

# Explicit Generalized Predictive Control of Speed and Position of PMSM Drives

Květoslav Belda and David Vošmik

**Abstract**—This paper deals with a specific explicit design of Generalized Predictive Control (GPC) for speed and position of 3-phase Permanent Magnet Synchronous Motors (PMSM). Predictive algorithms are designed with single and double integrations with respect to step and ramp reference signals. Needful field weakening and current limitation are solved by specific procedures based on a local indirect tuning/amplification of the relevant GPC parameters. The proposed solution due to low computational demands is suitable for real applications. Designed algorithms and procedures are demonstrated by figures and oscillogram screenshots of representative variables measured on the 10.7kW PMSM drive.

**Index Terms**—Permanent magnet synchronous motors, motion control, speed control, position control, predictive control, field weakening, current limitation.

## I. INTRODUCTION

3-phase Permanent Magnet Synchronous Motors (PMSM) are now frequently applied to advanced electrical drives. Due to minor demands on maintenance and only few mechanical elements, those motors are used in many industrial applications (machine tools, robots-manipulators) and in many traffic applications (transport vehicles, trams, trolley-buses). Their control consists in the design of appropriate amplitude and phase of stator voltage entering Pulse-Width-Modulation (PWM). PWM provides adequate amplitude and frequency of all Alternate Currents (AC) for the stator excitation.

A conventional approach is based on vector control realized by cascade configuration of PI controllers [1], [2]. That configuration represents a fixed coupling of separate PI controllers, where their mutual relations are considered as external disturbances. The cascade configuration consists of position, speed and current loops with respect to the application. Usually, constant setting of PI controllers corresponds only to some specific working range without any optimization for changing motor states. It is a limitation for applications with the dynamically changeable working conditions and ranges.

A promising alternative is a model-based approach that naturally consider all available information in the mathematical model of the motor. The base of the approach consists in the formulation of control task as a specific optimization, which takes into account the appropriate model, user reference values, requirements and physical constraints together.

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K. Belda is with the Institute of Information Theory and Automation, CAS, 182 08 Prague, Czech Republic, (e-mail: [belda@utia.cas.cz](mailto:belda@utia.cas.cz)).

D. Vošmik is with Regional Innovation Centre for Electrical Engineering, 306 14 Pilsen, Czech Republic (e-mail: [vosmik1@rice.zcu.cz](mailto:vosmik1@rice.zcu.cz)).

This paper focuses on Generalized Predictive Control (GPC) [3] as a one advanced control strategy of the model-based approach. The GPC strategy [4] generally represents a complex multi-objective optimization, which can involve different control targets [5], [6] e.g. compliance with specific constraints or flexible setting of a controller stiffness. In this context, the GPC is considered as a suitable control design of PMSM drives with the same functionality and universality as the conventional control approach. However, model-based GPC approach is inherently more flexible and efficient. The paper aim is to introduce a straightforward GPC design as a fully-equipped model-based continuous solution in comparison with widely used finite-set model predictive discrete solutions within specific regions as e.g. in [2], [7], [8].

The paper aims at fast explicit predictive control forms applied to the PMSM drives [9], [10]. The proposed GPC design is intended for the motion control, both speed control especially for traffic domain and position control for industrial robotics and machine tool applications. The suitability of the solution follows from its generality corresponding to the usual modern control theory [11].

The developed explicit forms employ fixed control laws with speed-variant gains. The gains are pre-computed off-line for a given specific range of electrical rotor speed of the motor. The rotor speed represents only one selecting parameter that determines the unique values of a mathematical model of the considered motor for off-line control gain calculation and values of the gains for on-line control action computation. The one parameter implies the fast real-time gain selection.

The designed algorithms realizing proposed explicit GPC are coupled with three front-end modules, which specifically adapt input signals entering the GPC algorithms. The modules, by a signal adaptation, enable GPC to solve a field weakening, current limitation and coping with a known load torque. The known load is usually combined with position control whereas other subtasks are predominantly employed in speed control. Stand-alone controllers based on developed algorithms and modules are demonstrated by real experiments on laboratory tandem of motors equipped with torque transducer between their shafts.

The paper is organized as follows. Section II outlines suitable mathematical models used for model-based control design. Section III introduces basic block diagram of predictive control. Section IV deals with front-end modules of GPC for torque-current control, field weakening and current limitation. Section V explains the principles of explicit GPC algorithms. Section VI presents real experiments by figures and directly captured oscillogram screenshots.



#### IV. FRONT-END MODULES

The front-end modules, i.e. torque-current, field weakening and current limitation modules shown in Fig. 1, enable GPC design to prove comparable and compact set of functionalities as usual cascade control without increasing complexity of the design. Torque-current module is needful if some nonzero reference torque  $\tau_{Lw}$  is required. Field weakening and current limitation modules are necessary for the limiting cases of high values of the speed and abrupt current peaks.

The requirement on complying with the reference torque can be solved via equilibrium torque equation. Field weakening can be achieved by adjustment of the flux current component [1]. The remaining current limitation problem can be solved inside as a part of GPC global optimization e.g. via a quadratic programming [11]. However, such usual solution leads to the algorithms unfeasible in real time in view of PMSM dynamics. Different ways consist in the direct penalization of possible current overshoots evaluated for a limited set of predictions within short prediction horizons [13], [14]. Similar idea will be considered here. However, the proposed solution will be involved into a usual GPC design without any restriction on a prediction horizon length.

