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State Estimation and Model Predictive Control for the Systems with Uniform Noise

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Abstract: This paper concerns the model predictive control applied to the systems with bounded uncertainties. These systems are described by a state-space model with uniformly distributed states and outputs with unknown bounds of respective distributions. The model matrices are assumed to be known. The approximate estimation of states and noise bounds is based on the Bayesian approach. A state-space generalised predictive control is selected as a suitable target model predictive control strategy. The proposed concept of the above mentioned estimation within generalised predictive control is illustrated by representative comparative simulation examples.

Keywords: Predictive control; bounded noise; probabilistic models; linear state-space models.

1. INTRODUCTION

Model predictive control is a popular method of process control Wang (2009), Camacho and Bordons (2007). It is usually formulated with state-space models, which can comfortably describe systems with a complex internal structure and many inputs and outputs. However, states of the systems are frequently unknown or unmeasurable. Therefore, a state estimation is indispensable for state-space model-based control design.

Since the control is applied to the real systems, stochastic disturbances entering the measurements have to be taken into account. They are usually described by noises with unbounded normal distributions. Then, the unknown states are estimated by well-known Kalman filter (KF) Anderson and Moore (1979) primarily. However, the Gaussian models do not respect specific boundaries of involved model variables, e.g. their physical ranges. For that reason, the KF algorithms have to be adapted Simon (2010). Moreover, KF needs relatively accurate setting of state and output covariance matrices, which is usually a difficult task in practise. Alternatively, non-probabilistic robust approach can be used, see e.g. Mayne et al. (2005), but at the cost of lowered control quality caused by the conservativeness of the approach.

This paper stays within a probabilistic framework but departs from the assumption of noise normality mentioned above. The involved noises are assumed to be bounded with unknown bounds. Specifically, a linear state-space model with uniform noise (LSU model) is utilised. The approximate Bayesian estimation of this model is proposed in Pavelková and Kárný (2014). It consists in the estimation of uniformly distributed system states and relevant noise parameters.

The paper deals with a novel interconnection of the generalized predictive control (GPC) Ordis and Clarke (1993) with the above mentioned LSU model. GPC is a powerful and flexible representative of model predictive control class which is able to take into account various mathematical calculus and model representations Clarke et al. (1987).

The LSU estimator and GPC were already successfully applied in individual way by authors in both linear and nonlinear settings, Belda and Böhm (2006), Pavelková (2011), Pavelková and Jirsa (2014), Belda and Vošmik (2016). The proposed utilization of the LSU model within GPC scheme enables user to decrease substantially the above mentioned effort connected with a tuning of covariance matrices while preserving the high control quality given up by robustness-oriented procedures and to cope with bounded models straightforwardly.

The paper is organized as follows. Section 2 introduces a probabilistic model of a controlled system that respects hard bounds on involved variables. Section 3 deals with the state estimation for considered models with uniform noises. In Section 4, the principles of model-based generalized predictive control are summarized. Section 5 demonstrates proposed approach by comparative simulation examples. The paper contributions are summed in the Section 6.

Throughout the paper, the following notation will be used: z_t is the value of a column vector z at a discrete-time instant $t \in t^\star \equiv \left\{\underline{t}, \cdots, \overline{t}\right\}$, $\underline{t} \geq 1$; $z_{t;i}$ is the i-th entry of z_t; ℓ_z is the length of the vector z; $\underline{z}$ and $\overline{z}$ are lower and upper bounds on z, respectively. The symbol $f(\cdot|\cdot)$ denotes a conditional probability density function (pdf); names of arguments distinguish respective pdfs; no formal distinction is made between a random variable, its realisation and an argument of the pdf.

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2. MODEL OF CONTROLLED SYSTEM

2.1 Probabilistic Model of Closed-Loop System

The closed-loop system that includes controller and controlled system can be described by the set ℓ_y-dimensional observable outputs y_t , ℓ_u-dimensional system inputs u_t, ℓ_x-dimensional system states x_t (possibly unobserved), $t \in t^\star$, and by the collection of time-invariant unknown parameters Θ. In the considered Bayesian set up Berger (1985); Kárný et al. (2005), the system is modelled by pdfs. The chain rule factorises the joint pdf of all involved variables as follows

$$
\begin{aligned}
& f\left(d_1, \cdots, d_{\bar{t}}, x_0, \cdots, x_{\bar{t}}, \Theta\right) \qquad (1)\\
&= \underbrace{f\left(x_0, \Theta\right)}_{\text{prior pdf}} \prod_{t \in t^\star} \underbrace{f\left(u_t | d_1, \cdots, d_{t-1}\right)}_{\text{controller}} \\
&\times \prod_{t \in t^\star} \underbrace{f\left(y_t | x_t, \Theta\right)}_{\text{observation model}} \times \underbrace{f\left(x_t | x_{t-1}, u_{t-1}, \Theta\right)}_{\text{state evolution model}} .
\end{aligned}
$$

This form assumes that Θ and x_t are unknown to the controller and that x_t contains entire information on the past evolution of the system. The sequence of data records $d_t = (y_t, u_t)$, $t \in t^\star$, is sequentially observed.

