Optimization
A Journal of Mathematical Programming and Operations Research

ISSN: 0233-1934 (Print) 1029-4945 (Online) Journal homepage: http://www.tandfonline.com/loi/gopt20

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To cite this article: Michal Červinka (2016) A note on stability of stationary points in mathematical programs with generalized complementarity constraints, Optimization, 65:5, 1049-1060, DOI: 10.1080/02331934.2015.1107560

To link to this article: http://dx.doi.org/10.1080/02331934.2015.1107560

Optimization, 2016
Vol. 65, No. 5, 1049–1060, http://dx.doi.org/10.1080/02331934.2015.1107560

A note on stability of stationary points in mathematical programs with generalized complementarity constraints

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(*Received 1 December 2014; accepted 28 September 2015*)

We consider parameter-dependent mathematical programs with constraints governed by the generalized non-linear complementarity problem and with additional non-equilibrial constraints. We study a local behaviour of stationarity maps that assign the respective C- or M-stationarity points of the problem to the parameter. Using generalized differential calculus rules, we provide criteria for the isolated calmness and the Aubin properties of stationarity maps considered. To this end, we derive and apply formulas of some particular objects of the third-order variational analysis.

Keywords: parameter-dependent mathematical programs with generalized equilibrium constraints; M-stationarity; C-stationarity; isolated calmness; Aubin property

AMS Subject Classifications: 90C31; 90C33; 49K40

1. Introduction

A *mathematical program with generalized complementarity constraints* (MPCC for short) is an optimization problem where a parameter-dependent generalized non-linear complementarity problem arises as part of constraints. MPCCs arise frequently in many fields of applied mathematics, e.g. in mechanics (contact problems with friction, optimal design problems, shape optimization) and robotics (motion planning models for robotic hands), as well as in economic modelling (economic planning problems, sector analysis, electricity/gas transmission problems, sparse portfolio optimization) and game theory (Stackelberg problems, bilevel cooperative or non-cooperative games). We refer the reader to books [1–3] for several other applications and theory of MPCCs and related class of bilevel programming problems.

In the past two decades, a number of different numerical methods have been proposed for the solution of MPCCs. These methods generally provide only a stationary point of MPCC of a certain type. Many such stationarity concepts were formally introduced and studied in [4]. Nowadays, S-, M- and C-stationarity concepts are frequently used in the MPCC literature both in numerics and study of optimality conditions.

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In this paper, we consider parameter-dependent MPCCs, where both the objective and the non-linear complementarity problem in the constraints are subject to perturbations in the form of a joint parameter. The main goal of this paper is to analyse the local behaviour of the multifunction that assigns stationarity points of some type to a parameter. Stability analysis of MPCCs has already been topic of several papers, namely stability of a value function, [5–7] stability of a feasible set, [8] stability of C-stationary points and associated multipliers [4,9] based on Kojima's result from [10], and stability of M-stationary points in a special subclass of MPCCs [11].

In this paper, we extend the results on stability of M-stationary points in [11] to a general MPCC and include corresponding results regarding stability of C-stationary points. Analogously to, [11] our main workhorses are the particular objects of the third-order variational analysis which enable us to state some useful stability statements.

The structure of the paper is as follows. In Section 2, we provide the formulation of the problem and introduce two types (C- and M-) of stationarity maps. In Section 3, we apply a technique to compute the tangent and limiting normal cones to the graphs of polyhedral multifunction which are related to C- and M-stationarity maps. These results are applied in Section 4, where we obtain, in particular, an upper approximations of the graphical derivative and the limiting coderivative of both types of the stationarity maps and state stability statements, illustrated by an academic example.

Our notation is basically standard. $\mathbb{B}$ denotes the unit ball. We use $\mathbb{R}_+$, $\mathbb{R}_-$, $\mathbb{R}_{++}$ and $\mathbb{R}_{--}$ to denote non-negative, non-positive, positive and negative reals, respectively. For $x, y \in \mathbb{R}^n$, $x \perp y$ stands for orthogonality of x and y, i.e. $x^\top y = 0$. For a set Ω, $\overline{\Omega}$ denotes its closure, and for a closed cone D with vertex at the origin, D° denotes its negative polar cone. By $x \xrightarrow{\Omega} \bar{x}$, we mean that $x \to \bar{x}$ with $x \in \Omega$. $T_\Omega(x)$ signifies the contingent (Bouligand–Severi, tangent) cone to Ω at x.

For the readers' convenience, we now state the definitions of several basic notions from modern variational analysis. For a set Ω and a point $\bar{x} \in \overline{\Omega}$, the *regular (Fréchet) normal cone* to Ω at $\bar{x}$ is defined by

$$\widehat{N}_\Omega(\bar{x}) := \left\{ x^* \in \mathbb{R}^n \;\middle|\; \limsup_{x \xrightarrow{\Omega} \bar{x}} \frac{\langle x^*, x - \bar{x} \rangle}{\| x - \bar{x} \|} \le 0 \right\} = (T_\Omega(\bar{x}))^\circ .$$

The *limiting (Mordukhovich) normal cone* to Ω at $\bar{x}$ is given by

$$N_\Omega(\bar{x}) = \operatorname*{Lim\,sup}_{x \xrightarrow{\Omega} \bar{x}} \widehat{N}_\Omega(x),$$

where the 'Lim sup' stands for the Painlevé–Kuratowski upper (or outer) limit. This limit is defined for a set-valued mapping $M[\mathbb{R}^n \rightrightarrows \mathbb{R}^m]$ at a point $\bar{x}$ by

$$\operatorname*{Lim\,sup}_{x \to \bar{x}} M(x) := \{ y \in \mathbb{R}^m \mid \exists x_k \to \bar{x}, \exists y_k \to y \text{ with } y_k \in M(x_k) \}.$$

For a convex set Ω, both normal cones N_Ω and $\widehat{N}_\Omega$ amount to the normal cone of convex analysis, for which we use simply the notation N_Ω.

