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Research paper

Herding, minority game, market clearing and efficient markets in a simple spin model framework

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ABSTRACT

We present a novel approach towards the financial Ising model. Most studies utilize the model to find settings which generate returns closely mimicking the financial stylized facts such as fat tails, volatility clustering and persistence, and others. We tackle the model utility from the other side and look for the combination of parameters which yields return dynamics of the efficient market in the view of the efficient market hypothesis. Working with the Ising model, we are able to present nicely interpretable results as the model is based on only two parameters. Apart from showing the results of our simulation study, we offer a new interpretation of the Ising model parameters via inverse temperature and entropy. We show that in fact market frictions (to a certain level) and herding behavior of the market participants do not go against market efficiency but what is more, they are needed for the markets to be efficient.

1. Introduction

Agent-based models (ABM) have attracted much attention in economics and finance in recent years [1–3] as they describe the reality better than simplified models of traditional economics and finance. The crucial innovation lies in assuming a boundedly rational economic agent [4,5] instead of a perfectly rational representative agent with homogeneous expectations [6,7]. In these models, agents make decisions without utility maximization but usually using simple heuristics. The resulting systems are usually driven endogenously, i.e. without exogenous shocks forcing the dynamics.

In finance, the founding contributions were laid by Brock and Hommes models [8,9] characteristic by strategy-switching agents and possible bifurcation dynamics. Essential contributions to the topic are the early papers of Lux and Marchesi [10] and Kaizoji [11] who introduce a possibility of generating the returns-like series from simple models based on interactions between multiple agents. They both serve as a starting point to an important branch of the ABMs which is based on a parallel between ferromagnetism and market dynamics, i.e. the Ising model adjusted for financial markets. In the models, economic agents participating in the market are spins of a magnet. In the same way as the spins, the agents are influenced by (make their decisions based on) their neighbors, or agents with similar beliefs, but also by the overall market sentiment and activity. The novel model of Bornholdt [12] combining the standard Ising model from physics with an additional term reminiscent of the minority game, i.e. the tendency of agents to lean away from the majority opinion when the majority prevails too much, has been shown to successfully mimic the basic financial stylized facts such as no serial correlation of re-

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turns, persistence and clustering of volatility, and non-Gaussian distribution of returns. Kaizoji et al. [13] expand the model of Bornholdt [12] by four additional parameters to allow for simulations of the traded volume through the balance between supply and demand. Implications for bull and bear markets together with bubbles occurrence is discussed there as well.

These founding papers have led to various adjusted and generalized models trying mainly to fit the market data or mimic the stylized facts. Sornette and Zhou [14] build a model with external news and expectations of the agents, who are able to adjust their expectations through learning. Their model is able to generate fat-tailed returns with exponentially decaying serial correlation structure, aggregate normality, volatility power-law decaying serial correlation and clustering as well as specific multi-fractal properties. Compared to the basic model of Bornholdt [12] with two parameters, the model of Sornette and Zhou [14] uses seven. Zhou and Sornette [15] present further results of the same model with a more direct connection to behavioral finance as the agents are allowed to be not fully rational. Yang et al. [16] utilize a similar model and try to explain the dynamics of the KOSPI stock market. They show that one of the parameters is directly proportional to the tail index of the distributions with power-law tails. However, it is not clear whether this holds for reasonable values of other parameters as well.

Queiros et al. [17] present a model combining the ideas of Lux and Marchesi [10], Bornholdt [12] and Sornette and Zhou [14], explicitly including a term depending on magnetization. Even though the model is able to produce most of the stylized facts, it is not able to replicate the important features of volatility – clustering and power-law autocorrelation decay. Sieczka and Hołyst [18] expand the model of Kaizoji et al. [13] by allowing the agents to take three instead of standard two positions – in addition to “buy” and “sell”, there is also a possibility to stay inactive. The model is able to reproduce many stylized facts but the ones connected to volatility. Denys et al. [19] further enhance the model of Sieczka and Hołyst [18] by having opinions in their willingness to buy or sell which are only then translated into actual buying or selling actions. This opinion dynamics enters the neighbor interactions part of the model. Even thought the main motivation of the paper is to correct the Sieczka and Hołyst [18] study, this enhanced model still does not mimic the power-law decay in the volatility autocorrelation function. The idea of opinion forming in the financial Ising models is further developed in Krause and Bornholdt [20] where the volatility clustering is obtained.