The resulting solution of mentioned problems will take part of a cost function optimization by a specific on-line excitation of the inputs adapted by front-end modules. It represents a specific indirect local amplification of the appropriate weight parameters of GPC cost functions considered in the optimization of the appropriate control actions.

##### A. Torque-Current Module

Torque-current (TC) module arises from torque equilibrium equation (3). The module represents only a simple rearrangement of this equation:

$$i_{Sqw} = \frac{2}{3} \frac{1}{p^2 \psi_M} \left( p \tau_{Lw} + J \frac{d}{dt} \omega_{ew} - B \omega_{ew} \right) \quad (8)$$

The equation (8) represents in essence only the conversion of the known deterministic torque reference  $\tau_{Lw}$  into reference of the  $q$ -current component  $i_{Sqw}$  that is simply processable in control design as a standard reference input.

##### B. Field Weakening Module

Field weakening is necessary if a further increase of the rotor speed is required and supply voltage cannot be increased onward in appropriate way, just due to voltage supply limits. Since the direct control of magnetic flux  $\Psi_M$  is not possible with respect to permanent magnets, the field weakening is provided by incorporating a negative  $d$ -current (flux) component  $i_{Sd}$  [1]. The adjustment of  $i_{Sd}$  is limited by the following condition

$$i_S = \sqrt{i_{Sd}^2 + i_{Sq}^2} \leq I_{Smax} \quad (9)$$

where  $I_{Smax}$  is a maximum admissible stator current of the motor.

The described idea can be used in similar way in GPC design. The adjustment of the  $d$ -current component represents a new specific reference value  $i_{Sdw}$  that is negative,

if the weakening property is necessary, otherwise is zero. The field weakening (FW) module preceding GPC block is based on this principle. The procedure of the FW module can be expressed as follows:

$$\begin{aligned} u_S &:= \sqrt{u_{Sd}^2 + u_{Sq}^2}; \\ \text{if } u_S &\geq U_{Smax}, \\ i_{Sdw} &:= (U_{Smax} - u_S) k_{fw}; \\ \text{if } |i_{Sdw}| &> I_{Smax}, i_{Sdw} &:= -I_{Smax} k_{i_{ub}}; \text{end} \\ \text{else} \\ i_{Sdw} &:= 0; \\ \text{end} \end{aligned} \quad (10)$$

This procedure gives non-zero reference  $i_{Sdw}$  (1<sup>st</sup>  $if$ -block) and suppresses its possible overshoot of  $I_{Smax}$  (2<sup>nd</sup>  $if$ -block).

##### C. Current Limitation Module

Current limitation is important for a safe use of PMSM drives. The current sum may not exceed admissible stator current  $I_{Smax}$ , otherwise current overshoot can cause a drive malfunction or activation of current breakers leading to an undesirable drive operation interruption. The procedure of the current limitation (CL) module boosts the magnitudes of  $d$ - $q$  current components in order to increase their weight greatly in the appropriate cost function during its optimization in GPC design or in a pre-computed explicit control law:

$$\begin{aligned} i_{SdE} &:= i_{Sd}; \\ i_{SqE} &:= i_{Sq}; \\ \text{if } |i_{Sq}| &\geq \sqrt{I_{Smax}^2 - i_{Sdw}^2}, i_{SqE} &:= \left( \frac{|i_{Sq}|}{I_{Smax}} \right)^{k_{sp}} i_{Sq}; \\ \text{elseif } |i_{Sd}| &> I_{Smax} k_{i_{ub}}, i_{SdE} &:= \left( \frac{|i_{Sd}|}{I_{Smax}} \right)^{k_{sp}} i_{Sd}; \\ \text{end} \end{aligned} \quad (11)$$

where  $k_{sp}$  is a suitably selected exponent, and currents  $i_{SdE}$  and  $i_{SqE}$  are modified inputs of GPC.

Outlined procedure causes that the appropriate real current components will be intensively suppressed by weighting matrices in appropriate design cost function or GPC control law. Thus, artificial proportional extension of appropriate current component appears as a big outlier against other function terms and a predictive control law strongly suppresses such outlier. From practical point of view, the changeable lower limit  $\sqrt{(I_{Smax}^2 - i_{Sdw}^2)}$  in (11) is reasonable to be held above some meaningful level so that the condition would be feasible. The proposed procedure (11) gives acceptable results, but it does not represent hard limitation. It is only soft limitation, which comes close to hard limitation.

From GPC design point of view, the proposed algorithms (10) and (11) including (8) do not change a control design complexity or tuning of control parameters. The algorithms only modify selected inputs to the GPC controller. In fact, it means a specific indirect local tuning of control parameters that are inseparable with the signals in appropriate products.

## V. PRINCIPLES OF GPC ALGORITHMS

### A. Preliminaries of Predictive Control Design

This section outlines essential design terms and key procedural steps of the design. A used notation considers several principal variables:

$$\Delta x, x, \Delta y, y, \Delta u, u, e, w.$$

All variables are considered and defined as vectors respecting general multidimensional definition. They represent increments and absolute values of the system state, system outputs, control actions, errors and references. Their definitions and utilization in GPC design will be introduced in the subsequent subsections that deal with a specific composition of the equations of predictions, used quadratic cost functions and adapted minimization control laws for control actions.