2.2 State-Space Model of Controlled System

The controlled system, considering (1), is given by the observation and state evolution model. Let us consider such controlled system that can be represented by a linear discrete time state-space model in the following form ($t \in t^\star$)

$$
\begin{aligned}
x_t &= Ax_{t-1} + Bu_{t-1} + \nu_t = \tilde{x}_t + \nu_t \qquad (2)\\
y_t &= Cx_t \quad + \quad n_t \qquad = \tilde{y}_t + n_t \qquad (3)
\end{aligned}
$$

where $\tilde{x}_t$, $\tilde{y}_t$ are deterministic parts of model, A, B, C are known state-space model matrices; ν_t, n_t are vectors of the state and output noises, of sizes ℓ_x and ℓ_y, respectively. Noises are assumed to be identically distributed and conditionally independent of past, having zero means, constant variances and uncorrelated entries.

The noises ν_t in (2) and n_t in (3) are usually assumed to be Gaussian with known covariance matrices. The paper departs from this standard and assumes that both the state noise ν_t and the observation noise n_t are uniformly distributed on a multivariate box with the centre at zero and with unknown half-widths ρ and r of the support intervals, respectively. Then, the noises are described as

$$
f\left(\nu_t | \rho\right) = \mathcal{U}_{\nu_t}\left(0_{\ell_x}, \rho\right), \quad f\left(n_t | r\right) = \mathcal{U}_{n_t}\left(0_{\ell_y}, r\right) \qquad (4)
$$

where $\mathcal{U}_z(\mu, \alpha)$ denotes a uniform pdf of a variable z given by the expectation μ and the half-width of the support α; ρ and r are time-invariant.

The equations (2), (3), and (4) define a linear state-space uniform model that corresponds to state evolution and observation factors in (1) considering $\Theta \equiv \left[\rho^T, r^T\right]^T$:

$$
\begin{aligned}
f\left(x_t | x_{t-1}, u_{t-1}, \Theta\right) &= \mathcal{U}_{x_t}\left(\tilde{x}_t, \rho\right) \qquad (5)\\
f\left(y_t | x_t, \Theta\right) &= \mathcal{U}_{y_t}\left(\tilde{y}_t, r\right) \qquad (6)
\end{aligned}
$$

3. STATE AND PARAMETER ESTIMATION USING UNIFORM STATE-SPACE MODEL

Standard Bayesian estimation of (5) and (6) on a window of a fixed length N_e works with the data D_t defined as

$$
D_t = \left[\, d_t^T, \, d_{t-1}^T, \, \cdots, \, d_{t-N_e}^T \right]^T \qquad (7)
$$

and internal X_t that collect both unobserved states and unknown parameters

$$
X_t \equiv \left[\, x_t^T, \, \cdots, \, x_{t-N_e}^T, \Theta^T \right]^T. \qquad (8)
$$

The estimation reduces to evaluation of characteristics of the posterior pdf $f(X_t|D_t)$ of X_t conditioned by the observed data D_t. A design of the Bayesian estimator requires the knowledge of the observation and state evolution models and a specification of the prior pdf. The prior information has the form of possible ranges of X_t, i.e.

$$
\underline{X}_t = \begin{bmatrix} \underline{x}_t \\ \vdots \\ \underline{x}_{t-N_e} \\ \underline{\Theta} \end{bmatrix} \leq X_t \leq \begin{bmatrix} \overline{x}_t \\ \vdots \\ \overline{x}_{t-N_e} \\ \overline{\Theta} \end{bmatrix} = \overline{X}_t. \qquad (9)
$$

The joint pdf of D_t and X_t, $t \in t^\star$, is

$$
\begin{aligned}
f\left(X_t, D_t\right) &= \left(\prod_{i=1}^{\ell_x} \frac{1}{2\,\rho_i} \prod_{j=1}^{\ell_y} \frac{1}{2\,r_j} \right)^{N_e} \prod_{j=1}^{\ell_\Theta} \frac{1}{\overline{\Theta}_j - \underline{\Theta}_j} \qquad (10)\\
&\times \prod_{k=t-N_e}^{t} \left(f\left(u_k | d_1, \cdots, d_{t-1}\right) \prod_{i=1}^{\ell_x} \frac{1}{\overline{x}_{k;i} - \underline{x}_{k;i}} \right) \chi(X_t; X_t^\star)
\end{aligned}
$$

The set indicator $\chi(X_t; X_t^\star)$ restricts the support of this pdf. It equals 1 if $X \in X^\star$ and 0 otherwise.