Krawiecki [21] enriches the Ising-type models with a random organic network structure. Through three uniformly distributed random variables and three parameters, the model replicates the basic stylized facts even for volatility. Krause and Bornholdt [22] move towards a macroscopic model using the original microscopic model of Bornholdt [12] and Takaishi [23] generalizes the whole framework for multiple assets showing that cross-correlated assets can be generated. However, other stylized facts are not covered in detail.

For a detailed treatment and history of the Ising-type models in financial economics together with other agent-based models, we suggest the current treatment of the topic by Sornette [24].

There are at least three interesting outcomes that can be inferred from the review above. First, the results and ability of models to recover the financial stylized facts are often very sensitive to the parameters choice. Only a narrow range of parametric values yields reasonable results and the models usually break down in a sense that they converge to a very stable magnetization and thus price which results in zero returns. Second, majority of papers dealing with the financial Ising-type models focus primarily on retrieving the stylized facts of returns and volatility (and sometimes traded volume) and touch the interpretation of parametric values only on surface. And third, vast majority of the reviewed models are not able to outperform the original Bornholdt [12] model in the sense of the stylized facts coverage. Note that all the expanded models add more parameters and often random variables to the basic model, yet there are not able to outperform the basic model significantly.

We contribute to the topical literature by inspecting the implications of the financial Ising model towards the capital markets efficiency. We focus on the model parameters and how they influence returns dynamics in the optics of the efficient market hypothesis. The attention is given to finding a combination of parameters which yields an efficient market or dynamics close to it. We thus regard the question “What combination of parameters yields returns and volatility mimicking the stylized facts?” as studied and answered in enough detail in the reviewed papers, implying that the structure and construction of the models are reasonable, and we focus on the question “What combination of parameters yields returns consistent with the efficient market hypothesis?”. We show that the effects of parameters are more complicated than one might expect and their influence is apparently non-linear with a special role of the critical temperature of the system and we discuss the implications for foundations of the efficient market hypothesis.

2. Ising model for financial markets

As a representative of the agent-based models applied to finance and financial economics, we opt for a simple Ising model adjusted for financial markets as proposed by Bornholdt [12]. There are two main reasons why this specific model is chosen. First, the model is able to mimic the most important stylized facts of financial returns. And second, the model has only two parameters which allows for a straightforward interpretation of the outcomes without a need for additional restrictions.

2.1. Model basics

The model builds on a combination of the standard Ising model of ferromagnetism with local field interactions [25] and a minority game behavior of market agents [26,27]. Financial market is represented by a square lattice (usually with torus-like neighborhoods, which we utilize as well) with a side of N, i.e. with N^2 elements representing market agents. These elements are referred to as spins due to their magnetization of either $+1$ or -1. This spin orientation is translated into a financial market as either a buy or a sell signal (decision), respectively. The spin orientation of agent i for a time period t is labelled as $S_i(t)$. For each agent i, the local field $h_i(t)$ for a time period t is defined as

$$h_i(t) = \sum_{j=1}^{N^2} J_{ij} S_j(t) - \alpha C_i(t) \frac{1}{N^2} \sum_{j=1}^{N^2} S_j(t), \tag{1}$$

