Online Tuned Model Predictive Control for Robotic Systems with Bounded Noise

Květoslav Belda and Lenka Pavelková
Department of Adaptive Systems
Institute of Information Theory and Automation of the CAS
Pod Vodárenskou věží 4, 182 08 Prague 8, Czech Republic
Emails: belda@utia.cas.cz, pavelkov@utia.cas.cz

***Abstract*—This paper deals with a discrete predictive control design for motion control of robotic systems. The design considers time-varying state-space robot model. It is assumed that used robot state has to be estimated from measured robot outputs. These outputs represent controlled quantities including a bounded noise. Considering this arrangement, the paper introduces a novel solution to the state and noise parameter estimations based on linear programming that is incorporated in the control design. Estimated states are utilised for updating state-dependent elements in the robot model and for control design itself. Estimated noise parameters are employed in advanced tuning of control parameters, namely penalisation matrices. The proposed theoretical outcomes are demonstrated on one multi-input multi-output robot-manipulator as a specific representative of robotic systems.**

I. Introduction

Robotic systems as industrial robots-manipulators realise their motion in environments containing various disturbances and noises. These uncertainties can cause inadequate demands for control actions or increase control errors. To overcome this problem, respective control design intended for a real robot motion should take the mentioned influences into account.

Model predictive control (MPC) is currently a very popular control method [1]. It offers several ways how to cope with the above mentioned uncertainties. For small disturbances, deterministic MPC design can be used. Then, the control actions are determined using the nominal model. Under certain conditions, the controlled system is robustly stable against a sufficiently small additive disturbance [2]. For an arbitrarily high but bounded disturbances, robust MPC is standardly used. The bounds are assumed to be known. The control actions are designed considering all possible disturbance realisations, see e.g. [3], [4]. However, robust approaches may be too conservative for high-dynamical tasks of the control of the robot motion. For stochastic disturbances, stochastic MPC is now developed. Contrary to robust MPC, in stochastic MPC, disturbances may not be necessarily bounded. The present constraints are softened, i.e. they are not required to be satisfied for all realisations of the disturbance, [5], [6]. Nevertheless, softening of a control constraints is not acceptable for all cases.

This paper deals with the above described control problem of robot motion influenced by a noise. The uncertainty is assumed to be bounded with unknown bounds. The paper focuses on output feedback MPC using a state-space model and considering unknown bounded noises and unmeasurable states. Therefore, state and noise parameter estimations are introduced. The respective estimates are obtained using an approximate Bayesian estimation based on the linear programming [7]. The state estimates are utilised in predictive control both to update state-dependent model matrices of considered state-space model and to compute control actions within predictive optimisation procedure. Furthermore, estimated noise parameters are used for a setting or tuning of the control parameters. Here, for simplicity, general unconstrained positional MPC is considered. However, this approach may generally be used for involvement of constraints [8]. Furthermore, it can be applied to offset-free MPC [9], proposed also by author in [10].

The paper is organised as follows. Section II introduces used stochastic linear discrete-time state-space model describing controlled system. In Section III, the used MPC is presented and tasks needed for control design and online tuning are defined. Section IV describes the state estimation that is required for the design of control actions. In Section V, noise parameter estimation is introduced including the use of obtained estimates for tuning of MPC. Section VI demonstrates the theoretical outcomes on a specific parallel robot-manipulator.

Throughout the paper, the following notation is used: z_k is the value of a column vector z at a discrete-time instant $k \in k^\star \equiv \{\underline{k}, \cdots, \overline{k}\}$, $\underline{k} \geq 1$; $z_{k;i}$ is the i-th entry of z_k; ℓ_z is the length of the vector z; $\underline{z}$ and $\overline{z}$ are lower and upper bounds on z, respectively.

II. Model of Controlled System

Modelling of robotic systems represents a complex task. These systems are generally described by non-linear models. An example of the specific robotic system is presented in Section VI. Let us consider that the relevant model of the controlled system can be adapted to a linear-like time-varying state-space model as follows

$$\begin{aligned} x_k &= A_k\, x_{k-1} + B_k\, u_{k-1} + \nu_k = \tilde{x}_k + \nu_k \\ y_k &= C\, x_k + n_k = \tilde{y}_k + n_k \end{aligned} \tag{1}$$

where subscript k denotes discrete time instants; x_k, y_k are system states and outputs; A_k and B_k are time-varying state and input matrices; C is a constant output matrix; $\tilde{x}_k$, $\tilde{y}_k$ are corresponding deterministic parts of x_k, y_k; ν_k, n_k are vectors of stochastic state and output noises, respectively. Such model can represent all necessary features and relations in a robotic system and it is suitable for control design and related estimations as well.