The GPC algorithms are derived as discrete (digital) procedures suitable for a usual digital implementation. They provide computation of control actions within one optimization calculation. Generally, the calculation employs predictions of expected future output values mathematically expressed by equations of predictions [3]. These equations are closely related to the form of a cost function [15]. As mentioned, at a predictive control design, the quadratic cost function is used in various forms. A particular form depends on used equations of predictions that correspond to required control targets. In this paper, the two specific forms are introduced. They differ in the number of included integrators. Their features correspond to the conventional cascade configuration.

### B. Composition of Equations of Predictions

The equations of predictions express the functions of future system outputs in relation to unknown future control actions. The composition of the equations is always initiated by a discretization of updated state-space model (6) considering current  $\omega_e$ . A consequent discrete state-space model is as follows

$$\begin{aligned} \hat{x}_{k+1} &= A x_k + B u_k \\ y_k &= C x_k \end{aligned} \quad (12)$$

where  $A = A_k(\omega_e(k)) = A_{k+i}$ ,  $i = 1, \dots, N$  for an optimization in the current time instant  $k$  along a current prediction horizon  $N$ . The next optimization in the instant  $k+1$  is initiated by update of model (6) and its discretization again. In addition to the discrete model (12), an evolution model of aggregated control error  $\bar{e}_k$  is taken into account

$$e_k = w_k - y_k, \quad \bar{e}_k = \bar{e}_{k-1} + e_k \quad (13)$$

To achieve integral property in the design, the model (12) can be written in an incremental form in the following way

$$\begin{aligned} \hat{x}_{k+1} - x_k &= A (x_k - x_{k-1}) + B (u_k - u_{k-1}) \\ \hat{y}_{k+1} - y_k &= C (x_k - x_{k-1}) \end{aligned} \quad (14)$$

and in a condensed incremental form as well

$$\begin{aligned} \Delta \hat{x}_{k+1} &= A \Delta x_k + B \Delta u_k \\ \Delta \hat{y}_{k+1} &= C \Delta x_k \end{aligned} \quad (15)$$

The model (15) represents the base of the incremental feature of the equations of predictions used in the proposed predictive control design. The sequence of their composition corresponds to the cost functions in sections V-C and V-D.

The procedures of the composition are based on recursive principle. It is involved in the initial equations (16), (17), (20) and (23) by the index  $j = 1, \dots, N$  that determines individual discrete time instants for the prediction horizon  $N$ .

The individual equations are defined as follows. The equation (16) expresses the incremental predictions of the system state  $\Delta \hat{x}$

$$\Delta \hat{x}_{k+j} = A^j \Delta x_k + \sum_{i=1}^j A^{i-1} B \Delta u_{k+j-i} \quad (16)$$

The equation (17) defines increments of system outputs  $\Delta \hat{y}$

$$\Delta \hat{y}_{k+j} = C A^j \Delta x_k + \sum_{i=1}^j C A^{i-1} B \Delta u_{k+j-i} \quad (17)$$

It can be written in the condensed matrix form (18)

$$\Delta \hat{Y} = F_1 \Delta x_k + G_1 \Delta U \quad (18)$$

where individual elements  $\Delta \hat{Y}$ ,  $\Delta U$ ,  $F_1$  and  $G_1$  are defined as follows

$$\begin{aligned} \Delta \hat{Y} &= [ \Delta \hat{y}_{k+1}^T \quad \dots \quad \Delta \hat{y}_{k+N}^T ]^T \\ \Delta U &= [ \Delta u_k^T \quad \dots \quad \Delta u_{k+N-1}^T ]^T \\ F_1 &= \begin{bmatrix} CA \\ \vdots \\ CA^N \end{bmatrix}, \quad G_1 = \begin{bmatrix} CB & \dots & 0 \\ \vdots & \ddots & \vdots \\ CA^{N-1}B & \dots & CB \end{bmatrix} \end{aligned} \quad (19)$$

The equation (20) represents evolution of the full-value predictions of the system outputs  $\hat{y}$

$$\hat{y}_{k+j} = y_k + \sum_{i=1}^j \Delta \hat{y}_{k+i} \quad (20)$$

The appropriate matrix notation of (20) is the following

$$\hat{Y} = F_1 y_k + F_2 \Delta x_k + G_2 \Delta U \quad (21)$$

where subsequent individual elements  $\hat{Y}$ ,  $F_1$ ,  $F_2$  and  $G_2$  are defined as

$$\begin{aligned} \hat{Y} &= [ \hat{y}_{k+1}^T \quad \dots \quad \hat{y}_{k+N}^T ]^T \\ F_1 &= [ I \quad \dots \quad I ]^T \\ F_2 &= \begin{bmatrix} CA \\ \vdots \\ \sum_{i=1}^N CA^i \end{bmatrix}, \quad G_2 = \begin{bmatrix} CB & \dots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^N CA^{i-1}B & \dots & CB \end{bmatrix} \end{aligned} \quad (22)$$

Finally, the equation (23) is for an aggregate control error  $\hat{e}$

$$\begin{aligned} \hat{e}_{k+j-1} &= \bar{e}_k + \sum_{i=1}^{j-1} \{w_{k+i}\} - (j-1) I y_k \\ &\quad - \sum_{i=1}^{j-1} \{ (j-i) C A^i \} \Delta x_k \\ &\quad + \sum_{l=1}^{j-1} \left\{ \sum_{i=l}^{j-1} \{ (j-i) C A^{i-l} B \} \Delta u_{k+l-1} \right\} \end{aligned} \quad (23)$$