The set $X_t^\star$ contains such X_t, for which the noise terms in (2) and (3), considering pdfs (4), are within the multivariate box defined by (9), as follows

$$
\begin{aligned}
X_t^\star = \{\, & X_t \colon (\, x_t, \, \Theta) \text{ meeting } (9) \qquad (11)\\
\text{and } & |x_{t;i} - \tilde{x}_{t;i}| \leq \rho_i, \; i = 1, \cdots, \ell_x, \\
& |y_{t;j} - \tilde{y}_{t;j}| \leq r_j, \, j = 1, \cdots, \ell_y \} \, .
\end{aligned}
$$

A point estimation is considered. Therefore, a series of maximum a posteriori (MAP) estimates $\hat{X}_t$ of the unknown X_t (8), $t \in t^\star$ is evaluated Berger (1985). Note that estimation does not require the knowledge of controller $f\left(u_t | d_1, \cdots, d_{t-1}\right)$; only the series of u_t is required to be known.

With the joint pdf (10), it holds for MAP estimates

$$
\hat{X}_t = \arg\max_{X_t \in X_t^\star} \left(\prod_{i=1}^{\ell_x} \frac{1}{2\,q_i} \prod_{i=1}^{\ell_y} \frac{1}{2\,\rho_i} \right)^{N_e} . \qquad (12)
$$

For completely known model matrices, the problem (12) can be reduced to a linear programming (LP) problem Pavelková and Kárný (2014) to find a vector X_t (8), $t \in t^\star$, that minimizes

$$\mathrm{f}^T X_t = \sum_{i=1}^{\ell_x} \rho_i + \sum_{j=1}^{\ell_y} r_j \tag{13}$$

subject to

$$\mathcal{A} X_t \le b_t, \quad \underline{X}_t \le X_t \le \overline{X}_t,$$

where $\mathrm{f}^T \equiv [0^T_{(\ell_{X_t}-\ell_x-\ell_y)}, 1^T_{(\ell_x+\ell_y)}]$ consists of the vectors of zeros and ones of the indicated lengths. $\underline{X}_t$, $\overline{X}_t$ are given by (9). $\mathcal{A}$ and b_t are the matrix and vector, respectively, constructed according to inequalities (11) as follows

$$\mathcal{A} = \begin{bmatrix} \mathcal{A}11 & \mathcal{A}12 \\ \mathcal{A}21 & \mathcal{A}22 \end{bmatrix}, \quad b_t = \begin{bmatrix} b1_t \\ b2_t \end{bmatrix},$$

where

$$\mathcal{A}11 = \begin{bmatrix} I & -A & 0 & \cdots & 0 & 0 \\ 0 & I & -A & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & I & -A \\ 0 & 0 & 0 & \cdots & 0 & I \end{bmatrix} \otimes K$$

$$\mathcal{A}12 = 1_{(2(N_e+1))} \otimes \left[-I_{(\ell_x,\ell_x)} \; 0_{(\ell_x,\ell_y)}\right]$$

$$\mathcal{A}21 = \begin{bmatrix} C & \cdots & 0_{(\ell_y,\ell_x)} & 0_{(\ell_y,\ell_x)} \\ \vdots & \ddots & \vdots & \vdots \\ 0_{(\ell_y,\ell_x)} & \cdots & 0_{(\ell_y,\ell_x)} & C \end{bmatrix} \otimes K$$

$$\mathcal{A}22 = 1_{(2(N_e+1))} \otimes \left[0_{(\ell_y,\ell_x)} \; -I_{(\ell_y)}\right]$$

$$b1_t = [(Bu_{t-1})^T, \cdots, (Bu_{t-N_e})^T, (Bu_{t-N_e-1} + A\hat{x}_{t-N_e-1})^T]^T \otimes K$$

$$b2_t = \left[y_t^T, \cdots, y_{t-N_e}^T\right]^T \otimes K$$

where $\otimes$ denotes Kronecker product, $K \equiv [1 \; -1]^T$, $I_{(\alpha,\beta)}$ and $0_{(\alpha,\beta)}$ are the identity and zero matrices of the indicated dimensions, respectively, in $\mathcal{A}11$, they are square with identical order ℓ_x, and $\hat{x}_{t-N_e-1}$ is the estimate that is obtained in the previous step.