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In the model (1), the noises ν_k and n_k are assumed to be identically distributed and conditionally independent of past, having zero means and constant variances. Unlike the usual assumption that noises ν_k and n_k are normally distributed with known covariance matrices, it is assumed that both the state noise ν_k and the output noise n_k are distributed uniformly on a multidimensional box with zero centre point and unknown half-widths ρ and r of the support intervals, respectively, i.e.

$$f(\nu_k|\rho) = \mathcal{U}_{\nu_k}(0_{\ell_x}, \rho), \quad f(n_k|r) = \mathcal{U}_{n_k}(0_{\ell_y}, r) \tag{2}$$

where $\mathcal{U}_z(\mu, \alpha)$ denotes a uniform probability density function (pdf) of a variable z given by the expectation μ and the half-width of the support α; parameters ρ and r are assumed to be time-invariant or slowly varying. The symbol $f(\cdot|\cdot)$ denotes a conditional pdf; names of arguments distinguish respective pdfs; no formal distinction is made between a random variable, its realisation and an argument of the pdf.

The equations (1) and (2) define a stochastic linear state-space uniform model (LSU model) used in further explanation.

III. Predictive Control Design

In this paper, a discrete positional generalised predictive control (GPC) [11] is considered. To design an optimal control action, GPC employs predictions of expected future outputs of controlled system represented by a state space model. Here, the above mentioned LSU model will be utilised. The future output values are expressed by equations of predictions [12]. The main design elements, i.e. equations of predictions and relevant quadratic cost function, together with interpretation and tuning of corresponding control parameters, are introduced in the following subsections.

A. Equations of Predictions

The equations of predictions enable the design to optimise control task within a specific finite horizon considering future reference values thus to generate optimal control actions. These equations express the relationship between predicted future outputs and unknown control actions. Here, they are composed using LSU model (1) with estimated state values as follows

$$\begin{aligned} \hat{y}_{k+1} &= CA_k\,\hat{x}_k + CB_k u_k \\ \hat{y}_{k+2} &= CA_k^2\,\hat{x}_k + CA_k B_k u_k + CB_k u_{k+1} \\ &\vdots \\ \hat{y}_{k+N_p} &= CA_k^{N_p}\hat{x}_k + CA_k^{N_p-1}B_k u_k \cdots + CB_k u_{k+N_p-1} \end{aligned} \tag{3}$$

where N_p is a prediction horizon and $\hat{x}_k$ is a point state estimate based on previous realised inputs and measured outputs (the estimation of x_k is described in Section IV). The equations (3) can be expressed in matrix form:

$$\hat{Y}_{k+1} = F_k\,\hat{x}_k + G_k\,U_k \tag{4}$$

$\hat{Y}_{k+1}$, and U_k in (4) are sequences of output predictions and searched control actions, respectively

$$\hat{Y}_{k+1} = \left[\hat{y}_{k+1}^T, \cdots, \hat{y}_{k+N_p}^T\right]^T, \; U_k = \left[u_k^T, \cdots, u_{k+N_p-1}^T\right]^T \tag{5}$$

and F_k and G_k are matrices defined as follows

$$F_k = \begin{bmatrix} CA_k \\ \vdots \\ CA_k^{N_p} \end{bmatrix}, \quad G_k = \begin{bmatrix} CB_k & \cdots & 0 \\ \vdots & \ddots & \vdots \\ CA_k^{N_p-1}B_k & \cdots & CB_k \end{bmatrix} \tag{6}$$

Note that noise terms in (1) are omitted due to the assumption of their zero mean and matrices A_k and B_k are considered to be constant within one prediction horizon with respect to unknown future states and the assumption of their low variations within the horizon.