where $t = 1, \ldots, T$ is discrete time. The first term is defined as a local Ising Hamiltonian with neighbor interactions J_{ij}. This is the reference to the standard Ising model. In the economic interpretation, this represents the potential herding behavior as agents are influenced by their closest neighbors and they might thus tend together, potentially forming speculative bubbles. The second term represents the global coupling as it depends on the total magnetization of the system $M(t) \equiv \frac{1}{N^2} \sum_{j=1}^{N^2} S_j(t)$ at time t with sensitivity α. From the economic perspective, this term is a built-in minority game. For $\alpha > 0$, there is a tendency to go against the overall magnetization and thus against the whole market dynamics. The strategy spin $C_i(t)$ allows for deviations from the minority game behavior of spin i, i.e. $C_i(t)$ is not necessarily equal to one. On the one hand, $C_i = -1$ implies that the agents align with the total magnetization so that they follow the market trend. Such agents are usually referred to as the trend followers or chartists. On the other hand, $C_i = 1$ suggests the minority game behavior of the agents as they oppose the sign of the total magnetization. These agents are standardly referred to as the fundamentalists.

The price and returns dynamics of the system is extracted directly from the magnetization dynamics so that

$$\begin{aligned} \log P(t) &= M(t) \equiv \frac{1}{N^2} \sum_{j=1}^{N^2} S_j(t), \\ r(t) &= \Delta \log P(t) = \Delta M(t) = M(t) - M(t-1). \end{aligned} \tag{2}$$

The logic behind such representation is based on taking the positive spins as demand and the negative spins as supply. Their sum, i.e. the difference between demand and supply, is taken as excess demand so that the difference between two consecutive excess demands is a change in price of an asset [12,13,28].[1]

Orientation of the spin i at time $t+1$ is given by the heat-bath dynamics transition function with probability $p_i(t)$ as

$$\begin{aligned} S_i(t+1) &= +1 \text{ with } p_i(t) = [1 + \exp(-2\beta h_i(t))]^{-1} \\ S_i(t+1) &= -1 \text{ with } 1 - p_i(t), \end{aligned} \tag{3}$$

which is directly connected to Eq. 1 with an additional sensitivity β, which is parallel to the inverse temperature[2] of the original Ising model, i.e. $\beta = \frac{1}{T}$, and it is essential as it controls the responsiveness of the spin change probability to the local field $h_i(t)$. The inverse temperature determines the system regime – either paramagnetic or ferromagnetic in the original Ising model terminology. For the paramagnetic phase with under-critical $\beta < \beta_C = \frac{1}{T_C}$ (i.e. over-critical $T > T_C$ where C stands for "critical" or "Curie"), the model dynamics leads to the paramagnet which is characteristic by erratic behavior. Reversely for the ferromagnetic phase with over-critical β (under-critical temperature T), the model converges to a stable state as a ferromagnet.[3]

2.2. Agent types and strategy spins

These two types of behavior can be easily inferred from the heat-bath dynamics in Eq. 3. For the paramagnetic regime, the transition function is rather flat so that the spin probability depends on the local field $h_i(t)$ only weakly. Decreasing β then leads to a weakening local interactions effect. For β close to zero (infinite temperatures), the spin change is completely random with probability $\frac{1}{2}$. For the ferromagnetic regime, the local interactions become more dominant forming large clusters of oriented spins, one of which eventually dominates and leads to the stable state of the model with $|M(t)|$

[1] More precisely, the return is a function of the difference between magnetizations (if we elaborate on the excess demand interpretation of McCauley [28]). We stick to the more prevalent view of returns as differences between the magnetizations without further adjustments to keep the results and interpretations comparable with other topical studies.

[2] In physics, the Ising model in the absence of external magnetic field has two well separated phases – below and above the critical temperature T_C. In the former phase, the spins align spontaneously, whereas for the latter, thermal fluctuations eliminate such alignment completely. As this change of behavior around the critical temperature is abrupt and discontinuous, it is a nice example of phase transition [25,29]. Temperature is thus a crucial characteristic of a magnet and it influences its behavior significantly.

[3] Note that there is no noise term added in the whole dynamics and the decision-making of agents as described in Eqs. 1–3. This distinguishes the financial Ising model from the other financial ABMs which usually utilize exogeneous shocks to the system. The Ising model here is able to produce the market-like dynamics endogenously.