4. MODEL PREDICTIVE CONTROL

Model predictive control depends on a mathematical model of the controlled system. By using a state-space model (2) and (3), the information required for prediction ahead is represented by the state variable at the current time. The computation is provided within one optimization calculation.

In this paper, a discrete positional generalized predictive control (GPC) Ordis and Clarke (1993) is considered as a suitable model predictive control representative for coupling with state estimation (13). It employs predictions of expected future output values based on the state estimates. The future output values are expressed by equations of predictions Wang (2009). The main design elements, i.e. equations of predictions, corresponding quadratic cost function and final control law, are described in the following subsections.

4.1 Equations of Predictions

The equations of predictions express the relationship between future outputs and unknown future control actions. The composition of these equations arises from discrete state-space model (2) and (3) as follows

$$\begin{aligned} \hat{y}_{t+1} &= CA\hat{x}_t + CBu_t \\ \hat{y}_{t+2} &= CA^2\hat{x}_t + CABu_t + CBu_{t+1} \\ \vdots \quad & \quad \vdots \\ \hat{y}_{t+N_p} &= CA^{N_p}\hat{x}_t + CA^{N_p-1}Bu_t \cdots + CBu_{t+N_p-1} \end{aligned} \tag{14}$$

where $\hat{x}_t$ is state estimate from (13) and N_p is prediction horizon. The equations can be expressed in matrix form:

$$\hat{Y}_{t+1} = F\,\hat{x}_t + G\,U_t \tag{15}$$

where $\hat{Y}_{t+1}$, and U_t are sequences of output predictions and searched control actions, respectively

$$\hat{Y}_{t+1} = \left[\hat{y}_{t+1}^T, \cdots, \hat{y}_{t+N_p}^T\right]^T \tag{16}$$

$$U_t = \left[u_t^T, \cdots, u_{t+N_p-1}^T\right]^T \tag{17}$$

and terms F and G are defined as follows

$$F = \begin{bmatrix} CA \\ \vdots \\ CA^{N_p} \end{bmatrix}, \quad G = \begin{bmatrix} CB & \cdots & 0 \\ \vdots & \ddots & \vdots \\ CA^{N_p-1}B & \cdots & CB \end{bmatrix} \tag{18}$$

4.2 Quadratic Cost Function

Quadratic cost function is an objective function that balances differences between predicted outputs and given references together with amount of input energy. For considered positional GPC algorithm, it is defined as follows

$$\begin{aligned} J_t &= E\sum_{j=1}^{N_p}\{\|Q_{yw}(\hat{y}_{t+j} - w_{t+j})\|_2^2 + \|Q_u u_{t+j-1}\|_2^2\} \\ &= E\left[(\hat{Y}_{t+1}-W_{t+1})^T Q_{YW}(\hat{Y}_{t+1}-W_{t+1}) + U_t^T Q_U U_t\right] \end{aligned} \tag{19}$$

where and E denotes the expected value, W_{t+1} represents sequences of references

$$W_{t+1} = \left[w_{t+1}^T, \cdots, w_{t+N_p}^T\right]^T \tag{20}$$

and penalty matrices Q_{YW} and Q_U are defined as

$$Q_\diamond = \begin{bmatrix} Q_*^T Q_* & & 0 \\ & \ddots & \\ 0 & & Q_*^T Q_* \end{bmatrix} \left| \begin{array}{l} \text{subscripts } \diamond, *: \\ \diamond \in \{YW, U\} \\ * \in \{yw, u\} \end{array} \right. \tag{21}$$

Minimization of (19) leads to a control action vector for whole prediction horizon N_p:

$$U_t = (G^T Q_{YW} G + Q_U)^{-1}\, G^T Q_{YW}\,(W_{t+1} - F\hat{x}_t) \tag{22}$$

4.3 Predictive Control Law

During the control, only the first control action from (22) is really applied to the controlled system. Therefore, the appropriate control actions can be expressed as follows

$$u_t = M(G^T Q_{YW} G + Q_U)^{-1}\, G^T Q_{YW}\, (W_{t+1} - F\hat{x}_t) \quad (23)$$

where a rectangular matrix M selects the appropriate first rows of control actions u corresponding to the time instant t by means of appropriate unit diagonal elements Belda and Vošmik (2013).