Given a set-valued mapping $M[\mathbb{R}^n \rightrightarrows \mathbb{R}^m]$ and a point $(\bar{x}, \bar{y})$ from its closed graph

$$\operatorname{Gph} M := \{(x, y) \in \mathbb{R}^n \times \mathbb{R}^m | y \in M(x)\},$$

the *graphical derivative* $DM(\bar{x},\bar{y})[\mathbb{R}^n \rightrightarrows \mathbb{R}^m]$ of M at $(\bar{x},\bar{y})$ is defined by

$$DM(\bar{x},\bar{y})(h) := \{k \in \mathbb{R}^m | (h,k) \in T_{\mathrm{Gph}M}(\bar{x},\bar{y})\},$$

the *regular (Fréchet) coderivative* $\widehat{D}^*M(\bar{x},\bar{y})[\mathbb{R}^m \rightrightarrows \mathbb{R}^n]$ of M at $(\bar{x},\bar{y})$ is defined by

$$\widehat{D}^*M(\bar{x},\bar{y})(y^*) := \{x^* \in \mathbb{R}^n | (x^*,-y^*) \in \widehat{N}_{\mathrm{Gph}M}(\bar{x},\bar{y})\},$$

and the *limiting (Mordukhovich) coderivative* $D^*M(\bar{x},\bar{y})[\mathbb{R}^m \rightrightarrows \mathbb{R}^n]$ of M at $(\bar{x},\bar{y})$ is defined by

$$D^*M(\bar{x},\bar{y})(y^*) := \{x^* \in \mathbb{R}^n | (x^*,-y^*) \in N_{\mathrm{Gph}M}(\bar{x},\bar{y})\}.$$

Further, given $x^* \in D^*M(\bar{x},\bar{y})(y^*)$, then the *second order limiting coderivative* of M at $(\bar{x},\bar{y},y^*,x^*)$ is defined by [11, Definition 1]

$$D^{**}M(\bar{x},\bar{y},y^*,x^*)(z^*) = \{(u^*,v^*,w^*) | (u^*,v^*,w^*,-z^*) \in N_{\mathrm{Gph}\, D^*M}(\bar{x},\bar{y},y^*,x^*)\}.$$

Finally, throughout the paper we employ the notions of Aubin property, isolated calmness and calmness.

A set-valued mapping $M[\mathbb{R}^n \rightrightarrows \mathbb{R}^m]$ is said to have the *Aubin (pseudo-Lipschitz, Lipschitz-like) property* around $(\bar{x},\bar{y}) \in \mathrm{Gph}\, M$ with modulus $\ell \geq 0$ if there are neighbourhoods $\mathcal{U}$ of $\bar{x}$ and $\mathcal{V}$ of $\bar{y}$ such that

$$M(x) \cap \mathcal{V} \subset M(u) + \ell ||x-u|| \mathbb{B}$$

for all $x, u \in \mathcal{U}$, where $\mathbb{B}$ is closed unit ball.

A multifunction $M[\mathbb{R}^n \rightrightarrows \mathbb{R}^m]$ is said to have the *isolated calmness (local upper Lipschitz, calmness on selections) property* (to be *isolatedly calm*) at $(\bar{x},\bar{y}) \in \mathrm{Gph}\, M$, provided there exist neighbourhoods $\mathcal{U}$ of $\bar{x}$ and $\mathcal{V}$ of $\bar{y}$ and a constant $\kappa \geq 0$ such that

$$M(x) \cap \mathcal{V} \subset \{\bar{y}\} + \kappa ||x-\bar{x}|| \mathbb{B} \quad \text{when } x \in \mathcal{U}.$$

A set-valued mapping $M[\mathbb{R}^n \rightrightarrows \mathbb{R}^m]$ is said to be *calm (pseudo upper Lipschitz)* at $(\bar{x},\bar{y}) \in \mathrm{Gph}\, M$ with modulus $L \geq 0$ if there are neighbourhoods $\mathcal{U}$ of $\bar{x}$ and $\mathcal{V}$ of $\bar{y}$ such that

$$M(x) \cap \mathcal{V} \subset M(\bar{x}) + L\|x-\bar{x}\|\mathbb{B} \quad \text{for all } x \in \mathcal{U}.$$

Clearly, both the Aubin and the isolated calmness properties imply calmness, whereas neither converse is true. There does not exist any direct relationship between calmness and isolated calmness of a multifunction. In the sequel, calmness will be utilized as a suitable qualification condition in the used rules of generalized differential calculus, cf. [12,13], whereas the Aubin and the isolated calmness properties will be considered as valuable stability concepts for the behaviour of stationary points with respect to the parameter.

2. Problem statement and preliminaries

Throughout the whole paper, we shall be concerned by the following parameter-dependent MPCC:

$$\begin{array}{ll} \text{minimize} & f(p,x,y) \\ \text{subject to} & \\ & 0 \in F(p,x,y) + N_{\mathbb{R}^m_+}(G(p,x,y)) \\ & x \in \omega,\ y \in \Omega \end{array} \tag{1}$$

where $f[\mathbb{R}^s \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}]$, $F[\mathbb{R}^s \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m]$ and $G[\mathbb{R}^s \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m]$ are at least twice continuously differentiable functions, ω and Ω closed sets.

Taking into consideration the geometry of the graph of $N_{\mathbb{R}^m_+}$, in fact, the generalized equation in the constraints of (1) reads as parameter-dependent generalized non-linear complementarity problem: for a given $p \in \mathbb{R}^s$ and $x \in \mathbb{R}^n$ find $y \in \mathbb{R}^m$ such that

$$0 \le F(p, x, y) \perp G(p, x, y) \ge 0.$$

For simplicity, we assume that a parameter p enters only the objective f and mappings F and G in the generalized complementarity problem, but not the geometric constraints $x \in \omega$ and $y \in \Omega$ as we do not impose any particular structure to sets ω and Ω.