The equation (23) can be expressed, as well as previous equations, in the following matrix form:

$$\hat{E} = F_{\text{I}} \bar{e}_k + W_s - F_{\text{II}} y_k - F_3 \Delta x_k - G_3 \Delta U \quad (24)$$

where remaining elements  $\hat{E}$ ,  $W_s$ ,  $F_{\text{II}}$ ,  $F_3$  and  $G_3$  are

$$\hat{E} = [ \bar{e}_k^T \quad \cdots \quad \hat{e}_{k+N-1}^T ]^T$$

$$W_s = [ 0^T \quad w_{k+1}^T \quad (w_{k+1} + w_{k+2})^T \cdots (\sum_{i=1}^{N-1} \{w_{k+i}\})^T ]^T$$

$$F_{\text{II}} = [ 0 \quad I \quad 2I \quad \cdots \quad (N-1)I ]^T$$

$$F_3 = [ 0^T \quad (CA)^T \quad \cdots \quad (\sum_{i=1}^{N-1} (N-i)CA^i)^T ]^T \quad (25)$$

$$G_3 = \begin{bmatrix} 0 & \cdots & 0 \\ CB & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^{N-1} (N-i)CA^{i-1}B & \cdots & CB & 0 \end{bmatrix}$$

### C. GPC with Single Integrator (1<sup>st</sup> GPC Algorithm)

Let us consider a quadratic cost function in the form

$$J_k = \sum_{j=1}^N \left\{ \|(\hat{y}_{k+j} - w_{k+j}) Q_{yw}\|_2^2 + \|\Delta \hat{y}_{k+j} Q_{\Delta y}\|_2^2 + \|\Delta u_{k+j-1} Q_{\Delta u}\|_2^2 \right\} \quad (26)$$

$$= (\hat{Y} - W)^T Q_{YW} (\hat{Y} - W) + \Delta \hat{Y}^T Q_{\Delta Y} \Delta \hat{Y} + \Delta U^T Q_{\Delta U} \Delta U$$

where  $W$ ,  $Q_{YW}$ ,  $Q_{\Delta Y}$  and  $Q_{\Delta U}$  are defined as

$$W = [ w_{k+1}^T \quad w_{k+2}^T \quad \cdots \quad w_{k+N}^T ]^T$$

$$Q_{\diamond} = \begin{bmatrix} Q_*^T Q_* & & 0 \\ & \ddots & \\ 0 & & Q_*^T Q_* \end{bmatrix} \quad \begin{array}{l} \text{subscripts } \diamond, * : \\ \diamond \in \{YW, \Delta Y, \Delta U\} \\ * \in \{yw, \Delta y, \Delta u\} \end{array} \quad (27)$$

The minimization of the function (26) by searching for its local minimum leads to the control law (28)

$$\Delta u_k = M(G_2^T Q_{YW} G_2 + G_1^T Q_{\Delta Y} G_1 + Q_{\Delta U})^{-1} \times \left[ G_2^T Q_{YW} (W - F_1 y_k) - \{G_2^T Q_{YW} F_2 + G_1^T Q_{\Delta Y} F_1\} \Delta x_k \right] \quad (28)$$

where a rectangular matrix  $M$  is a matrix, which selects the appropriate first rows of control action increments  $\Delta u_k = M \Delta U$  corresponding to the current time instant  $k$ .

To simplify the computation,  $w_{k+j} = w_k$  for  $j = 1, \dots, N$  can be considered with regard to a fast dynamics of PMSM drives. Then, the algorithm with integration of  $\Delta u$  and penalization of output increments  $\Delta y$  will be composed as follows:

$$\begin{aligned} e_k &:= w_k - y_k \\ \Delta x_k &:= x_k - x_{k-1} \\ \Delta u_k &:= K_e F_1 e_k - K_{\Delta x} \Delta x_k \\ u_k &:= u_{k-1} + \Delta u_k \end{aligned} \quad (29)$$

Individual matrix gains  $K_e$  and  $K_{\Delta x}$  in (29) are pre-computed off-line for whole range of  $\omega_e$ . During a control process, they are only selected by current  $\omega_e$ . The gains correspond to the itemized form of the control law (28) as follows

$$\begin{aligned} K_e &= M(\cdot)^{-1} \times G_2^T Q_{YW} \\ K_{\Delta x} &= M(\cdot)^{-1} \times \{\cdot\} \end{aligned} \quad (30)$$

### D. GPC with Double Integrator (2<sup>nd</sup> GPC Algorithm)

Let us consider another form of the quadratic cost function

$$J_k = \sum_{j=1}^N \left\{ \|(\hat{y}_{k+j} - w_{k+j} - \hat{e}_{k+j-1}) Q_{yw}\|_2^2 + \|\Delta \hat{y}_{k+j} Q_{\Delta y}\|_2^2 + \|\Delta u_{k+j-1} Q_{\Delta u}\|_2^2 \right\} \quad (31)$$

$$= (\hat{Y} - W - \hat{E})^T Q_{YW} (\hat{Y} - W - \hat{E}) + \Delta \hat{Y}^T Q_{\Delta Y} \Delta \hat{Y} + \Delta U^T Q_{\Delta U} \Delta U$$

The minimization of the function (31) by searching for the local minimum leads to the following control law (32)

$$\begin{aligned} \Delta u_k &= M(G^T Q_{YW} G + G_1^T Q_{\Delta Y} G_1 + Q_{\Delta U})^{-1} \\ &\times \left[ G^T Q_{YW} \bar{e}_k + G^T Q_{YW} W + G^T Q_{YW} W_s - G^T Q_{YW} (F_1 + F_{\text{II}}) y_k - \{G_2^T Q_{YW} (F_2 + F_3) + G_1^T Q_{\Delta Y} F_1\} \Delta x_k \right] \end{aligned} \quad (32)$$

where  $G = G_2 + G_3$ .