B. Cost Function and Its Minimisation

Quadratic cost function is an objective function that balances control errors, i.e. differences between predicted outputs and given references, against amount of input energy. For considered positional predictive algorithm, the cost function is defined as follows

$$\begin{aligned} J_k &= \mathcal{E}\sum_{j=1}^{N_p}\left\{\|Q_{yw}(\hat{y}_{k+j} - w_{k+j})\|_2^2 + \|Q_u u_{k+j-1}\|_2^2\right\} \\ &= \mathcal{E}\left\{(\hat{Y}_{k+1} - W_{k+1})^T Q_{YW}^T Q_{YW}(\hat{Y}_{k+1} - W_{k+1}) + U_k^T Q_U^T Q_U U_k\right\} \end{aligned} \tag{7}$$

where $\mathcal{E}$ denotes the expected value, $\|.\|_2$ is the quadratic norm, W_{k+1} represents a sequence of references

$$W_{k+1} = \left[w_{k+1}^T, \cdots, w_{k+N_p}^T\right]^T \tag{8}$$

and Q_{YW} and Q_U are penalisation matrices defined as

$$Q_\diamond^T Q_\diamond = \begin{bmatrix} Q_*^T Q_* & & 0 \\ & \ddots & \\ 0 & & Q_*^T Q_* \end{bmatrix} \left| \begin{array}{l} \text{subscripts } \diamond, *: \\ \diamond \in \{YW,\, U\} \\ * \in \{yw,\, u\} \end{array} \right. \tag{9}$$

Minimisation of (7) can be provided in efficient and numerically stable square-root form considering the minimisation of vector square-root J_k instead of J_k, scalar product of J_k:

$$\min_{U_k} J_k = \min_{U_k} \mathsf{J}_k^T \mathsf{J}_k \;\rightarrow\; \min_{U_k} \mathsf{J}_k \tag{10}$$

$$\min_{U_k} \mathsf{J}_k = \min_{U_k} \mathcal{E}\left\{\begin{bmatrix} Q_{YW} & 0 \\ 0 & Q_U \end{bmatrix}\begin{bmatrix} \hat{Y}_{k+1} - W_{k+1} \\ U_k \end{bmatrix}\right\} \tag{11}$$

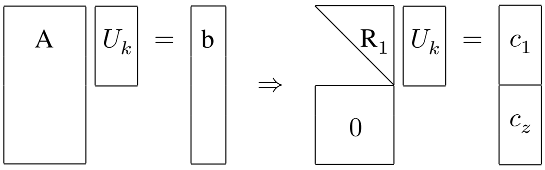
which leads to the over-determined algebraic system

$$\begin{bmatrix} Q_{YW}\,G_k \\ Q_U \end{bmatrix} U_k = \begin{bmatrix} Q_{YW}(W_{k+1} - F_k\hat{x}_k) \\ 0 \end{bmatrix} \tag{12}$$

The system (12) can be written in general form [13]

$$\begin{aligned} \mathrm{A}\, U_k &= \mathrm{b} && (13)\\ \mathrm{Q}^T \mathrm{A}\, U_k &= \mathrm{Q}^T \mathrm{b} \ \text{ with respect to } \ \mathrm{A} = \mathrm{Q}\,\mathrm{R} \\ \mathrm{R}_1\, U_k &= c_1 && (14) \end{aligned}$$

Thus, orthogonal matrix Q^T transforms matrix A to an upper triangle R_1 as indicated in the following block diagram



(15)

In (15), vector c_z is a loss vector. Its Euclidean norm $||c_z||$ equals to the square-root of the optimal minimum of the cost function $\sqrt{J}$ i.e. $J = c_z^T c_z$.

Note that the final control action u_k, intended for realisation, is the first sub-vector from the overall vector U_k as indicated in (5).

C. Control Parameters and Their Tuning

Predictive control usually operates under appropriate constant control parameters N_p, Q_{yw}, Q_u. Constant prediction horizon N_p is chosen usually with respect to constant order of the system. Regarding penalisations Q_{yw}, Q_u, they are usually constant and diagonal matrices. Nevertheless, their suitable on-line tuning can improve the control process.

Let us consider stochastic disturbances influencing the system. In this way, a discrepancy between used model and controlled system may occur. It can cause an increase of control errors by the using of a fixed predictive design.

The mentioned discrepancy or a decrease of the model accuracy can partially be solved by a suitable tuning of the control parameters Q_{yw} and Q_u in (7). These parameters control stiffness or hardness and softness of the given predictive controller. Thus, on-line tuning (adaptation) of these control parameters can moderate sudden changes within control action profiles.

The proposed tuning is based on the evaluation of reliability of a used model, i.e. "level of information capability". It utilises the following link between Q_{yw}, Q_u and covariance matrices [14], [15], [16]

$$Q_{yw} \propto C_y^{-1}, \quad Q_u \propto C_u^{-1} \tag{16}$$

where C_y and C_u are output and input covariance matrices, respectively; $\propto$ means a proportionality. Due to mutual dependency of control parameters, it is sufficient to tune only one parameter e.g. Q_{yw} and leave Q_u to the chosen constant value.