There are various stationarity concepts for MPCCs (S-, M-, C-, A-, weak, etc.) and many papers study their mutual relationships, applications in numerics etc., cf. e.g. [4,14,15]. In this paper, we shall be concerned by the following two stationarity concepts.

The *C-stationarity conditions* to (1) [4] may be written down in the form

$$\begin{aligned} 0 &= \nabla_x f(p, x, y) + (\nabla_x F(p, x, y))^\top \nu + (\nabla_x G(p, x, y))^\top \mu + N_\omega(x) \\ 0 &= \nabla_y f(p, x, y) + (\nabla_y F(p, x, y))^\top \nu + (\nabla_y G(p, x, y))^\top \mu + N_\Omega(x) \\ &(G(p, x, y), -F(p, x, y), \nu, \mu) \in \operatorname{Gph} \Phi. \end{aligned} \tag{2}$$

The mapping Φ in the above formula is defined componentwise such that $(G_i(p, x, y), -F_i(p, x, y), \nu_i, \mu_i) \in \Lambda$, $i = 1, \dots, m$, with $\Lambda = \Lambda_1 \cup \Lambda_2 \cup \Lambda_3 \cup \Lambda_4$,

$$\begin{aligned} \Lambda_1 &= \mathbb{R}_+ \times \{0\} \times \mathbb{R} \times \{0\}, \\ \Lambda_2 &= \{0\} \times \mathbb{R}_- \times \{0\} \times \mathbb{R}, \\ \Lambda_3 &= \{0\} \times \{0\} \times \mathbb{R}_- \times \mathbb{R}_-, \\ \Lambda_4 &= \{0\} \times \{0\} \times \mathbb{R}_+ \times \mathbb{R}_+. \end{aligned}$$

The multipliers $\nu \in \mathbb{R}^m$ and $\mu \in \mathbb{R}^m$ are the so-called MPCC-multipliers.

The *M-stationarity conditions* of (1) may be written down as follows [16, Theorem 3.1]:

$$\begin{aligned} 0 &= \nabla_x f(p, x, y) + (\nabla_x F(p, x, y))^\top \nu + (\nabla_x G(p, x, y))^\top \mu + N_\omega(x) \\ 0 &= \nabla_y f(p, x, y) + (\nabla_y F(p, x, y))^\top \nu + (\nabla_y G(p, x, y))^\top \mu + N_\Omega(x) \\ \mu &\in D^* N_{\mathbb{R}^m_+}(G(p, x, y), -F(p, x, y))(\nu) \end{aligned} \tag{3}$$

Note that $\operatorname{Gph} D^* N_{\mathbb{R}_+} = \Lambda_1 \cup \Lambda_2 \cup \Lambda_3$.

Stationarity conditions in the form (3) originate from [17, Theorem 3.2]. The terminology 'C-stationarity' and 'M-stationarity' come from the fact that these conditions are obtained by means of application of generalized differential calculus of Clarke and Mordukhovich, respectively. Similarly to [11], we do not address any constraint qualification guaranteeing that, at a local minimum of (1), there exists MPCC-multipliers satisfying these necessary optimality conditions. This has already been extensively discussed in the literature, see e.g. [4,16,18]. Here, we are interested in analysis of local behaviour of solutions to (2) and (3) around the reference point $(\bar{p}, \bar{x}, \bar{y}, \bar{\nu}, \bar{\mu})$ and so the existence of this reference point is implicitly assumed.

Denoting

$$\Psi(p,x,y,\nu,\mu) = \begin{bmatrix} x \\ -\nabla_x f(p,x,y) - (\nabla_x F(p,x,y))^\top \nu - (\nabla_x G(p,x,y))^\top \mu \\ y \\ -\nabla_y f(p,x,y) - (\nabla_y F(p,x,y))^\top \nu - (\nabla_y G(p,x,y))^\top \mu \\ G(p,x,y) \\ -F(p,x,y) \\ \nu \\ \mu \end{bmatrix},$$

we can rewrite (2) and (3) to (respectively)

$$\Psi(p,x,y,\nu,\mu) \in \operatorname{Gph} N_\omega \times \operatorname{Gph} N_\Omega \times \operatorname{Gph} \Phi, \tag{4}$$

$$\Psi(p,x,y,\nu,\mu) \in \operatorname{Gph} N_\omega \times \operatorname{Gph} N_\Omega \times \operatorname{Gph} D^* N_{\mathbb{R}^m_+}. \tag{5}$$

We can associate (2) with a *C-stationarity map* $\mathcal{S}_C$

$$\mathcal{S}_C(p) = \{(x,y,\nu,\mu) | \Psi(p,x,y,\nu,\mu) \in \operatorname{Gph} N_\omega \times \operatorname{Gph} N_\Omega \times \operatorname{Gph} \Phi\}.$$

and (3) with a *M-stationarity map* $\mathcal{S}_M$

$$\mathcal{S}_M(p) = \left\{(x,y,\nu,\mu) | \Psi(p,x,y,\nu,\mu) \in \operatorname{Gph} N_\omega \times \operatorname{Gph} N_\Omega \times \operatorname{Gph} D^* N_{\mathbb{R}^m_+}\right\}.$$

Example 1 [11] Let $p, x, y \in \mathbb{R}$ and consider the following parameter-dependent generalized MPCC

$$\begin{aligned} &\text{minimize} \;\; -x - (y+p)^2 \\ &\text{subject to} \\ &\qquad\qquad 0 \in x - p + N_{\mathbb{R}_+}(y - p). \end{aligned}$$

The corresponding C-stationarity conditions (2)

$$\begin{aligned} &0 = -1 + \nu \\ &0 = -2y - 2p + \mu \\ &(y - p, -x + p, \nu, \mu) \in \Lambda, \end{aligned}$$

have a unique solution $(\bar{x}, \bar{y}) = (\bar{p}, -\bar{p})$, with multipliers $\bar{\nu} = 1, \bar{\mu} = 0$ for $\bar{p} \le 0$; and $(\bar{x}, \bar{y}) = (\bar{p}, \bar{p})$, with multipliers $\bar{\nu} = 1, \bar{\mu} = 4\bar{p}$ for $\bar{p} \ge 0$.