To simplify the computation, the references  $w_{k+j} = w_k$  for  $j = 1, \dots, N$  can be considered as well as in the previous 1<sup>st</sup> algorithm. Then, the algorithm with integration of  $\Delta u$  and  $\bar{e}$  including penalization of output increments  $\Delta y$  will be composed as follows:

$$\begin{aligned} e_k &:= w_k - y_k \\ \bar{e}_k &:= \bar{e}_{k-1} + e_k \\ \Delta x_k &:= x_k - x_{k-1} \\ \Delta u_k &:= K_e \bar{e}_k + K_e F_1 w_k + K_e F_{\text{II}} w_k - K_y y_k - K_{\Delta x} \Delta x_k \\ u_k &:= u_{k-1} + \Delta u_k \end{aligned} \quad (33)$$

Individual matrix gains  $K_e$ ,  $K_y$  and  $K_{\Delta x}$  in (33) are pre-computed off-line. Then, in a control process, they are only selected by particular current  $\omega_e$  as well. The gains correspond to the itemized form of the control law (32) as follows

$$\begin{aligned} K_e &= M(\cdot)^{-1} \times G^T Q_{YW} \\ K_y &= K_e (F_1 + F_{\text{II}}) \\ K_{\Delta x} &= M(\cdot)^{-1} \times \{\cdot\} \end{aligned} \quad (34)$$

Note that if current limitation and field weakening conditions are activated, then appropriate current elements of vectors  $y_k$  and  $w_k$  in the 1<sup>st</sup> algorithm with single integrator (29) or in the 2<sup>nd</sup> algorithm with double integrator (33) are replaced by modified, adapted inputs  $y_{k(1:2,1)} = [i_{sdE}, i_{sqE}]^T$  and  $w_k = [i_{sdw}, i_{sqw}, \omega_{ew}]^T$  or  $w_k = [i_{sdw}, i_{sqw}, \vartheta_{ew}]^T$  according to selected control task and appropriate procedures described in Sections III and IV respectively.

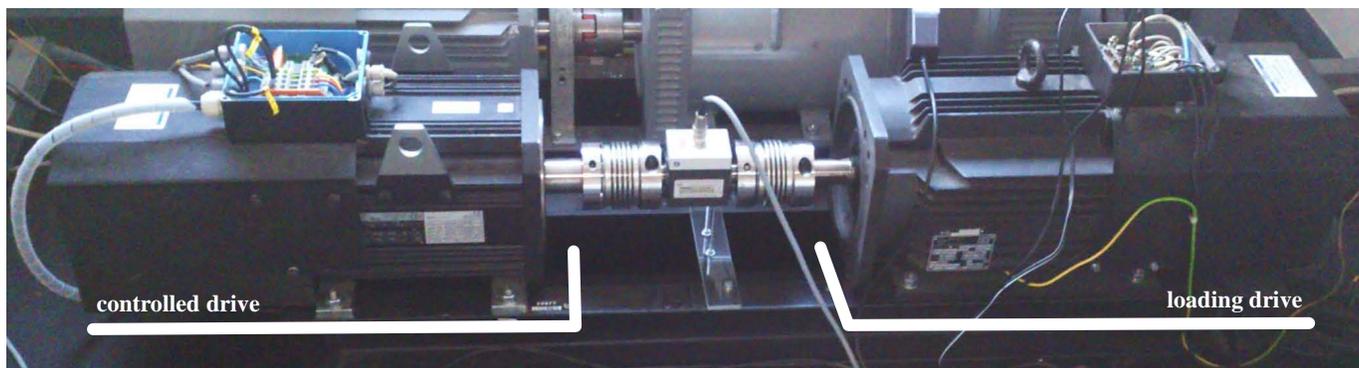


Fig. 2. Testing stand: controlled PMSM drive - 10.7 kW (left), torque transducer (middle) and coupled loading induction motor drive - 14.5 kW (right).

## VI. REAL EXPERIMENTS

This section demonstrates the proposed algorithms implemented in the control system based on DSP TMS320f28335. The control system is connected to the laboratory PMSM drive of rated power 10.7 kW and to the torque transducer T20WN/100NM. The main parameters of the motor are listed in the Table I at the end of the section. The testing stand with PMSM drive, torque transducer and coupled loading motor is shown in Fig. 2.

The optimal gains employed in control laws of GPC algorithms (29) or (33) are tuned and computed off-line by computer simulation for whole admissible range of the parameter  $\omega_e$ . The values of the individual elements of the gains are stored as fixed interpolating functional approximations, which follow their trends for considered admissible  $\omega_e$  range [9]. The trends of absolute values of the gains are symmetric with vertical symmetry axis  $\omega_e = 0$ . It simplifies the storing of gain values. As an example, the representative trends for matrix gain  $K_e$  are plotted in Fig. 3.

During real control experiments, the gains are only selected from the approximations just according to current  $\omega_e$ . Thus, the control actions are computed with front-end modules (TC), (FW) and (CL) and explicit control algorithms (29) or (33) including continuously changed gains. The computation is fast and achievable at considered sampling  $T_s = 125\mu s$  (8 kHz).

The algorithms (29) or (33) are applicable to both speed and position PMSM control tasks, just the relevant gain variables, signals and reference values have to be properly selected with respect to models (4) and (5).

