RECOGNITION OF PATTERNS IN VECTOR FIELDS BY GAUSSIAN-HERMITE INVARIANTS

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ABSTRACT

We propose a method for the recognition of vector field patterns under an unknown rotation. The rotation is modeled as a total transformation, which is applied on both spatial coordinates and field values. The invariants are constructed from orthogonal Gaussian-Hermite moments. Their numerical stability and recognition power are shown to be better than those of the invariants published so far.

Index Terms— Vector field, total rotation, invariants, Gaussian-Hermite moments, template matching.

1. INTRODUCTION

Vector fields are a type of multidimensional data that appear in many engineering areas. They may describe particle velocity, wind velocity, optical/motion flow, image gradient, and similar phenomena. Unlike traditional images, they behave differently under spatial transformations and require developing of special algorithms.

In 2D, vector field $\mathbf{f}(\mathbf{x}) = (f_1(x, y), f_2(x, y))$ can be interpreted as a pair of scalar images f_1 and f_2. If the field is rotated, the spatial rotation is always coupled with the rotation of the vector values by the same angle as $\mathbf{f'}(\mathbf{x}) = \mathbf{R}_\alpha \mathbf{f}(\mathbf{R}_{-\alpha}\mathbf{x})$, where $\mathbf{R}_\alpha$ is a rotation matrix. This is called *total rotation* which differs from *inner rotation* $\mathbf{f'}(\mathbf{x}) = \mathbf{f}(\mathbf{R}_{-\alpha}\mathbf{x})$, commonly applied to scalar images. Hence, to detect patterns of interest such as vortices and saddle points in a vector field, independently of their orientation, we cannot apply traditional rotation invariants known from image processing, because they are not invariant to total rotation. Instead, special invariants of vector fields must be used.

The first paper on rotation invariants of vector fields was published by Schlemmer et al. [1], who adapted the scalar moment invariants proposed by Mostafa and Psaltis [2] and Flusser [3, 4], and designed invariants composed of complex moments of the field. Rotation invariants of vector fields have found several applications. Liu and Ribeiro [5] used them to detect singularities on meteorological satellite images showing wind velocity. Liu and Yap [6] applied them to indexing and recognition of fingerprint images. Bujack et al. [7, 8] generalized the previous works and showed that the invariants can be derived also by means of the field normalization approach. They studied the use of the invariants in fluid dynamics, particularly in simulations of a von Kármán vortex street.

In this paper, we use orthogonal Gaussian-Hermite moments instead of geometric moments to design rotation invariants of vector fields with better numerical stability of the high-order moments. This effect has been known for scalar images [9] and we demonstrate it propagates to vector fields as well.

2. GAUSSIAN-HERMITE MOMENTS

Hermite polynomials can be defined and evaluated by means of the three-term recurrence relation

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x) \tag{1}$$

with initial conditions $H_0(x) = 1$ and $H_1(x) = 2x$. Hermite polynomials have a high dynamic range and poor localization. To overcome this, we modulate them with a Gaussian function and scale them. It yields *Gaussian-Hermite (GH) polynomials*

$$\mathcal{H}_n(x, \sigma) = H_n(x/\sigma)e^{-\frac{x^2}{2\sigma^2}} \ . \tag{2}$$

We use GH polynomials as the base functions of the moments. The GH moments η_{pq} were introduced to image analysis by

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Shen [10, 11]. They were proven to be robust to additive noise [12, 13]. They have been successfully employed in detection of moving objects in videos [14], in license plate recognition [15], in image registration as landmark descriptors [16], in fingerprint classification [17], in face recognition [18], and as directional feature extractors [19].

The GH moments exhibit an interesting property. They change under an in-plane rotation by angle α in the same way as do the monomials $x^p y^q$ [20, 21, 9]. This allows an easy construction of rotation invariants, which is known as the *Yang's theorem* for scalar images: If there exists a rotation invariant $I(m_{p_1 q_1}, m_{p_2 q_2}, \ldots, m_{p_d q_d})$ of geometric moments

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)\mathrm{d}x\mathrm{d}y, \tag{3}$$

the same function of the corresponding Hermite moments $I(\eta_{p_1 q_1}, \eta_{p_2 q_2}, \ldots, \eta_{p_d q_d})$ is also a rotation invariant (see [21] for the detailed proof). For numerical stability reasons, we normalize the GH moments as

$$\hat{\eta}_{pq} = \frac{1}{\sigma\sqrt{\pi(p+q)!2^{p+q}}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_p(x,\sigma)H_q(y,\sigma)f(x,y)\mathrm{d}x\mathrm{d}y,$$

which keeps the range of values in a reasonable interval and does not violate the Yang's theorem.