Similarly, the M-stationarity conditions (3)

$$\begin{aligned} &0 = -1 + \nu \\ &0 = -2y - 2p + \mu \\ &(y - p, -x + p, \nu, \mu) \in \operatorname{Gph} D^* N_{\mathbb{R}_+}, \end{aligned}$$

have a unique solution $(\bar{x}, \bar{y}) = (\bar{p}, -\bar{p})$ with multipliers $\bar{\nu} = 1, \bar{\mu} = 0$ for $\bar{p} \le 0$, while there is no solution for $\bar{p} > 0$.

3. Generalized derivatives of Φ and $D^*N_{\mathbb{R}^m_+}$

The formulation of the problem (1) contains first-order variational problem, both (first-order) stationarity conditions (2) and (3) involve objects that can be considered of the second-order variational analysis. Clearly, in order to analyse local behaviour of $\mathcal{S}_C$ and $\mathcal{S}_M$ via generalized derivatives, one needs to introduce the corresponding third-order variational objects.

In particular, to analyse graphical derivative and coderivative of $\mathcal{S}_C$, we will use in the sequel formulas of Gph $T_{\text{Gph }\Phi}$ and of Gph $N_{\text{Gph }\Phi}$. Analogously, for generalized derivatives of $\mathcal{S}_M$, we need formulas of Gph $T_{\text{Gph }D^*N_{\mathbb{R}^m_+}}$ and Gph $N_{\text{Gph }D^*N_{\mathbb{R}^m_+}}$. Formulas of the latter two objects were already derived in [11]. We shall derive here formulas of the other two objects using the same technique developed in [11, Section 3].

Using [11, Lemma 1], we first partition $\Lambda = \cup_{i=1}^4 \Lambda_i$ to sets

$$\Lambda_I = \left(\bigcap_{i\in I} \Lambda_i\right) \setminus \left(\bigcup_{i\in\{1,\dots,4\}\setminus I} \Lambda_i\right), \emptyset \neq I \subset \{1,\dots,4\}.$$

In our case, one obtains the following sets

$$\begin{aligned}
\Lambda_{\{1\}} &= \mathbb{R}_{++} \times \{0\} \times \mathbb{R} \times \{0\},\\
\Lambda_{\{2\}} &= \{0\} \times \mathbb{R}_{--} \times \{0\} \times \mathbb{R},\\
\Lambda_{\{3\}} &= \{0\} \times \{0\} \times \mathbb{R}_{--} \times \mathbb{R}_{--},\\
\Lambda_{\{4\}} &= \{0\} \times \{0\} \times \mathbb{R}_{++} \times \mathbb{R}_{++},\\
\Lambda_{\{13\}} &= \{0\} \times \{0\} \times \mathbb{R}_{--} \times \{0\},\\
\Lambda_{\{14\}} &= \{0\} \times \{0\} \times \mathbb{R}_{++} \times \{0\},\\
\Lambda_{\{23\}} &= \{0\} \times \{0\} \times \{0\} \times \mathbb{R}_{--},\\
\Lambda_{\{24\}} &= \{0\} \times \{0\} \times \{0\} \times \mathbb{R}_{++},\\
\Lambda_{\{1234\}} &= \{0\} \times \{0\} \times \{0\} \times \{0\},\\
\Lambda_{\{12\}} &= \emptyset, \quad \Lambda_{\{34\}} = \emptyset,\\
\Lambda_{\{123\}} &= \emptyset, \quad \Lambda_{\{124\}} = \emptyset,\\
\Lambda_{\{134\}} &= \emptyset, \quad \Lambda_{\{234\}} = \emptyset.
\end{aligned}$$

By application of [11, Theorem 1], we can derive formulas of Gph T_Λ and Gph N_Λ. In order to derive formulas for the general case of Gph $T_{\text{Gph}\Phi}$ and of Gph $N_{\text{Gph}\Phi}$, let us associate with each pair $(G(p,x,y), -F(p,x,y)) \in \text{Gph } N_{\mathbb{R}^m_+}$ the *index sets* (of inactive, strongly active and weakly active/biactive inequalities) related to the complementarity constraints in (1)

$$\begin{aligned}
L(p,x,y,\nu,\mu) &:= \{i \in \{1,\dots,m\} | (G_i(p,x,y), -F_i(p,x,y)) \in \mathbb{R}_{++} \times \{0\}\}\\
I_+(p,x,y,\nu,\mu) &:= \{i \in \{1,\dots,m\} | (G_i(p,x,y), -F_i(p,x,y)) \in \{0\} \times \mathbb{R}_{--}\}\\
I_0(p,x,y,\nu,\mu) &:= \{i \in \{1,\dots,m\} | (G_i(p,x,y), -F_i(p,x,y)) \in \{0\} \times \{0\}\}.
\end{aligned}$$