The formula of the computation of control actions (23) can be expressed by the following compact control law

$$u_t = -k_x\hat{x}_t + K_W W_{t+1} \quad (24)$$

where individual gains are defined as

$$\begin{aligned} K_W &= M(G^T Q_{YW} G + Q_U)^{-1}\, G^T Q_{YW} \\ k_x &= K_W\, F \end{aligned}$$

The obtained control law (24) represents a suitable form for the intended implementation.

5. EXPERIMENTS

In this section, the proposed concept of coupled model-based control and LSU estimation is illustrated on a representative simulative example and compared with the same concept but with KF estimation.

5.1 Simulation Setup

The illustrative examples were realized in MATLAB [1] with a single-input single-output continuous-time model of second order representing the controlled system

$$\ddot{\hat{y}}(\tau) = -2\dot{\hat{y}}(\tau) - \hat{y}(\tau) + u(\tau) \quad (25)$$

$$y(\tau) = \hat{y}(\tau) + n(\tau) \quad (26)$$

where τ denotes continuous time. This model corresponds to the following discrete-time ARX model with the sampling period $Ts = 0.1s$

$$\begin{aligned} y_t = \;& 1.8097y_{t-1} - 0.8187y_{t-2} \\ &+ 0.0047u_{t-1} + 0.0044u_{t-2} + n_t \end{aligned} \quad (27)$$

which can be transformed into the state-space model form (2) and (3)

$$x_{t+1} = \begin{bmatrix} 0.8144 & -0.0905 \\ 0.0905 & 0.9953 \end{bmatrix} x_t + \begin{bmatrix} 0.0905 \\ 0.0047 \end{bmatrix} u_t + \nu_t \quad (28)$$

$$y_t = [\, 0 \quad 1 \,] x_t + n_t \quad (29)$$

An additive measurement uniform noise was simulated with the pdf $f(n_t|r) = \mathcal{U}_{n_t}(0, 0.1)$. It was a source of uncertainty for the controlled system represented by the continuous model (25) and (26).

[1] www.mathworks.com/products/matlab

The state estimates were computed according to (13) using the function "linprog" from MATLAB Optimisation Toolbox with the following settings: $N_e \in \{25, 200\}$, $\underline{x} = [-\infty, -\infty]^T$, $\overline{x} = [\infty, \infty]^T$, $\underline{\rho} = [\varepsilon, \varepsilon]^T$, $\overline{\rho} = [10, 10]^T$, $\underline{r} = \varepsilon$, $\overline{r} = 10$; where $\varepsilon = 10^{-15}$ corresponds to the meaningful (guaranteed) accuracy of the computation.

For comparison, the state estimation was additionally performed by KF in the standard form

$$\hat{x}_t = (I - KC)A\hat{x}_{t-1} + (I - KC)Bu_{t-1} + Ky_t \quad (30)$$

where I is the identity matrix of appropriate dimension, and K is the Kalman gain computation of which requires the knowledge of covariance matrices of measurement and process noises. KF run using the function "kalman" from MATLAB Control System Toolbox for the settings:

(i) "ideal" setting based on results of LSU estimation (13) with covariance matrix for x_t: $C_x = diag([\frac{1}{3}\,\hat{\rho}^2_{t;1}, \frac{1}{3}\,\hat{\rho}^2_{t;2}])$, variance for y_t, $\sigma^2_y = \frac{1}{3}\hat{r}^2$ and initial error covariance matrix $P = diag([100,\, 100])$;

(ii) general "imperfect" setting with covariance matrix for x_t: $C_x = diag([\frac{1}{3}\cdot 1^2,\, \frac{1}{3}\cdot 1^2])$, variance for y_t: $\sigma^2_y = \frac{1}{3}\cdot 1^2$ and initial error covariance matrix $P = diag([100,\, 100])$.

Note that setting for the case (i) uses Gaussian approximation Gelman et al. (2004) where the mean and covariance of such approximation can be computed by matching the first two moments of the original distribution. Due to assumption of the zero means, the variances are as follows

$$\sigma^2_{\triangle\,\mathrm{KF}} \equiv \sigma^2_{\triangle\,\mathrm{LSU}} = \frac{1}{3}\,\triangle^2 \quad (31)$$

where $\triangle$ denotes ρ or r.

The gains of the GPC control law (24) were computed with the model (28) and (29) considering the following control parameters: $N_p = 10$, $Q_y = 1$, $Q_u = 0.01$.