The representative illustrations of the experiments include:

- speed control
  - slow high triangular speed signal  $\omega_{ew}$  (Fig. 4) with parameters in columns (A)
  - fast low triangular speed signal  $\omega_{ew}$  (Fig. 5) with the parameters in columns (B)

where (A) and (B) are columns of the Tables II and III;

- position control
    - position step signal  $\vartheta_{ew}$  (Fig. 6)
    - position sine signal  $\vartheta_{ew}$  (Fig. 7)
- both for zero load (left) and changing load (right) with parameters listed in the Table IV.

Fig. 4 demonstrates the algorithms when high triangular speed profile is required. Speed reference profile passes through field weakening region bounded approx. by 900 rpm. The proposed GPC algorithms, bounded by current  $I_{smax}$ , ensure reliable function in full speed operating range. The 1<sup>st</sup> algorithm slowly drifts from the reference slope. It is caused by only one integrator involved in the algorithm. The 2<sup>nd</sup> algorithm follows ramp segments more closely owing to double integrator. The both algorithms cannot solve speed error fully under field weakening and current limitation that are caused by excessive requirements of high command speed  $\omega_{ew} = \pm 2000$  rpm but insufficient (limited) power input.

Fig. 5 shows proposed GPC algorithms at fast low triangular speed reference signal. It is evident that double integrator (2<sup>nd</sup> algorithm) has positive influence. For triangular or ramp reference signals, the asymptotic tracking of the 2<sup>nd</sup> algorithm is obvious unlike the 1<sup>st</sup> algorithm with a steady offset. It corresponds to different behaviour of  $q$ -current components. The 2<sup>nd</sup> algorithm shows sharper current slopes, rectangular-like shape of  $i_{sq}$  signal. Since no field weakening is appeared, the needful  $i_{sq}$  current can be used for the motion.

Finally, Fig. 6 and Fig. 7 demonstrate the behavior of the 1<sup>st</sup> GPC algorithm for positional motion control. In the figures on the left, a control process is uniform with zero load, whereas on the right, the process is influenced by nonzero load that is manually switched (load sw.) from +13.4Nm to -11.5Nm with manually switched controller ahead (controller sw.) so that the switch would be perceptible.

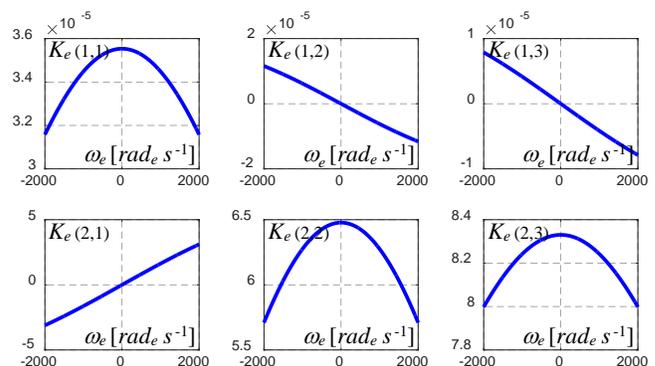


Fig. 3. Example of trends for elements of the gain  $K_e = f(\omega_e)$  in (29).

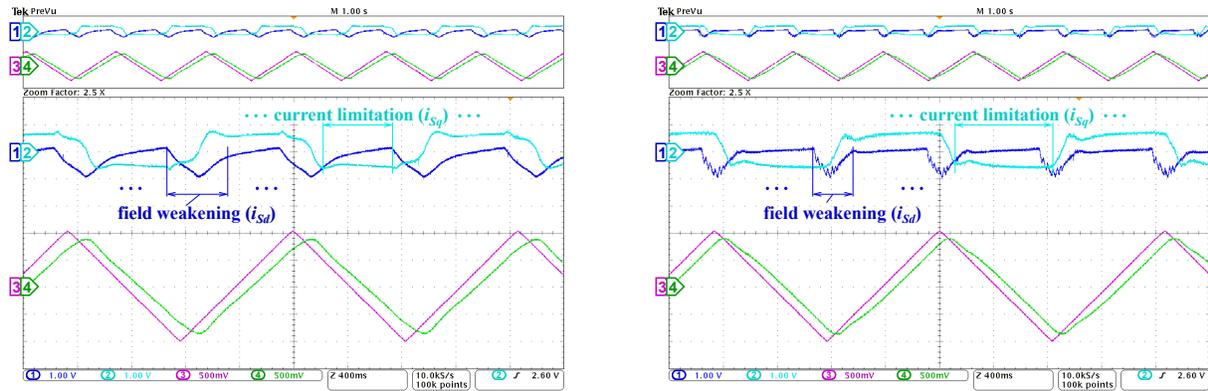


Fig. 4. Speed control: comparison of the 1<sup>st</sup> (left) and 2<sup>nd</sup> (right) GPC algorithms for high triangular command speed  $\pm 2000$  rpm; ch1:  $i_{Sd}$  current (25A/1V), ch2:  $i_{Sq}$  current (25A/1V), ch3:  $\omega_{ew}$  command el. rotor speed (135Hz/1V), ch4:  $\omega_e$  measured el. rotor speed (135Hz/1V).