Similarly, we shall introduce the disjunct decomposition of the set of indices of weakly active inequalities associated with quadruple $(G(p,x,y), -F(p,x,y), \nu, \mu) \in \text{Gph } \Phi$

$$I_0^{--}(p,x,y,\nu,\mu) := \{i \in I_0(p,x,y,\nu,\mu) | (\nu_i, \mu_i) \in \mathbb{R}_{--} \times \mathbb{R}_{--}\}$$

$$
\begin{aligned}
I_0^{++}(p,x,y,\nu,\mu) &:= \{i \in I_0(p,x,y,\nu,\mu) | (\nu_i,\mu_i) \in \mathbb{R}_{++} \times \mathbb{R}_{++}\}\\
I_0^{-0}(p,x,y,\nu,\mu) &:= \{i \in I_0(p,x,y,\nu,\mu) | (\nu_i,\mu_i) \in \mathbb{R}_{--} \times \{0\}\}\\
I_0^{0-}(p,x,y,\nu,\mu) &:= \{i \in I_0(p,x,y,\nu,\mu) | (\nu_i,\mu_i) \in \{0\} \times \mathbb{R}_{--}\}\\
I_0^{+0}(p,x,y,\nu,\mu) &:= \{i \in I_0(p,x,y,\nu,\mu) | (\nu_i,\mu_i) \in \mathbb{R}_{++} \times \{0\}\}\\
I_0^{0+}(p,x,y,\nu,\mu) &:= \{i \in I_0(p,x,y,\nu,\mu) | (\nu_i,\mu_i) \in \{0\} \times \mathbb{R}_{++}\}\\
I_0^{00}(p,x,y,\nu,\mu) &:= \{i \in I_0(p,x,y,\nu,\mu) | (\nu_i,\mu_i) \in \{0\} \times \{0\}\}.
\end{aligned}
$$

Note that these sets cover all possibilities for values of $(G(p,x,y), -F(p,x,y), \nu, \mu)$ with respect to Gph Φ. We shall omit the arguments of these sets whenever it cannot cause any confusion.

Thus, application of [11, Theorem 1] yields the following desired formulas. Consider $(\alpha,\beta,\gamma,\delta) \in T_{\mathrm{Gph}\,\Phi}(y, -F(p,x,y), \nu, \mu)$ and $(u^*, v^*, w^*, -z^*) \in N_{\mathrm{Gph}\,\Phi}(y, -F(p,x,y), \nu, \mu)$; then for each $i \in \{1\ldots, m\}$

$$
\begin{array}{lll}
\text{for } i \in L & \alpha_i \in \mathbb{R}, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i = 0 & u_i^* = 0, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R}\\
\text{for } i \in I_+ & \alpha_i = 0, \beta_i \in \mathbb{R}, \gamma_i = 0, \delta_i \in \mathbb{R} & u_i^* \in \mathbb{R}, v_i^* = 0, w_i^* \in \mathbb{R}, z_i^* = 0\\
\text{for } i \in I_0^{--} & \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i \in \mathbb{R} & u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0\\
\text{for } i \in I_0^{++} & \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i \in \mathbb{R} & u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0\\
\text{for } i \in I_0^{-0} & \begin{cases} \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i \in \mathbb{R}_- \text{ or}\\ \alpha_i \in \mathbb{R}_+, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i = 0 \end{cases} & \begin{cases} u_i^* = 0, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R} \text{ or}\\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0 \text{ or}\\ u_i^* \in \mathbb{R}_-, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R}_- \end{cases}\\
\text{for } i \in I_0^{0-} & \begin{cases} \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}_-, \delta_i \in \mathbb{R} \text{ or}\\ \alpha_i = 0, \beta_i \in \mathbb{R}_-, \gamma_i = 0, \delta_i \in \mathbb{R} \end{cases} & \begin{cases} u_i^* \in \mathbb{R}, v_i^* = 0, w_i^* \in \mathbb{R}, z_i^* = 0 \text{ or}\\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0 \text{ or}\\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}_+, w_i^* \in \mathbb{R}_+, z_i^* = 0 \end{cases}\\
\text{for } i \in I_0^{+0} & \begin{cases} \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i \in \mathbb{R}_+ \text{ or}\\ \alpha_i \in \mathbb{R}_+, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i = 0 \end{cases} & \begin{cases} u_i^* = 0, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R} \text{ or}\\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0 \text{ or}\\ u_i^* \in \mathbb{R}_-, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R}_+ \end{cases}\\
\text{for } i \in I_0^{0+} & \begin{cases} \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}_+, \delta_i \in \mathbb{R} \text{ or}\\ \alpha_i = 0, \beta_i \in \mathbb{R}_-, \gamma_i = 0, \delta_i \in \mathbb{R} \end{cases} & \begin{cases} u_i^* \in \mathbb{R}, v_i^* = 0, w_i^* \in \mathbb{R}, z_i^* = 0 \text{ or}\\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0 \text{ or}\\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}_+, w_i^* \in \mathbb{R}_-, z_i^* = 0 \end{cases}\\
\text{for } i \in I_0^{00} & \begin{cases} \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}_-, \delta_i \in \mathbb{R}_- \text{ or}\\ \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}_+, \delta_i \in \mathbb{R}_+ \text{ or}\\ \alpha_i \in \mathbb{R}_+, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i = 0 \text{ or}\\ \alpha_i = 0, \beta_i \in \mathbb{R}_-, \gamma_i = 0, \delta_i \in \mathbb{R} \end{cases} & \begin{cases} u_i^* = 0, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R}, \text{ or}\\ u_i^* \in \mathbb{R}, v_i^* = 0, w_i^* \in \mathbb{R}, z_i^* = 0 \text{ or}\\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0 \text{ or}\\ u_i^* \in \mathbb{R}_-, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R}, \text{ or}\\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}_+, w_i^* \in \mathbb{R}, z_i^* = 0, \text{ or}\\ u_i^* \in \mathbb{R}_-, v_i^* \in \mathbb{R}_+, w_i^* = 0, z_i^* = 0 \end{cases}
\end{array}
\tag{6}
$$

Remark 1 Non-empty sets $\Lambda_I, \emptyset \neq I \subset \{1, \ldots, 4\}$ above form a so-called *normally admissible stratification* of Λ, cf. [19, Definition 2] for the definition. Alternatively to a technique proposed in [11], one can apply [19, Theorem 2] to obtain the desired formula of the graph of $N_{\mathrm{Gph}\,\Phi}$.