All simulation experiments were performed under identical conditions with the same random noise realisation.

5.2 Results and Discussion

The experiment results are depicted in Fig. 1 – 5 for LSU estimator (left), for KF set according to estimates $\hat{\rho}$ and $\hat{r}$ obtained by LSU estimator (middle) and for a imperfectly set KF (right). The number in the bracket after variable name informs about memory length N_e. Fig. 1 shows the courses of the reference and controlled output. Fig. 2 depicts the course of control input. Fig. 3 displays the count histograms of y_t values for constant part of reference w_t, i.e., for $\tau \in \langle T_1, T_2\rangle = \langle 110s, 200s\rangle$. In Fig. 4 – 5, the count histograms for particular entries of x_t are depicted for the above mentioned constant part of reference w_t.

Table 1 compares (i) mean absolute controller error $\bar{e}$ and (ii) standard deviation σ_u of the series of control inputs u_t for the constant part of reference w_t within time interval $\langle T_1, T_2\rangle = \langle 110s, 200s\rangle$.

Table 2 shows the influence of various lengths of the LSU estimation memory N_e on noise bounds estimates and corresponding KF gain $K = K_{N_e}$.

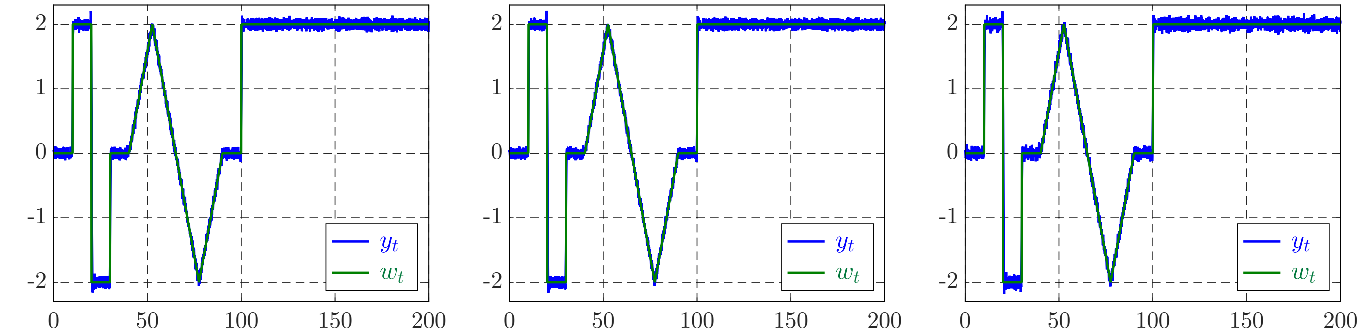

Fig. 1. Time histories $[s]$ of outputs y_t and references w_t for LSU estimator, well set KF and imperfectly set KF

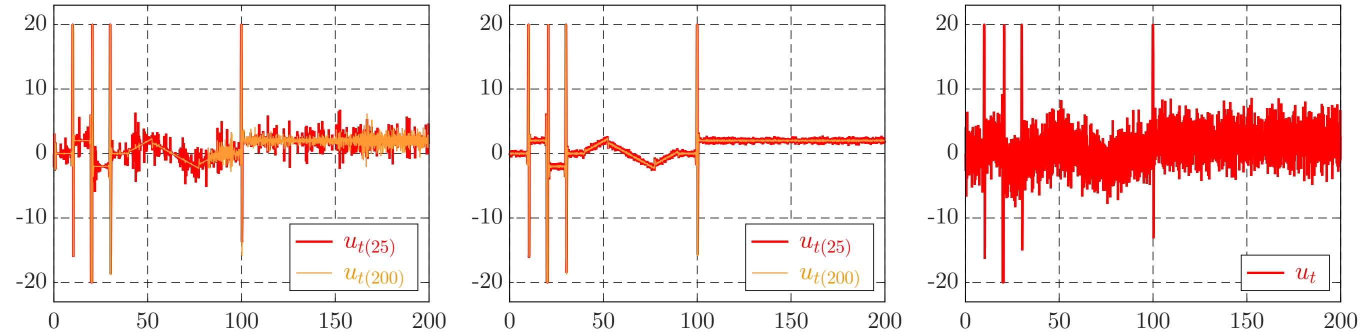

Fig. 2. Time histories $[s]$ of control actions u_t for LSU estimator, well set KF and imperfectly set KF