$\approx$ 1. Kramers and Wannier [29] show that for the original Ising model, i.e. with $\alpha = 0$, the critical temperature is equal to $T_C = \frac{2}{\ln(1+\sqrt{2})} \approx 2.269$ which gives $\beta_C = \frac{1}{T_C} \approx 0.441$.

The strategy term $C_i(t)$[4] is given as a general term in Eq. 1 which can be further specified. A popular choice is to highlight the minority game behavior of the spin by allowing the strategy to change with respect to the total magnetization and the spin's own orientation. This specification also allows for more strategy types. Bornholdt [12] proposes the following dynamics

$$C_i(t+1) = -C_i(t) \text{ if } \alpha S_i(t) C_i(t) \sum_{j=1}^{N^2} S_j(t) < 0 \tag{4}$$

for $i = 1, \ldots, N^2$. The practical implications of such rule are the following. The second term of the local field (Eq. 1) of all majority agents (who are $C_i(t) = 1$) has an opposite sign compared to the total magnetization. This forces the agents to swap their strategy. Similarly for the minority agents (with $C_i(t) = -1$), the second term of the local field has the same sign as the total magnetization, which forces them to change their strategy as well. As the total magnetization $M(t)$ is a part of the second term of the local field, the tendency towards switching strategies strengthens with the total magnetization deviating from zero, which is parallel to the equilibrium price of the asset. The further the magnetization (price) deviates from zero (equilibrium) the more agents will oppose it. In practice, this protects the model from deviating towards $\pm$ 1 and stabilizing there while still remaining well in the logic of how the market works and how the agents behave.

A simple alternative is to have the strategy spin update immediately, which reduces the local field equation to

$$h_i(t) = \sum_{j=1}^{N^2} J_{ij} S_j(t) - \alpha S_i(t) \left| \frac{1}{N^2} \sum_{j=1}^{N^2} S_j(t) \right|, \tag{5}$$

for $i = 1, \ldots, N^2$, i.e. it does not depend on the strategy of any spin at all [12]. The second term thus motivates an agent to change its spin orientation (i.e. the minority game behavior) with an increasing absolute value of magnetization $|M(t)|$.

3. Efficient market hypothesis

The efficient market hypothesis (EMH) has been a cornerstone of modern financial economics for decades. Even though its validity has been challenged on many fronts, it still remains the firm theoretical basis of the financial economics theory [30,31]. In the fundamental paper, Fama [32] summarizes the empirical validations of the theoretical papers of Fama [33] and Samuelson [34]. The theory is revised and made clearer in Fama [35].

From mathematical standpoint, the historical papers [33,34] are more important as they provide specific model forms of an efficient market. Specifically, Fama [33] connects the (logarithmic) price process of an efficient market to a random walk and Samuelson [34] specifies it as a martingale. Implications for the statistical properties of the (logarithmic) returns process, i.e. the first differences of (logarithmic) prices, of the efficient market are straightforward. For the former, the (logarithmic) returns are expected to be serially uncorrelated and follow the Gaussian (normal) distribution, i.e. the dynamics follows the Gaussian noise, which implies serial independence. For the latter, only the serial uncorrelatedness is implied as the martingale difference process is expected for the (logarithmic) returns [36,37]. We thus have two straightforward implications of the market efficiency – (asymptotically) normally distributed (for the random walk definition) and serially uncorrelated (for both the random walk and the martingale definitions) returns as serial independence implies no serial correlation.

In what follows, we focus on the ability of the financial Ising model to generate returns which would be considered as the returns of the efficient market in the sense of the efficient market hypothesis. We thus approach the model from a different perspective than majority of other studies which focus on its ability to mimic the stylized facts about returns and volatility. Our interest lays in inspecting how the parameters of the model interact with the assumptions of the Gaussian distribution and serial uncorrelatedness of returns. As the parameters represent local and global coupling of agents, one might expect that none of these are essential for market efficiency (but rather on the contrary).