For the sake of completeness, we present here also the formulas of Gph $T_{\mathrm{Gph}\, D^*N_{\mathbb{R}^m_+}}$ and Gph $N_{\mathrm{Gph}\, D^*N_{\mathbb{R}^m_+}}$. Consider $(\alpha, \beta, \gamma, \delta) \in T_{\mathrm{Gph}\, D^*N_{\mathbb{R}^m_+}}(y, -F(p, x, y), \nu, \mu)$ and $(u^*, v^*, w^*, -z^*) \in N_{\mathrm{Gph}\, D^*N_{\mathbb{R}^m_+}}(y, -F(p, x, y), \nu, \mu)$; then for each $i \in \{1. \dots, m\}$

$$
\begin{array}{lll}
\text{for } i \in L & \alpha_i \in \mathbb{R}, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i = 0 & u_i^* = 0, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R} \\
\text{for } i \in I_+ & \alpha_i = 0, \beta_i \in \mathbb{R}, \gamma_i = 0, \delta_i \in \mathbb{R} & u_i^* \in \mathbb{R}, v_i^* = 0, w_i^* \in \mathbb{R}, z_i^* = 0 \\
\text{for } i \in I_0^{--} & \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i \in \mathbb{R} & u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0 \\
\text{for } i \in I_0^{-0} & \begin{cases} \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i \in \mathbb{R}_- \text{ or} \\ \alpha_i \in \mathbb{R}_+, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i = 0 \end{cases} & \begin{cases} u_i^* = 0, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R} \text{ or} \\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0 \text{ or} \\ u_i^* \in \mathbb{R}_-, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R}_- \end{cases} \\
\text{for } i \in I_0^{0-} & \begin{cases} \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}_-, \delta_i \in \mathbb{R} \text{ or} \\ \alpha_i = 0, \beta_i \in \mathbb{R}_-, \gamma_i = 0, \delta_i \in \mathbb{R} \end{cases} & \begin{cases} u_i^* \in \mathbb{R}, v_i^* = 0, w_i^* \in \mathbb{R}, z_i^* = 0 \text{ or} \\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0 \text{ or} \\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}_+, w_i^* \in \mathbb{R}_+, z_i^* = 0 \end{cases} \\
\text{for } i \in I_0^{+0} & \alpha_i \in \mathbb{R}_+, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i = 0 & \begin{cases} u_i^* = 0, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R} \text{ or} \\ u_i^* \in \mathbb{R}_-, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R} \end{cases} \\
\text{for } i \in I_0^{0+} & \alpha_i = 0, \beta_i \in \mathbb{R}_-, \gamma_i = 0, \delta_i \in \mathbb{R} & \begin{cases} u_i^* \in \mathbb{R}, v_i^* = 0, w_i^* \in \mathbb{R}, z_i^* = 0 \text{ or} \\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}_+, w_i^* \in \mathbb{R}, z_i^* = 0 \end{cases} \\
\text{for } i \in I_0^{00} & \begin{cases} \alpha_i = 0, \beta_i = 0, \gamma_i \in \mathbb{R}_-, \delta_i \in \mathbb{R}_- \text{ or} \\ \alpha_i \in \mathbb{R}_+, \beta_i = 0, \gamma_i \in \mathbb{R}, \delta_i = 0 \text{ or} \\ \alpha_i = 0, \beta_i \in \mathbb{R}_-, \gamma_i = 0, \delta_i \in \mathbb{R} \end{cases} & \begin{cases} u_i^* = 0, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R}, \text{ or} \\ u_i^* \in \mathbb{R}, v_i^* = 0, w_i^* \in \mathbb{R}, z_i^* = 0 \text{ or} \\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* = 0 \text{ or} \\ u_i^* \in \mathbb{R}_-, v_i^* \in \mathbb{R}, w_i^* = 0, z_i^* \in \mathbb{R}, \text{ or} \\ u_i^* \in \mathbb{R}, v_i^* \in \mathbb{R}_+, w_i^* \in \mathbb{R}, z_i^* = 0, \text{ or} \\ u_i^* \in \mathbb{R}_-, v_i^* \in \mathbb{R}_+, w_i^* = 0, z_i^* = 0 \end{cases}
\end{array}
\tag{7}
$$

4. Generalized derivatives and qualitative stability of stationarity maps

In this section, by using the standard rules of the generalized differential calculus, we compute upper approximations of the graphical derivative and limiting coderivative of the stationarity maps $\mathcal{S}_C$ and $\mathcal{S}_M$, respectively. These upper approximations can be found useful in deriving sufficient (and sometimes also necessary) conditions for isolated calmness and Aubin property for both types of stationarity maps. Let us first introduce the *perturbation mappings*

$$
\begin{aligned}
M_C(z) &:= \{(p, x, y, \nu, \mu) | \Psi(p, x, y, \nu, \mu) + z \in \mathrm{Gph}\, N_\omega \times \mathrm{Gph}\, N_\Omega \times \mathrm{Gph}\, \Phi\}; \\
M_M(z) &:= \{(p, x, y, \nu, \mu) | \Psi(p, x, y, \nu, \mu) + z \in \mathrm{Gph}\, N_\omega \times \mathrm{Gph}\, N_\Omega \times \mathrm{Gph}\, D^*N_{\mathbb{R}^m_+}\}.
\end{aligned}
$$

To unburden the notation in the following formulas, let us introduce

$$
\begin{aligned}
H_x(p, x, y, \nu, \mu) &:= \nabla_x f(p, x, y) + (\nabla_x F(p, x, y))^\top \nu + (\nabla_x G(p, x, y))^\top \mu, \\
H_y(p, x, y, \nu, \mu) &:= \nabla_y f(p, x, y) + (\nabla_y F(p, x, y))^\top \nu + (\nabla_y G(p, x, y))^\top \mu,
\end{aligned}
$$

and

$$
h := (h_p, h_x, h_y, h_\nu, h_\mu).
$$

For simplicity, whenever it cannot cause any confusion, we do not state the arguments of maps in formulas.