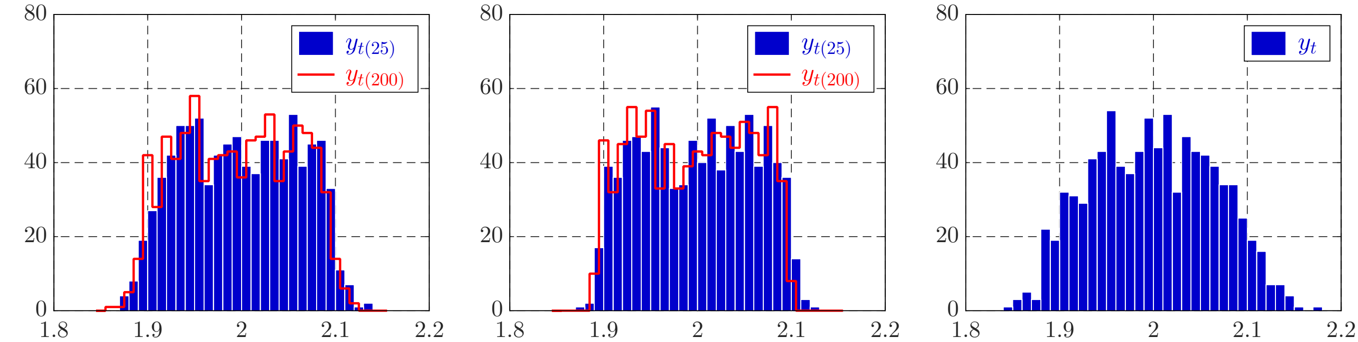

Fig. 3. Count histograms of system outputs y_t for LSU estimator, well set KF and imperfectly set KF

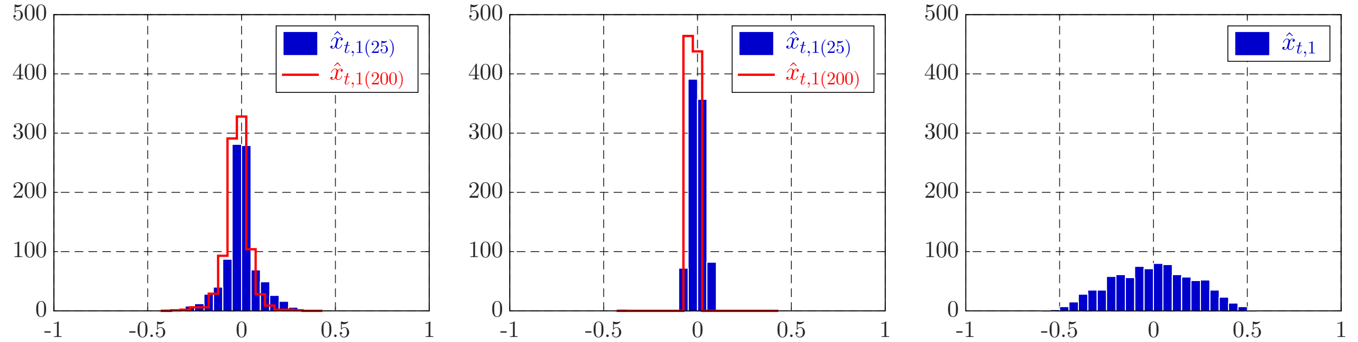

Fig. 4. Count histograms of estimated state variable $\hat{x}_{t,1}$ for LSU estimator, well set KF and imperfectly set KF

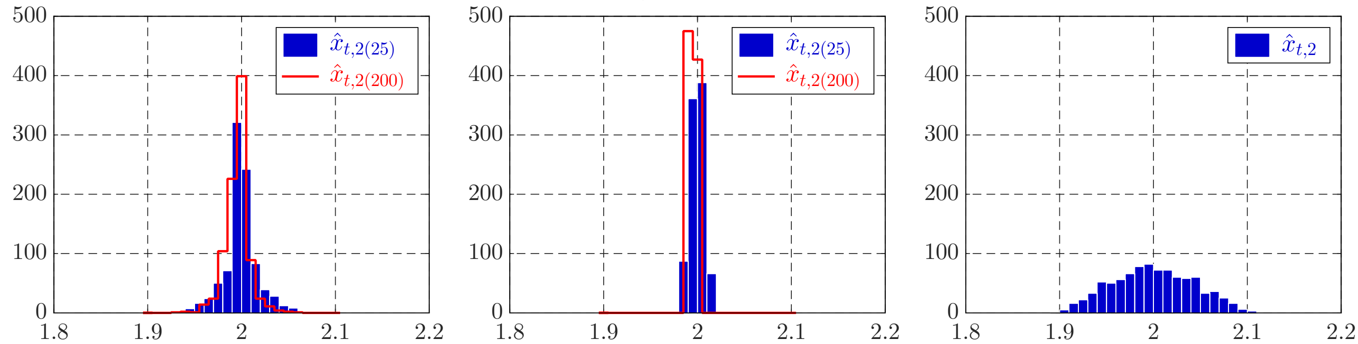

Fig. 5. Count histograms of estimated state variable $\hat{x}_{t,2}$ for LSU estimator, well set KF and imperfectly set KF