In [16], Q_{yw} is tuned simply according to the evolution of model precision matrix, i.e. inverse of covariance matrix. For this method, some setting of the forgetting factor is required. Here, we propose the more convenient solution where the required covariance matrices are obtained using the noise parameter estimates. For details on the covariance estimation, see (28) in Section V.

IV. State Estimation Using LSU Model

The construction of equations of predictions (3) requires the point state estimates $\hat{x}_k$ of the model (1). They are obtained by the Bayesian estimation of

$$\mathcal{X}_k \equiv [\, x_k^T,\, \cdots,\, x_{k-N_e}^T,\, \rho^T,\, r^T]^T \tag{17}$$

on a moving window of the length N_e using the observed data $D_k = [\, d_k^T,\, \cdots,\, d_{k-N_e}^T]^T$, $d_\kappa = [u_\kappa^T,\, y_\kappa^T]^T$. To obtain the point estimates, the Bayesian estimation reduces to evaluation of characteristics of the posterior pdf $f(\mathcal{X}_k|D_k)$ of $\mathcal{X}_k$ and a series of maximum a posteriori (MAP) estimates $\hat{\mathcal{X}}_k$ of the unknown $\mathcal{X}_k$, $k \in k^\star$, is evaluated [17]. For known model matrices, the problem the MAP estimation corresponds to the solving a linear programming (LP) problem [7] to find a vector $\mathcal{X}_k$, $k \in k^\star$, that minimises

$$\mathrm{f}^T \mathcal{X}_k = \sum_{i=1}^{\ell_x} \rho_i + \sum_{j=1}^{\ell_y} r_j \tag{18}$$

subject to

$$\mathcal{A}_k \mathcal{X}_k \le b_k,\ \underline{\mathcal{X}}_k \le \mathcal{X}_k \le \overline{\mathcal{X}}_k,$$

where $\mathrm{f}^T \equiv [0^T_{(\ell_{\mathcal{X}_k} - \ell_x - \ell_y)}, 1^T_{(\ell_x + \ell_y)}]$ consists of the vectors of zeros and ones of the indicated lengths. $\underline{\mathcal{X}}_k$ and $\overline{\mathcal{X}}_k$ are user given lower and upper bounds on $\mathcal{X}_k$. $\mathcal{A}_k$ and b_k are the matrix and vector, respectively, constructed according to inequalities arising from (1) with substituted noise bounds (2), i.e. (for $\kappa = k,\, k-1,\, \ldots,\, k-N_e$)

$$\begin{aligned} -\rho \le\ & x_\kappa - A\, x_{\kappa-1} - B\, u_{\kappa-1} \le \rho \\ -r \le\ & y_\kappa - C\, x_\kappa \le r \end{aligned} \tag{19}$$

Note that the construction of $\mathcal{A}_k$ and b_k for $\mathcal{X}_k$ (17) is described in detail in [18].

For future references, the above described state estimation is denoted as LP1.

V. Noise Parameter Estimation

The proposed penalisation tuning (16) requires a knowledge of covariance matrix C_y. It can be simply obtained using the estimated noise bounds as present below.

To describe the noise more precisely, we extend the original model (1) to the following form

$$\begin{aligned} Lx_k &= A_k\, x_{k-1} + B_k\, u_{k-1} + \nu_k = \tilde{x}_k + \nu_k \\ Py_k &= C\ x_k + n_k = \tilde{y}_k + n_k \end{aligned} \tag{20}$$

where $L = I + \Lambda$, $P = I + \Pi$ are upper triangular matrices with unit diagonal, I denotes an identity matrix; the nonzero elements in Λ and Π model a possible correlation of respective noise components [7]. Note that if Λ and Π are zero matrices, then the model is equal to (1).

The noise distributions have an identical form with (2) with generally different values of noise bounds

$$f\left(\nu_k|\breve{\rho}\right) = \mathcal{U}_{\nu_k}\left(0_{\ell_x}, \breve{\rho}\right), \quad f\left(n_k|\breve{r}\right) = \mathcal{U}_{n_k}\left(0_{\ell_y}, \breve{r}\right) \tag{21}$$

A. Estimation of $\breve{\rho}$, $\breve{r}$, L and P

The estimates of noise parameters $\breve{\rho}$, $\breve{r}$, and matrices Λ, Π including the state estimates can be obtained using LP similarly to LP1. In [7], the estimation of $\mathcal{X}_k \equiv [x_\kappa^T, \cdots, x_{\kappa-\breve{N}_e}^T, \mathrm{col}(\Lambda)^T, \mathrm{col}(\Pi)^T, \breve{\rho}^T, \breve{r}^T]^T$ is presented where the mapping $\mathrm{col}(Z)$ transforms the non-zero elements of the matrix Z into a column vector, $Z \in \{\Lambda, \Pi\}$. The relevant LP has the form (18) but with different $\mathcal{A}_k$, b_k.