4. Simulation setting

We are interested in the ability of the Ising model defined between Eqs. 1–5 to meet the criteria attributed to the efficient capital market, i.e. normality and serial uncorrelatedness of returns. To test these, we use the Jarque–Bera test [38] and Ljung–Box test [39], respectively.

There are two crucial parameters in the model – α and β – which can influence the prices and returns dynamics emerging from the model. We vary these two parameters and study how they influence the rejection rate of normality and uncorrelatedness of the respective tests. In other words, we are interested in a proportion of times these tests reject (with a significance level of 0.90) market efficiency of series generated by the financial Ising model with the specified parameters.

[4] The strategy spin is standardly given as a discrete number, usually either 1 or -1. However, nothing limits the parameter to be set as continuous [12].

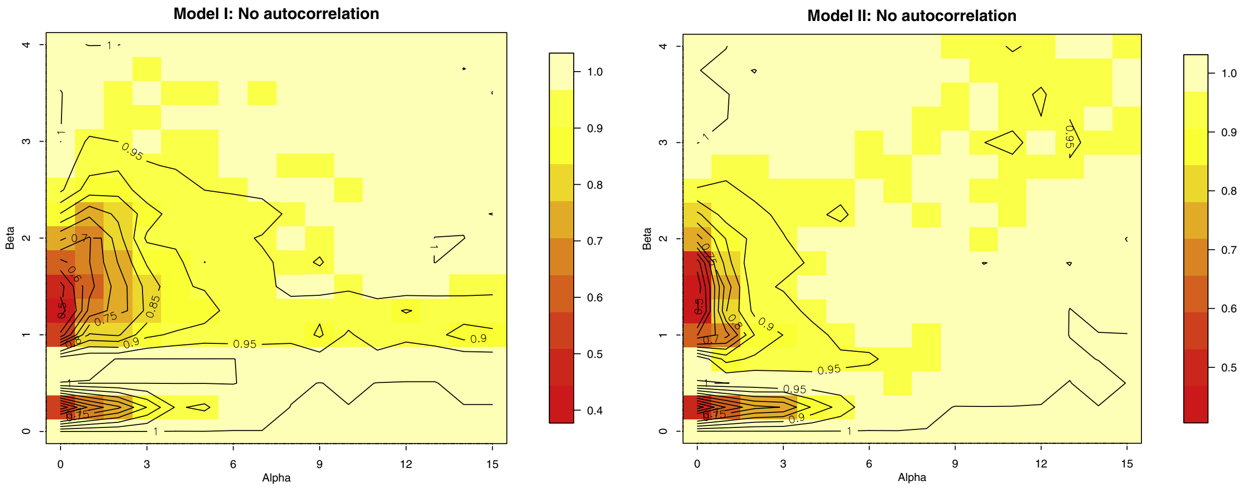

Fig. 1. Rejection rates of no serial correlation hypothesis for Model I according to Eq. 5 and Model II according to Eqs. 1 and 4. Parameter α varies between 0 and 15 with a step of 1, and parameter β between 0 and 4 with a step of 0.25. Other parameters are set at $T = 1000$ and $N = 25$, neighborhood interactions J_{ij} are set to the nearest neighbors and the spin itself with a weight of 1, and 0 otherwise. Heat maps are not smoothed, contour plots are provided for better orientation in the relationship between parameters and rejection rates.

Based on findings of previous research [12], we manipulate α between 0 and 15 with a step of 1 and β between 0 and 4 with a step of 0.25. We fix the time series length to $T = 1000$ and the number of agents in the market to $N^2 = 25^2 = 625$. The neighborhood influence J_{ij} is set equal to 1 for the nearest neighbors and the spin's own position (five spins in total), and 0 otherwise. For each setting, we perform 222 simulations.[5] Two specifications are studied – Model I given by Eq. 5, i.e. with the instant strategy spin decision, and Model II given by Eq. 1, i.e. with the standard variable strategy spins.