THEOREM 1

(i) *Let* $(\bar{x}, \bar{y})$ *be the C-stationary pair of* (1) *with a corresponding MPCC-multipliers* $(\bar{\nu}, \bar{\mu})$. *Then*

$$D\mathcal{S}_C(\bar{p}, \bar{x}, \bar{y}, \bar{\nu}, \bar{\mu})(h_p) \subset \left\{ (h_x, h_y, h_\nu, h_\mu) \;\middle|\; \begin{array}{l} \begin{pmatrix} h_x \\ -(\nabla H_x)h \end{pmatrix} \in T_{Gph\ N_\omega}(\bar{x}, -H_x) \\ \begin{pmatrix} h_y \\ -(\nabla H_y)h \end{pmatrix} \in T_{Gph\ N_\Omega}(\bar{y}, -H_y) \\ \begin{pmatrix} (\nabla G)h \\ -(\nabla F)h \\ h_\nu \\ h_\mu \end{pmatrix} \in T_{Gph\ \Phi}(G, -F, \bar{\nu}, \bar{\mu}) \end{array} \right\}. \tag{8}$$

(ii) *Let* $(\bar{x}, \bar{y})$ *be the M-stationary pair of* (1) *with a corresponding MPCC-multiplier* $(\bar{\nu}, \bar{\mu})$. *Then*

$$D\mathcal{S}_M(\bar{p}, \bar{x}, \bar{y}, \bar{\nu}, \bar{\mu})(h_p) \subset \left\{ (h_x, h_y, h_\nu, h_\mu) \;\middle|\; \begin{array}{l} \begin{pmatrix} h_x \\ -(\nabla H_x)h \end{pmatrix} \in T_{Gph\ N_\omega}(\bar{x}, -H_x) \\ \begin{pmatrix} h_y \\ -(\nabla H_y)h \end{pmatrix} \in T_{Gph\ N_\Omega}(\bar{y}, -H_y) \\ \begin{pmatrix} (\nabla G)h \\ -(\nabla F)h \\ h_\nu \\ h_\mu \end{pmatrix} \in T_{Gph\ D^* N_{\mathbb{R}^m_+}}(G, -F, \bar{\nu}, \bar{\mu}) \end{array} \right\}. \tag{9}$$

Proof 1 Both statements follow directly from [20, Theorem 6.31]. $\square$

THEOREM 2

(i) *Let* $(\bar{x}, \bar{y})$ *be the C-stationary pair of (1) with a corresponding MPCC-multipliers* $(\bar{\nu}, \bar{\mu})$. *Further, let* M_C *be calm at* $(0, \bar{p}, \bar{x}, \bar{y}, \bar{\nu}, \bar{\mu})$. *Then*

$$D^*\mathcal{S}_C\ (\bar{p}, \bar{x}, \bar{y}, \bar{\nu}, \bar{\mu})(x^*, y^*, \nu^*, \mu^*) \subset \left\{ p^* \;\middle|\; \begin{array}{ll} p^* & = (\nabla_p H_x)^\top f^* + (\nabla_p H_y)^\top h^* + (\nabla_p G)^\top u^* - (\nabla_p F)^\top v^* \\ -x^* & = e^* + (\nabla_x H_x)^\top f^* + (\nabla_x H_y)^\top h^* + (\nabla_x G)^\top u^* - (\nabla_x F)^\top v^* \\ -y^* & = (\nabla_y H_x)^\top f^* + g^* + (\nabla_y H_y)^\top h^* + (\nabla_y G)^\top u^* - (\nabla_y F)^\top v^* \\ -\nu^* & = (\nabla_\nu H_x)^\top f^* + (\nabla_\nu H_y)^\top h^* + w^* \\ -\mu^* & = (\nabla_\mu H_x)^\top f^* + (\nabla_\mu H_y)^\top h^* - z^* \\ e^* & \in D^* N_\omega(\bar{x}, -H_x)(f^*) \\ g^* & \in D^* N_\Omega(\bar{y}, -H_y)(h^*) \\ (u^*, & v^*, w^*) \in D^*\Phi(G, -F, \bar{\nu}, \bar{\mu})(z^*) \end{array} \right\}. \tag{10}$$

(ii) *Let* $(\bar{x}, \bar{y})$ *be the M-stationary pair of* (1) *with a corresponding MPCC-multipliers* $(\bar{\nu}, \bar{\mu})$. *Further, let* M_M *be calm at* $(0, \bar{p}, \bar{x}, \bar{y}, \bar{\nu}, \bar{\mu})$. *Then*

$$
D^*\mathcal{S}_M\ (\bar{p},\bar{x},\bar{y},\bar{\nu},\bar{\mu})(x^*,y^*,\nu^*,\mu^*)
\subset \left\{ p^* \;\middle|\; \begin{array}{ll}
p^* & = (\nabla_p H_x)^\top f^* + (\nabla_p H_y)^\top h^* + (\nabla_p G)^\top u^* - (\nabla_p F)^\top v^* \\
-x^* & = e^* + (\nabla_x H_x)^\top f^* + (\nabla_x H_y)^\top h^* + (\nabla_x G)^\top u^* - (\nabla_x F)^\top v^* \\
-y^* & = (\nabla_y H_x)^\top f^* + g^* + (\nabla_y H_y)^\top h^* + (\nabla_y G)^\top u^* - (\nabla_y F)^\top v^* \\
-\nu^* & = (\nabla_\nu H_x)^\top f^* + (\nabla_\nu H_y)^\top h^* + w^* \\
-\mu^* & = (\nabla_\mu H_x)^\top f^* + (\nabla_\mu H_y)^\top h^* - z^* \\
e^* & \in D^* N_\omega(\bar{x}, -H_x)(f^*) \\
g^* & \in D^* N_\Omega(\bar{y}, -H_y)(h^*) \\
(u^*, & v^*, w^*) \in D^{**} N_{\mathbb{R}^m_+}(G, -F, \bar{\nu}, \bar{\mu})(z^*)
\end{array} \right\}. \tag{11}
$$