Here, the state estimates are supposed to be at disposal from LP1. Therefore, the full vector $\mathcal{X}_k$ mentioned above is reduced to

$$\mathcal{X}_k \equiv [\mathrm{col}(\Lambda)^T, \mathrm{col}(\Pi)^T, \breve{\rho}^T, \breve{r}^T]^T \tag{22}$$

Analogically to LP1, the task of the estimation of $\mathcal{X}_k$ is formulated as the LP problem to find a vector $\mathcal{X}_k$, $k \in k^\star$, that minimises

$$\mathrm{f}^T \mathcal{X}_k = \sum_{i=1}^{\ell_x} \breve{\rho}_i + \sum_{j=1}^{\ell_y} \breve{r}_j \tag{23}$$

subject to $\quad \mathcal{A}_k \mathcal{X}_k \leq b_k, \; \underline{\mathcal{X}}_k \leq \mathcal{X}_k \leq \overline{\mathcal{X}}_k$.

Here, $\mathcal{A}_k$ and b_k are constructed using the following inequalities (for $\kappa = k, k-1, \cdots, k-\breve{N}_e$)

$$\begin{aligned} -\breve{\rho} \leq (I+\Lambda)\,\hat{x}_\kappa - A\,\hat{x}_{\kappa-1} - B\,u_{\kappa-1} &\leq \breve{\rho} \\ -\breve{r} \leq (I+\Pi)\,y_\kappa - C\,\hat{x}_\kappa &\leq \breve{r} \end{aligned} \tag{24}$$

Then,

$$\mathcal{A}_k = \begin{bmatrix} \hat{X}_k \otimes K & 0_{(2\ell_x,\beta_y)} & -1_{2\ell_x} & 0_{2\ell_x} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{X}_{k-\breve{N}_e} \otimes K & 0_{(2\ell_x,\beta_y)} & -1_{2\ell_x} & 0_{2\ell_x} \\ 0_{(2\ell_y,\beta_x)} & Y_k \otimes K & 0_{2\ell_y} & -1_{2\ell_y} \\ \vdots & \vdots & \vdots & \vdots \\ 0_{(2\ell_y,\beta_x)} & Y_{k-\breve{N}_e} \otimes K & 0_{2\ell_y} & -1_{2\ell_y} \end{bmatrix} \tag{25}$$

where $\beta_x = \sum_{i=1}^{l_x-1}(l_x - i)$ and $\beta_y = \sum_{i=1}^{l_y-1}(l_y - i)$,

$\hat{X}$, Y are computed in the same way as follows:

$$Z_\kappa = \begin{bmatrix} Z_\kappa^{(1)} & 0_{\ell_z-2}^T & \cdots & 0_2^T & 0 \\ 0_{\ell_z-1}^T & Z_\kappa^{(2)} & \cdots & 0_2^T & 0 \\ \vdots & & \ddots & & \vdots \\ 0_{\ell_z-1}^T & 0_{\ell_z-2}^T & \cdots & 0_2^T & Z_\kappa^{(\ell_z-1)} \\ 0_{\ell_z-1}^T & 0_{\ell_z-2}^T & \cdots & 0_2^T & 0 \end{bmatrix}, \tag{26}$$

$Z_\kappa^{(i)} = [z_{\kappa;i+1}, \cdots, z_{\kappa;\ell_z}]$, $i = 1, \cdots, \ell_z - 1$,

where $Z \in \{\hat{X}, Y\}$, $z \in \{\hat{x}, y\}$;

$$b_k = \begin{bmatrix} A\hat{x}_{k-1} + Bu_{k-1} - \hat{x}_k \\ \vdots \\ A\hat{x}_{k-N_e-1} + Bu_{k-N_e-1} - \hat{x}_{k-N_e} \\ y_k - C\hat{x}_k \\ \vdots \\ y_{k-\breve{N}_e} - C\hat{x}_{k-N_e} \end{bmatrix} \otimes K, \tag{27}$$

$\otimes$ denotes Kronecker product; $K \equiv [1 \;\; -1]^T$; $0_{(\alpha,\beta)}$ are zero matrices of appropriate dimensions; 0_α and 1_α are column vectors of zeros and ones of length α, respectively.

