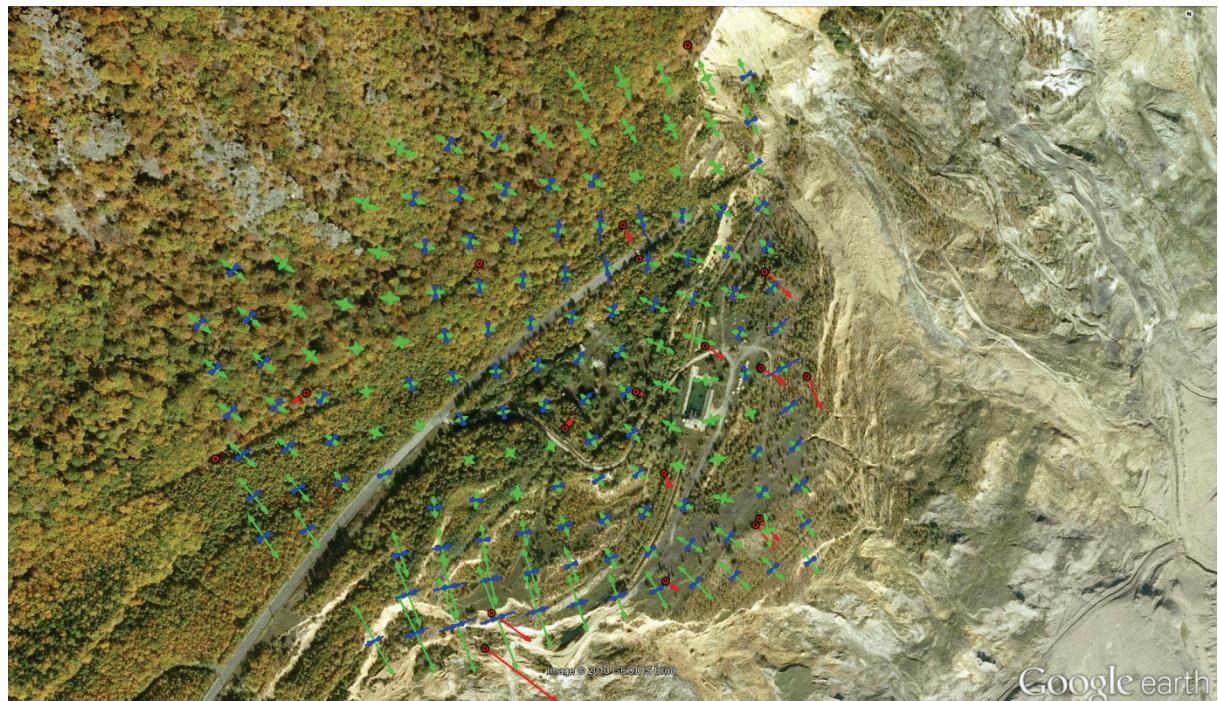


Fig. 15 Resulting values of the principal axes of deformation tensors and displacements on background of an aerial image with landslides during 11 years after these surveys and their evaluation.

Acknowledgments

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