Proof 2 Both statements follow directly from [12, Theorem 4.1]. □

Moreover, (8)–(11) become equalities, whenever $\nabla\Psi$ is surjective at $(\bar{p},\bar{x},\bar{y},\bar{\nu},\bar{\mu})$, cf. [20, Exercise 6.7]. Note, that e.g. for $\omega = \mathbb{R}^n$, for any $\bar{p},\bar{x},\bar{y},\bar{\nu},\bar{\mu}$, one has $T_{\mathrm{Gph}\, N_\omega}(\bar{x}, -H_x) = \mathbb{R}^n \times \{0\}$, while $e^* = 0$ and $f^* \in \mathbb{R}^n$. Similarly for Ω.

Verification of calmness conditions stated in Theorem 2 can be a difficult task. Whenever the perturbation mapping has the Aubin property around the reference point, the calmness condition is also satisfied. Verification of calmness condition via Aubin property can be useful in some special cases of MPCC, see e.g. [11, Example 1]. However, even verification of conditions that implies the Aubin property of the perturbation mapping may fail.

The above results can be easily applied to obtain sufficient conditions for isolated calmness and Aubin property of stationarity maps. Stationarity map $\mathcal{S}_C$ possesses the isolated calmness property at $(\bar{p},\bar{x},\bar{y},\bar{\nu},\bar{\mu})$ if and only if

$$
D\mathcal{S}_C(\bar{p},\bar{x},\bar{y},\bar{\nu},\bar{\mu})(0) = \{0,0,0,0\}, \tag{12}
$$

see, e.g. [21, Theorem 4C.1]). The Mordukhovich criterion [22] provides a characterization of the Aubin property through knowledge of the respective coderivative: a set-valued mapping $\mathcal{S}_C$ has Aubin property around $(\bar{p},\bar{x},\bar{y},\bar{\nu},\bar{\mu})$ if and only if

$$
D^*\mathcal{S}_C(\bar{p},\bar{x},\bar{y},\bar{\nu},\bar{\mu})(0,0,0,0) = \{0\}.
$$

Analogously for $\mathcal{S}_M$. These criteria become both sufficient and necessary whenever $\nabla\Psi$ is surjective at $(\bar{p},\bar{x},\bar{y},\bar{\nu},\bar{\mu})$.

We conclude with continuation of Example 1 and show effectiveness of the our condition for the isolated calmness property of both C- and M-stationarity mapping at $p = 0$.

Example 1 *(continued)* Consider the C-stationary point $(\bar{p},\bar{x},\bar{y}) = (0,0,0)$ with multipliers $(\bar{\nu},\bar{\mu}) = (1,0)$. From (8), we get the following upper approximation of the graphical derivative of the stationarity map

$$
\begin{aligned}
&D\mathcal{S}_C(0,0,0,1,0)(h_p)\\
&\subset \left\{ (h_x,h_y,h_\nu,h_\mu) \;\middle|\; \begin{array}{r} h_x \in \mathbb{R}\\ h_\nu = 0\\ h_y \in \mathbb{R}\\ -2h_p - 2h_y + h_\mu = 0\\ -h_p + h_y \geq 0\\ h_p - h_x = 0\\ h_\nu \in \mathbb{R}\\ h_\mu = 0 \end{array} \text{ or } \begin{array}{r} h_x \in \mathbb{R}\\ h_\nu = 0\\ h_y \in \mathbb{R}\\ -2h_p - 2h_y + h_\mu = 0\\ -h_p + h_y = 0\\ h_p - h_x = 0\\ h_\nu \in \mathbb{R}\\ h_\mu \geq 0 \end{array} \right\}\\
&= \begin{cases} \{(h_p, -h_p, 0, 0)\} & \text{if } h_p \leq 0,\\ \{(h_p, h_p, 0, 4h_p)\} & \text{if } h_p > 0. \end{cases}
\end{aligned}
$$

By the means of (12), the C-stationarity map is isolatedly calm at $(0,0,0,1,0)$. And thus it is also calm at $(0,0,0,1,0)$. Note that the formula above gives the upper approximation of the graphical derivative of mapping S_C both for the directions $h_p \geq 0$ and $h_p \leq 0$ and recovers both parts of the C-stationary solutions around the reference point.

Consider the same, M-stationary point $(\bar{p}, \bar{x}, \bar{y}) = (0,0,0)$ with multipliers $(\bar{\nu}, \bar{\mu}) = (1,0)$. This time, (9) yields the following upper approximation of the graphical derivative of the stationarity map

$$
\begin{aligned}
D\mathcal{S}_M(0,0,0,1,0)(h_p) &\subset \left\{ (h_x,h_y,h_\nu,h_\mu) \;\middle|\; \begin{array}{r} h_x \in \mathbb{R}\\ h_\nu = 0\\ h_y \in \mathbb{R}\\ -2h_p - 2h_y + h_\mu = 0\\ -h_p + h_y \geq 0\\ h_p - h_x = 0\\ h_\nu \in \mathbb{R}\\ h_\mu = 0 \end{array} \right\}\\
&= \begin{cases} \{(h_p, -h_p, 0, 0)\} & \text{if } h_p \leq 0,\\ \emptyset & \text{if } h_p > 0. \end{cases}
\end{aligned}
$$

By the means of (12), also the M-stationarity map is isolatedly calm (and calm) at $(0,0,0,1,0)$.

Acknowledgements

The author would like to thank both referees for their very detailed comments which helped to improve the presentation of the paper.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the Grant Agency of the Czech Republic [grant number P402/12/1309].

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