For future references, the above described noise parameter estimation is denoted as LP2.

B. Computation of Covariance Matrices

Considering the output equation in (20) and using the estimates of P and $\breve{r}$, the output covariance matrix $C_y \equiv \mathrm{cov}(y, y \,|\, \tilde{y}, \breve{r})$ is computed as follows

$$\begin{aligned} C_y &= \mathcal{E}\{yy^T \,|\, \tilde{y}, \breve{r}\} - \mathcal{E}\{y \,|\, \tilde{y}, \breve{r}\}\, \mathcal{E}^T\{y \,|\, \tilde{y}, \breve{r}\} \\ &= \tfrac{1}{3}\, P^{-1} D D^T (P^{-1})^T \end{aligned} \tag{28}$$

where D is a square matrix with the main diagonal elements $D_{ii} = \breve{r}_i$ and zeros elsewhere; $\mathcal{E}\{y \,|\, \tilde{y}, \breve{r}\} = P^{-1}\tilde{y}$; $\mathcal{E}\{y^T \,|\, \tilde{y}, \breve{r}\} = P^{-1}\big(\tilde{y}\tilde{y}^T + \tfrac{1}{3} D D^T\big)\,(P^{-1})^T$. Full derivation of (28) can be found in [19].

Note that state covariance matrix $C_x \equiv \mathrm{cov}(x, x \,|\, \tilde{x}, \breve{\rho})$ can be obtained in the same way using the state evolution equation in (20), i.e. $C_x = \frac{1}{3} L^{-1} D D^T (L^{-1})^T$, where D is a diagonal square matrix for $D_{ii} = \breve{\rho}_i$ and $D_{ij} = 0, i \neq j$.

VI. Experiments

To illustrate achieved theoretical outcomes, the specific parallel robot-manipulator "Moving Slide" (Fig. 1) is used.

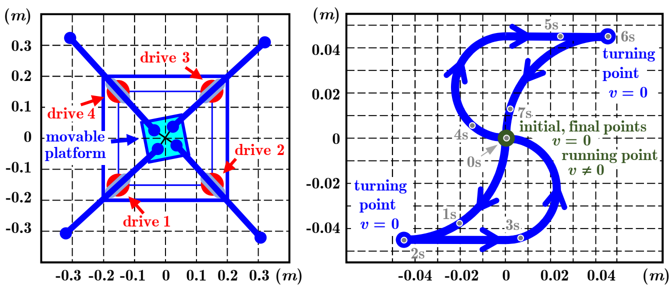


Fig. 1. Scheme of the robot-manipulator 'Moving Slide', and used testing 'S'-shape trajectory with time marks.

The robot consists of four kinematic chains with rotational, positional and rotational joint configuration, respectively. It represents a special planar robotic system with four inputs (torques) and three outputs (two position coordinates x_c, y_c, and rotation angle ψ_c of movable platform), [20]. It can be described by a nonlinear state-space model based on the Lagrange equations with a continuous state $x(t) = [y(t), \dot{y}(t)]^T$ composed from outputs and their derivatives [21]:

$$\begin{aligned} \dot{x}(t) &= F(x(t)) + B(x(t))u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{29}$$

For considered control design, the model (29) has to be linearised. Utilising a specific decomposition according to [22] demonstrated in [23], the model (29) can be transformed to the linear-like state-dependent form. After the subsequent time discretisation, the following model is obtained

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k \\ y_k &= C x_k \end{aligned} \tag{30}$$