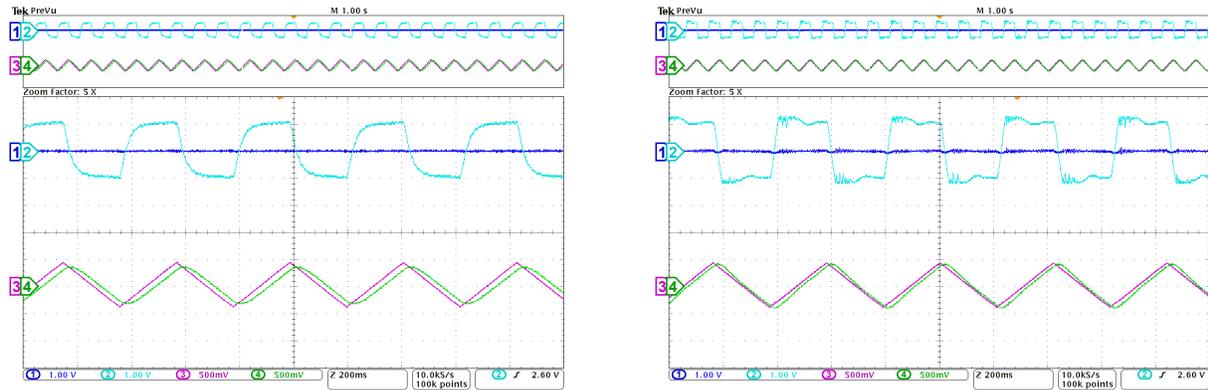


Fig. 5. Speed control: comparison of the 1<sup>st</sup> (left) and 2<sup>nd</sup> (right) GPC algorithms for low triangular command speed  $\pm 800$  rpm; ch1:  $i_{Sd}$  current (25A/1V), ch2:  $i_{Sq}$  current (25A/1V), ch3:  $\omega_{ew}$  command el. rotor speed (135Hz/1V), ch4:  $\omega_e$  measured el. rotor speed (135Hz/1V).

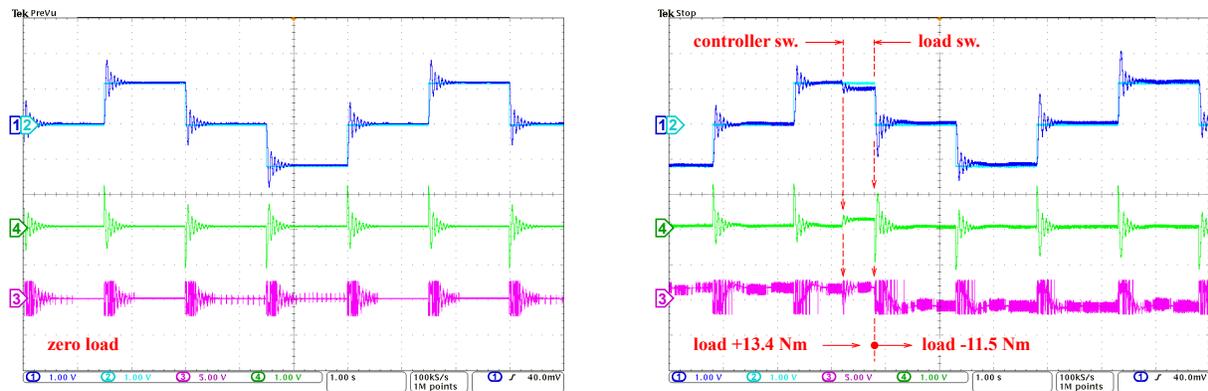


Fig. 6. Position control: comparison of the 1<sup>st</sup> GPC algorithm for step signal  $([0, +1.5, 0, -1.5, 0])$  rad<sub>e</sub> with zero load (left) and changing load (right); ch1:  $v_e$  (1.256 rad<sub>e</sub>/1V), ch2:  $v_{ew}$  (1.256 rad<sub>e</sub>/1V), ch3:  $i_{Sq}$  current (8A/1V), ch4:  $v_{ew} - v_e$  (1.256 rad<sub>e</sub>/1V).

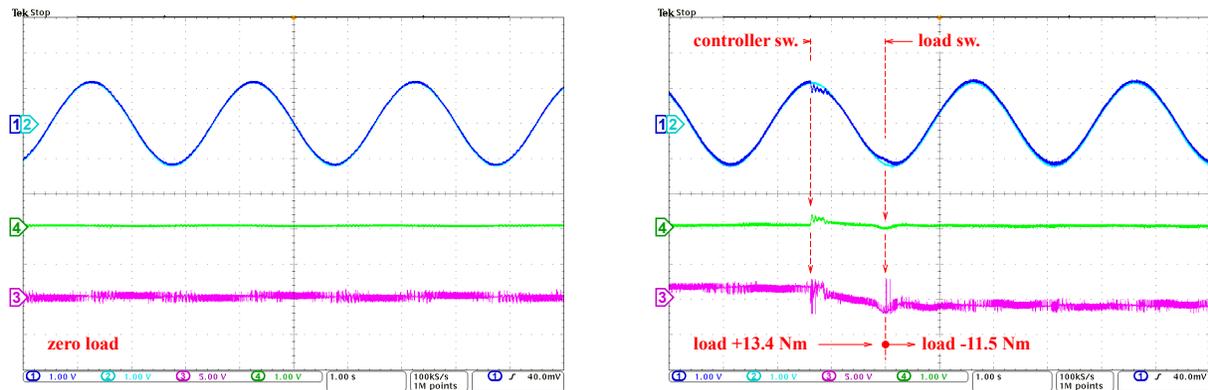


Fig. 7. Position control: comparison of the 1<sup>st</sup> GPC algorithm for sine signal (amplitude  $\pm 1.5$  rad<sub>e</sub>) with zero load (left) and changing load (right); ch1:  $v_e$  (1.256 rad<sub>e</sub>/1V), ch2:  $v_{ew}$  (1.256 rad<sub>e</sub>/1V), ch3:  $i_{Sq}$  current (8A/1V), ch4:  $v_{ew} - v_e$  (1.256 rad<sub>e</sub>/1V).