5. Results and discussion

We examine the effect of different combinations of parameters α and β on the returns generated by the Ising model, namely its two local field specifications given by Eq. 5 (Model I) and Eq. 1 (Model II). The former model is a simplified version which attributes a global minority game behavior to all agents (for $\alpha > 0$) whereas the latter one allows the agents to switch their global strategy between the minority game and trend following. Both models keep their local interactions so that their decision is influenced by their nearest neighbors (for $\beta > 0$). For both models and their specifications given by the parameter setting, we run 222 simulations and for each, we test whether the generated returns are serially correlated and distributed according to the Gaussian distribution. Fig. 1 illustrates the results for the "no autocorrelation" null hypothesis of the Ljung–Box test and Fig. 2 shows the results for the "Gaussian" null hypothesis of the Jarque–Bera test for both models.[6] For both tests, we present the rejection rate of the test with a significance level set to 90%, i.e. the proportion of simulations which generate returns inconsistent with the efficient market hypothesis. The lower the rate (or rather the closer the rate to 0.1), the closer the model specification simulates the efficient market (with respect to either serial uncorrelatedness or normality of returns).

As the no serial correlation condition is common for both specifications of the market efficiency, we start with its results. Fig. 1 summarizes the simulation results for both models as heat maps and contour plots for better visualization. We observe that the outcomes are qualitatively very similar for both models. The plane is practically split into two which are separated by $\beta = 0.5$. Note that this value is close to the critical inverse temperature $\beta_C \approx 0.441$. In both parts, we find a strongly non-linear dependence between β and the rejection rate. For the models above the critical temperature (below the critical inverse temperature), we find the minimum rejection rate of around 0.5 for $\beta = 0.25$. For the models below the critical temperature (above the critical inverse temperature), the minimum rejection rate of around 0.4 is found for $\beta = 1.25$. For specifications where the local fields plays no role ($\beta = 0$), the null hypothesis of no serial correlation (and thus the efficient market hypothesis) is rejected in practically all the cases. The dependence of the rejection rate on α is much more straightforward as the higher the α parameter is, the higher the rejection rate is as well. Even though the relationship is not linear either, it is monotone. Situations closest to the efficient capital market are thus found for $\alpha = 0$. The rejection rates of the no autocorrelation hypothesis are in general higher for Model II, i.e. the model with more heterogeneous agents able to switch their strategy spin.

[5] The R code is available upon request.

[6] We have tried various options for testing serial correlation and normality and the results remain qualitatively very similar. We report only these two tests for brevity.