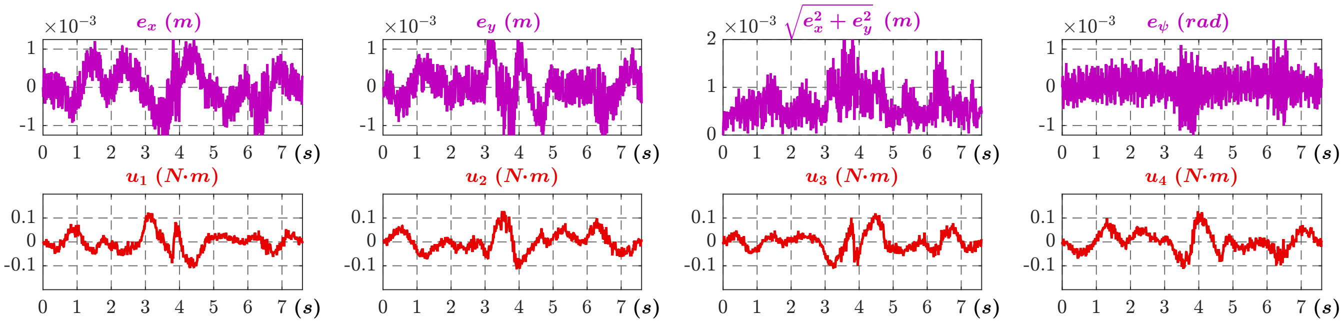


Fig. 2. Time histories of control errors (up) and control actions (down) for the case with constant Q_{yw}

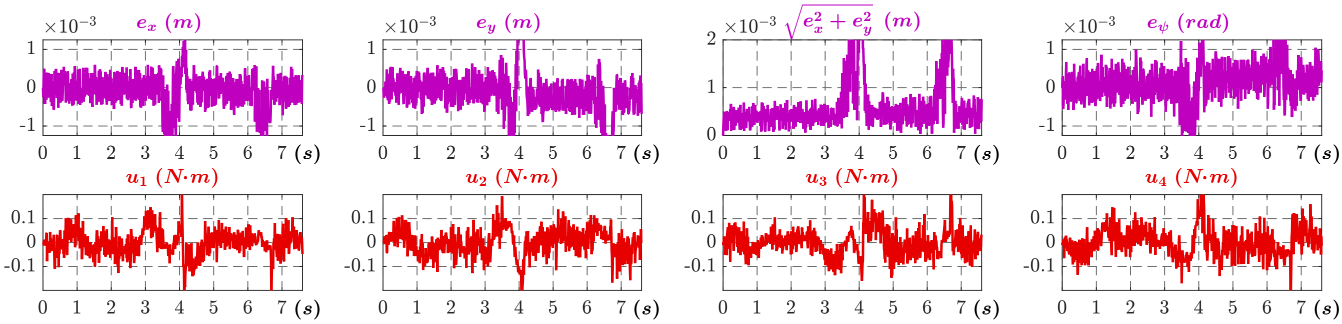


Fig. 3. Time histories of control errors (up) and control actions (down) for the case with tuned Q_{yw}

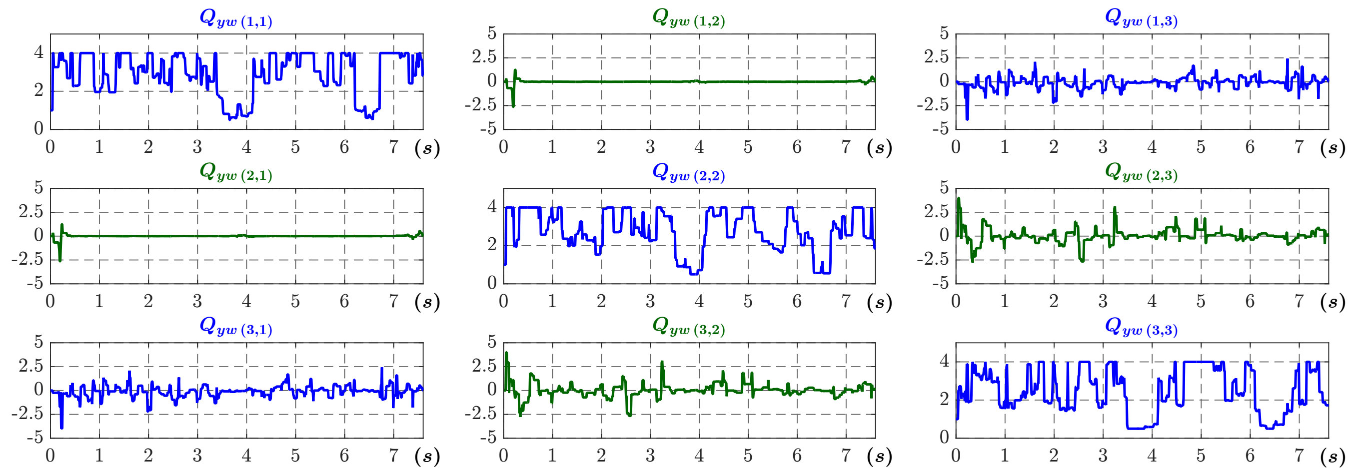


Fig. 4. Time histories of individual elements of penalisation matrix Q_{yw} for the online tuning

The matrices A_k and B_k are state dependent, i.e. their current values are determined by the current state values. The model (30) represents deterministic part of the controlled system. Considering real conditions, additive process and measurement noises are added to the model (30). In this way, the form (1) is obtained.