Comparison of diagrams in Fig. 1 for particular estimators indicates that the output courses are almost identical. It can be explained by stabilising properties of the closed loop of used GPC controller. Nevertheless, the control actions differ significantly (Fig. 2). Table 1 confirms that the best results were obtained for control with KF set according to LSU estimates of the noise bounds while the imperfectly set KF caused the worst control actions. Results for the closed control loop with the LSU estimator depend on the length of N_e. The higher value of N_e provides more accurate estimation leading to the smoother behaviour of controlled input. However, this estimation consumes more computational time, which may be disadvantageous in real applications. Nevertheless, satisfying control results were obtained as well for lower N_e values.

Count histograms in Fig. 4 – 5 show that control loops both with LSU estimator and KF estimator, set according to ρ and r estimates, preserves the boundedness of the outputs and the states while the control loop with imperfectly set KF tends to more flat distributions of both output and states.

6. CONCLUSIONS

The paper presents a novel interconnection of GPC with the LSU model intended for the control of stochastic systems with bounded states and outputs. The following conclusions can be derived from simulative experiments:

(i) without initial setting, LSU estimator outperforms KF; LSU estimator needs no initial setting because the noise bounds estimation is a part of the estimation algorithm – the only optional parameter is a length of the estimation N_e that is not crucial Pavelková and Kárný (2014);

(ii) LSU estimator can be also used within a standard control setting with KF estimator (30) as a simple and efficient tuner of the respective covariance matrices; in this way, the performance of KF can be rapidly improved with a minimal effort.

Table 1. Mean absolute controller error $\bar{e}$ and standard deviation σ_u for $\langle T_1{=}110s, T_2{=}200s\rangle$, where $\bar{e} = (\Sigma_t |y_t - w_t|)\, Ts/(T_2 - T_1 + Ts)$.

LSU, $N_e = 25$	LSU, $N_e = 200$	well set KF	imperfect KF
$\bar{e} = 0.0117$ $\sigma_u = 1.1966$	$\bar{e} = 0.0097$ $\sigma_u = 1.0992$	$\bar{e} = 0.0028$ $\sigma_u = 0.8496$	$\bar{e} = 0.0362$ $\sigma_u = 2.7741$

Table 2. KF gain values K_{N_e} and covariance matrix parameters from LSU estimation

KF, $N_e = 25$	KF, $N_e = 50$	KF, $N_e = 100$	KF, $N_e = 200$
$K_{25} = \begin{bmatrix} -0.025446 \\ 0.094630 \end{bmatrix}$ $\hat{\rho}_1 = 2.8291e\text{-}9$ $\hat{\rho}_2 = 0.0096$ $\hat{r} = 0.0777$	$K_{50} = \begin{bmatrix} -0.019323 \\ 0.064405 \end{bmatrix}$ $\hat{\rho}_1 = 1.8676e\text{-}9$ $\hat{\rho}_2 = 0.0077$ $\hat{r} = 0.0838$	$K_{100} = \begin{bmatrix} -0.015765 \\ 0.049760 \end{bmatrix}$ $\hat{\rho}_1 = 6.8564e\text{-}10$ $\hat{\rho}_2 = 0.0067$ $\hat{r} = 0.0876$	$K_{200} = \begin{bmatrix} -0.010217 \\ 0.030492 \end{bmatrix}$ $\hat{\rho}_1 = 0.0013$ $\hat{\rho}_2 = 0.0050$ $\hat{r} = 0.0907$

These conclusions confirm that the tuning effort connected with KF can indeed be diminished without giving up the achievable control quality.

For the sake of simplicity, the paper presents simulation experiments with simple model of controlled system only. Nevertheless, the utilised LSU estimator is designed and applicable to more complex systems represented by multi-input multi-output models with correlated noises or with partially unknown model matrices.

The future research will focus on real applications, for instance in the robotics area, by using LSU models including cases with correlated noise. Furthermore, a faster, "Kalman-like", version of LSU estimator is being currently developed where precision of estimates would not depend on the length of estimation window.

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