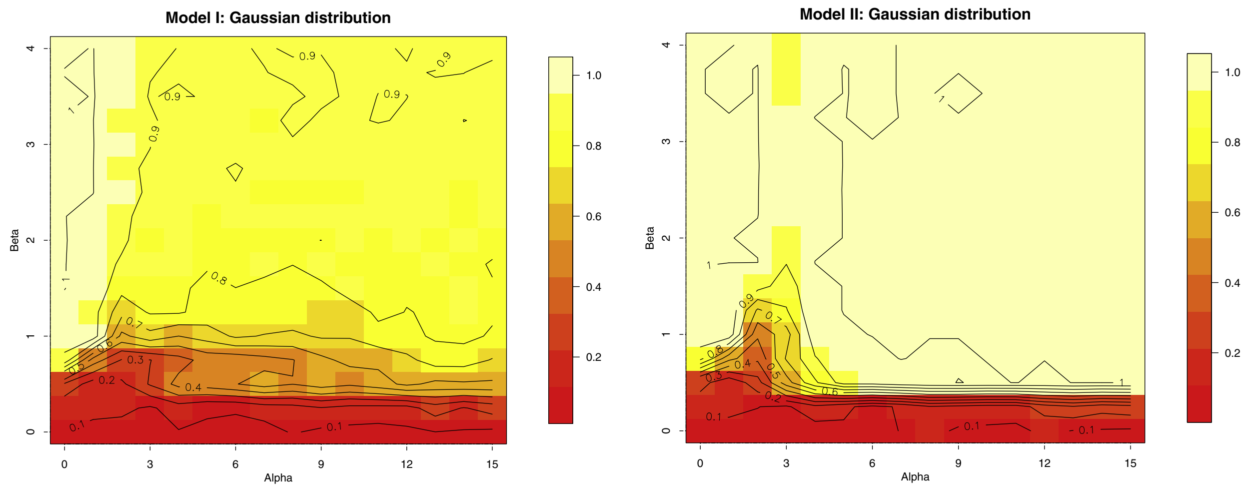

Fig. 2. Rejection rates of the Gaussian distribution hypothesis for Model I according to Eq. 5 and Model II according to Eqs. 1 and 4. Parameter α varies between 0 and 15 with a step of 1, and parameter β between 0 and 4 with a step of 0.25. Other parameters are set at $T = 1000$ and $N = 25$, neighborhood interactions J_{ij} are set to the nearest neighbors and the spin itself with a weight of 1, and 0 otherwise. Heat maps are not smoothed, contour plots are provided for better orientation in the relationship between parameters and rejection rates.

The results for rejection rates of the Gaussian distribution are much less complex. In Fig. 2, we find that the rejection rates attain low values only for the inverse temperatures β below the critical value. Above the critical inverse temperature β_C, the rejection rates quickly jump towards high values. This is true for both specifications of the Ising model analyzed here, even though the rejection rates are again lower for Model I. These findings are only mildly dependent on the global coupling parameter α. For Model I, the rejection rates form a weak U-shape, i.e. the rejection rates are the highest for very low and very high levels of α, and they remain relatively lower in between. For Model II, the lowest levels are obtained for $1.5 \leq \alpha \leq 3$.

The results suggest that the model is able to generate serially uncorrelated and normally distributed returns only for a rather narrow range of parameters. Interestingly, the serially uncorrelated returns are found also for $\beta > \beta_C$ which is a new finding not discussed in the literature which usually focuses only on $\beta < \beta_C$. The model dynamics for the inverse temperatures above the critical value is thus not as uninteresting as usually claimed. From the perspective of the Gaussian distribution, though, the inverse temperatures above the critical one are not interesting.

Let us now focus on the results through the optics of the efficient market hypothesis. If we focus on the martingale version of the hypothesis, we are interested only in the serial correlation of returns. For these, we find the minimum rejection rates at $\{\alpha, \beta\} = \{0, 0.25\}$ and $\{\alpha, \beta\} = \{0, 1.25\}$. If we stick to the classical interpretation of α and β as the intensities of the global and the local coupling, respectively, we can argue that the efficient market is found for no global coupling but some local coupling. The latter part of the claim is very interesting as it suggests that some form of herding is necessary for the market to be efficient. For no local coupling with $\beta = 0$, the market is identified as inefficient for all values of α practically always. This is well in hand with an intuitive feeling that markets would not work if they were completely random, which would be the case for $\{\alpha, \beta\} = \{0, 0\}$ when the agents make their decisions on the 50-50 basis. The effect of the global coupling is rather intuitive as well – the stronger the tendency towards the minority game behavior, the lower the efficiency. The slight differences between Model I and Model II suggest that the higher heterogeneity of the agents leads to lower efficiency. When we add the Gaussian distribution into the mix, the situations when $\beta > \beta_C$ are discarded and we are left only with the under-critical inverse temperatures which are consistent with the efficient market. The interpretation as presented above is not touched by this change.