The following two experiments are demonstrated:

(i) predictive control with state estimation LP1 (Sec. IV) and constant penalisation Q_{yw};

(ii) predictive control with state estimation LP1 and online tuned penalisation Q_{yw} according to LP2 (Sec. V-A).

The above described robotic system was simulated by (29) and a measurement noise n_k was added to the generated output. State estimates were obtained using model (1).

Utilising the state estimate from previous step, the model matrices in (1) were updated. Then, the current state value was estimated using LP1. In experiment (ii), the state estimate from LP1 was used for subsequent noise parameter estimation LP2 and penalisation Q_{yw} was updated according to (16) with the covariance estimate (28). Finally, the control input was designed according to (15).

The experiment setup was as follows: horizon of prediction $N_p = 10$; length of windows for LP1 $N_e = 3$ and for LP2 $\check{N}_e = 14$; constant penalisation matrices $Q_u = 10^{-2} \cdot I_{\ell_u}$ and $Q_{yw} = I_{\ell_y}$, where I_α is the identity matrix of order α.

The measurement noise n_k (2) was simulated having distribution $\mathcal{U}(0_{\ell_y}, r)$, $r = 5 \cdot 10^{-4} \cdot 1_{\ell_y}$. Two abrupt increases of the noise $\mathcal{U}(0_{\ell_y}, 2\,r)$ were simulated in the time intervals $\langle 3.4\mathrm{s}, 4\mathrm{s}\rangle$ and $\langle 6.2\mathrm{s}, 6.6\mathrm{s}\rangle$.

For online tuning of penalisation matrix according to (16), a scalar constant of proportionality $k_p = 10^{-7}$ was chosen, $Q_{yw} = k_p C_y^{-1}$. In accord with usual constant values, hard bounds for penalisation were chosen as follows: lower bound for diagonal elements of Q_{yw}, $\underline{b} = 0.5$, upper bound for all elements of Q_{yw}, $\overline{b} = 4$. The lower bounds guarantee that the controller will generate some control actions even in the case when a high noise decreases overly the quality of state estimates. The upper bounds prevent too rapid control actions.

Fig. 2 and Fig. 3 show the control results for the case with the constant and tuned penalisation matrix Q_{yw}, respectively. The case without tuning presents a smoother course of control actions with higher control errors while in the tuned case, the more balanced control errors are obtained but with the more excited control actions. In the tuned case, the control errors oscillate around zero with only two departures caused by the noise increases.

Time history of individual online tuned elements of penalisation Q_{yw} based on noise parameter estimation LP2 is in Fig. 4. On the courses of diagonal elements, the sections with increased noise can be clearly recognised. There, the penalisation was released as the state estimates precision was lowered. The courses of non-diagonal elements oscillating around zero indicate that output components are mutually uncorrelated.

Note that the reason for simultaneous estimations LP1 and LP2 is following. LP1 provides both noise bounds and state estimates but it does not consider a correlation of noise components. LP2 in its full variant provides also state estimates but it requires an additional linearisation as original inequalities entering LP problem contain the product Λx_τ [7]. In the presented experiments, the best results were obtained by joint use of LP1 and LP2.

VII. Conclusion

The paper proposes a novel solution to the state and noise parameter estimation incorporated in the model predictive control including the online tuning of control parameters. It is intended for the cases where an additive noise is bounded.

The paper confirms and extends the preliminary experiments concerning the interconnection of the predictive control and the linear uniform state-space model. The mentioned experiments were presented on one simple benchmark model in [18]. Here, the time-varying linearised model is used for the control of a nonlinear robotic system. So, the state estimates are used not only for control design itself but also for the update of state dependent model matrices. Unlike the standardly used estimation based on Kalman filter, the used uniform model needs no setting of covariance matrices because the noise parameter estimation is a part of the algorithm. Further contribution of presented algorithm consists in using of noise parameter estimates for the online tuning of control parameters, namely penalisation elements.

The proposed solution realises state estimation and tuning for model-based control design in general. It respects the reliability of used model of controlled system. The paper considers a general unconstrained positional MPC.

The following research will concentrate on the extension to the constrained cases according to [8] and to the offset-free solution according to [10].

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