Fig. 6 shows the sharp motion control along position steps  $\vartheta_{ew} \in [0, -1.5, 0, +1.5, 0]$  rad<sub>e</sub>, which correspond to mechanical turns  $\pm 21.5$  deg<sub>m</sub>. The sharp  $\vartheta_{ew}$  steps serve as a testing boundary reference signals with respect to a robotic application intensification of motion control. Smooth reference signals are usually dominant in motion control, thus sine reference is used additionally. Fig. 7 demonstrates self-possessed GPC control along a sine reference signal with amplitude  $\pm 1.5$  rad<sub>e</sub>  $\approx \pm 21.5$  deg<sub>m</sub> as for the step signal in Fig. 6.

The reference positions  $\vartheta_{ew}$  (step and sine signals) are continuously followed by real rotor position  $\vartheta_e$  that converges to zero control error both for zero and changing load. From practical point of view, the motion control usually works with smooth trajectories with acceleration and deceleration as well as real load torques are usually less abrupt without a sign alteration against shown experiments, especially in Fig. 6.

TABLE I  
PARAMETERS OF THE USED PMSM DRIVE

Symbol	Description	Value
$P$	rated power	10.7 kW
$R_S$	stator resistance	0.28 $\Omega$ (Ohm)
$L_S$	stator inductance	0.003465 H (Henry)
$\Psi_M$	PM rotor magnetic flux	0.1989 Wb (Weber)
$B$	viscous friction coef.	0 kg m <sup>2</sup> s <sup>-1</sup>
$p$	number of pole pairs	4
$J$	moment of inertia	0.04 kg m <sup>2</sup>

TABLE II  
SPEED CONTROL: PARAMETERS OF THE 1<sup>ST</sup> GPC ALGORITHM

Symbol	Description	Value (A)	Value (B)
$N$	horizon of prediction		4
$Q_{yw}$	output penalization	diag(2, 1, 2)	diag(2, 1, 2)
$Q_{\Delta y}$	output incr. penalization	diag(10 <sup>2</sup> , 20, 2)	diag(20, 80, 5)
$Q_{\Delta u}$	input incr. penalization	diag(14, 7)	diag(5, 3.5)
$T_s$	sampling period	0.000125 s	
$k_{fw}$	field weakening gain	10 <sup>4</sup>	
$k_{iub}$	margin coefficient	0.9	
$k_{sp}$	current lim. exponent	40	

TABLE III  
SPEED CONTROL: PARAMETERS OF THE 2<sup>ND</sup> GPC ALGORITHM

Symbol	Description	Value (A)	Value (B)
$N$	horizon of prediction		4
$Q_{yw}$	output penalization	diag(10, 5, 4)	diag(10, 5, 10)
$Q_{\Delta y}$	output incr. penalization	diag(140, 80, 1)	diag(20, 80, 1)
$Q_{\Delta u}$	input incr. penalization	diag(20, 8)	diag(10, 8)
$T_s$	sampling period	0.000125 s	
$k_{fw}$	field weakening gain	4 $\times$ 10 <sup>4</sup>	
$k_{iub}$	margin coefficient	0.9	
$k_{sp}$	current lim. exponent	100	50

TABLE IV  
POSITION CONTROL: PARAMETERS OF THE BOTH GPC ALGORITHMS

Symbol	Description	Value
$N$	horizon of prediction	10
$Q_{yw}$	output penalization	diag(10 <sup>-2</sup> , 10 <sup>-2</sup> , 5)
$Q_{\Delta y}$	output incr. penalization	diag(10 <sup>-3</sup> , 10 <sup>-2</sup> , 600)
$Q_{\Delta u}$	input incr. penalization	diag(10 <sup>-3</sup> , 10 <sup>-3</sup> )
$T_s$	sampling period	0.000125 s

## VII. CONCLUSION

The paper presents novel algorithms of model-based Predictive Control [15] as an alternative to the conventional cascade control [1]. Real experiments on laboratory 10.7kW PMSM drive demonstrate a flexibility of the proposed algorithms under various working conditions. Fast responses of the algorithms (sampling 8 kHz) follow from the explicit GPC formulation enabling off-line pre-computation (pre-optimization) and fast selection of control law gains according to only one parameter. Thus, the proposed algorithms have an evident potential for future development in PMSM motion control.

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**Kvĕtoslav Belda** received the M.Sc. degree in automatic control and engineering informatics (1999), the Ph.D. degree in technical cybernetics (2003) and 2<sup>nd</sup> Ph.D. degree in electrical engineering and informatics (2006), from the Czech Technical University in Prague. Since 1999, he has been a Researcher with the Institute of Information Theory and Automation of the CAS. His research interests include model-based control design, drives, industrial robotics and mechatronics.



**David Vošmik** received the M.Sc. degree in electrical engineering from the University of West Bohemia, Pilsen, in 2006, and the Ph.D. degree in electronics from the University of West Bohemia, Pilsen, in 2014, Czech Republic. He is a member of Regional Innovation Centre for Electrical Engineering, RICE. His research interests are power electronics and drives including parameter identification and signal estimation.