However, there is an alternative way how to interpret the interplay between α and β. Going back to the definition of the local field in Eq. 1 and the buy-sell decision probability in Eq. 3, we observe that the β parameter is present only in Eq. 3 and not in Eq. 1. Its interpretation as the intensity of local coupling (herding) is thus rather far-fetched. If we take the local field definition as an interaction between the local (first term) and global (second term) coupling, then the α parameter becomes a weight of how much more important the global coupling is compared to the local one. The higher the α parameter is, the more influence the global coupling compared to the local coupling has. If $\alpha = 0$, the dynamics is driven solely by the local coupling, and if $\alpha \gg 1$, the dynamics is driven solely by the global coupling. The fact that the generated returns are closer to the efficient market for low values of α underlines that some level of local interactions goes well in hand with market efficiency. The high values of α and thus high influence of global coupling goes directly against the market efficiency.

Such interpretations are not much different from the ones made using the standard interpretation of α and β. However, we are able to make such claims using only one of the parameters. To look deeper into the interpretation of β, we use the idea presented by McCauley [28] who discusses the market efficiency in the sense of market clearing, i.e. clearing of supply and demand, and its connection to entropy of the market. We will refer to this type of efficiency as the technical efficiency of the market. If market clears perfectly, it is technically efficient. Going back to the parallel of the original Ising model towards financial applications, we further explore possible connections between the physics model and its financial application. In practice, 100% efficiency is impossible. However, the efficient market hypothesis assumes perfect market clearing and thus the 100% efficiency. Such level of efficiency suggests that there is no energy loss in the system and as such, the entropy of the system does not increase (in general, it either increases or keeps its level). If there is energy coming into the system (i.e. agents take actions), it is only possible to have no change in entropy if the temperature of the system approaches infinity. This yields zero inverse temperature β. For positive inverse temperatures, the system entropy increases and it is not technically efficient. This gives us a new interpretation of the β parameter in the financial Ising model.

The results clearly show that the markets are not efficient in the EMH perspective for $\beta = 0$ which is parallel to the perfect market clearing. This suggests that at least some market frictions are necessary for the market to be efficient. Note that such claim does not go completely against the notion of the market efficiency as laid down by Fama [32] who states three sufficient conditions for efficient markets – no transaction costs, all available information freely available to all agents, and all agents agree on implications of such information and future distributions of the traded assets. Fama refers to these conditions as "a frictionless market". However, he adds that these are sufficient but not necessary conditions. As specifically noted by Fama [32], such assumptions do not reflect the real financial markets. Violating these assumptions does not necessarily imply inefficiency but it is a potential source. Our results suggest that not only the frictions do not always go against efficiency, but they mainly suggest that frictions are needed for the market to be efficient in the EMH sense. To reach the efficient market, there need to be frictions.

6. Conclusion

We present a novel approach towards the financial Ising model. Most studies utilize the model to find settings which generate returns closely mimicking the financial stylized facts such as fat tails, volatility clustering, volatility persistence, and others. We tackle the model utility from the other side and look for the combination of parameters which yields return dynamics of the efficient market in the view of the efficient market hypothesis. Working with the Ising model, we are able to present nicely interpretable results as the model is based on only two parameters. Apart from showing the results of our simulation study, we offer a new interpretation of the Ising model parameters. The main outcomes of our study are the following. First, there is an important interplay between the local interactions and the global coupling. The more the agents lean towards the minority game behavior, the less efficient the market is. Reversely, the more the local interactions dominate the minority game influence, the closer the markets are to efficiency. Second, if the dynamics is driven solely by the local interactions ($\alpha = 0$), i.e. local herding, the markets are the most efficient. However, if there is no herding ($\beta = 0$) or strong herding ($\beta \to \beta_C$), the markets become inefficient as well. Some level of herding is thus necessary for market efficiency. Third, the technical efficiency of the market in the sense of market clearing is not necessary for market efficiency in the sense of EMH. On the contrary, some level of market frictions is essential for the efficient markets. These results shed a new light on the efficient market hypothesis which is usually presented as a hypothesis with unrealistic assumptions. However, as noted already by Fama [32], these assumptions are sufficient but not necessary. We show that in fact market frictions (to a certain level) and herding behavior of the market participants do not go against market efficiency but what is more, they are needed for the markets to be efficient.

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