3. GAUSSIAN-HERMITE ROTATION INVARIANTS OF VECTOR FIELDS

We can treat the vector field as a field of complex numbers

$$\mathbf{f}(x,y) = f_1(x,y) + i f_2(x,y).$$

Any moment M_{pq} (geometric, GH, or any other) is then

$$M_{pq}^{(\mathbf{f})} = M_{pq}^{(f_1)} + i M_{pq}^{(f_2)}.$$

Since the outer rotation (i.e. the rotation of the vector values) can be modeled as a multiplication of the vector field by a constant factor $e^{-i\alpha}$, any moment M_{pq} satisfies

$$M'_{pq} = e^{-i\alpha} M_{pq}.$$

It means the Yang's theorem is valid also for the total rotation of vector fields.

The rotation invariants of scalar images [22] are commonly derived via *complex moments*

$$c_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+iy)^p (x-iy)^q f(x,y)\mathrm{d}x\mathrm{d}y. \tag{4}$$

The complex moments change under the inner rotation by angle α as $c'_{pq} = e^{-i(p-q)\alpha} c_{pq}$ and under a total rotation as $c_{pq}^{(\mathbf{f}')} = e^{-i(p-q+1)\alpha} c_{pq}^{(\mathbf{f})}$. The link between the geometric and the complex moments

$$c_{pq} = \sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} (-1)^{q-j} i^{p+q-k-j} m_{k+j,p+q-k-j} \tag{5}$$

yields the possibility of applying the Yang's theorem. When replacing the c_{pq}'s by the corresponding functions of the GH moments

$$d_{pq} = \sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} (-1)^{q-j} i^{p+q-k-j} \hat{\eta}_{k+j,p+q-k-j}\ , \tag{6}$$

the behavior under a total rotation must be preserved, which leads to

$$d_{pq}^{(\mathbf{f}')} = e^{-i(p-q+1)\alpha} \cdot d_{pq}^{(\mathbf{f})}. \tag{7}$$

Analogously to the scalar case [3, 22], we can cancel the rotation parameter by a multiplication of proper powers of the d_{pq}'s. Let $\ell \geq 1$ and k_i, p_i, and q_i $(i = 1, \ldots, \ell)$ be non-negative integers such that $\sum_{i=1}^{\ell} k_i(p_i - q_i + 1) = 0$. Then the product

$$I = \prod_{i=1}^{\ell} d_{p_i q_i}^{k_i} \tag{8}$$

is invariant with respect to total rotation. This statement may generate an infinite number of rotation invariants. It is desirable to work with an independent and complete subset (basis). A simple basis can be obtained by

$$\Phi(p,q) \equiv d_{pq} d_{q_0 p_0}^{p-q+1}, \tag{9}$$

where $p_0 - q_0 = 2$ and $d_{q_0 p_0} \neq 0$. To get a complete system, we set by definition $\Phi(q_0, p_0) \equiv |d_{q_0 p_0}|$. The choice of the basis is not unique and it is determined by the choice of $d_{q_0 p_0}$.

4. EXPERIMENTS

The aim of the experiments is to compare GH invariants of vector fields to the invariants composed of geometric/complex moments. The *geometric invariants* are formally defined by the same equation as (9), but complex moments c_{pq} are used in place of d_{pq}:

$$\Psi(p,q) \equiv c_{pq} c_{q_0 p_0}^{p-q+1}. \tag{10}$$

4.1. Numerical Precision

In this experiment, we evaluated numerical properties of GH and geometric moment invariants up to the order $p+q = 160$. We took a sample vector field, rotated it by $\pi/4$ using the total rotation, and calculated the relative error of each invariant. The relative errors of the geometric invariants are visualized in Fig. 1(a) using the color map on the right. White points correspond to NaN values of the invariants. We can see limited possibility of working with the geometric invariants if $p - q > 20$ or $p, q > 80$.

The relative errors of the GH invariants are visualized in the same way in Fig. 1(b). The main difference is that all investigated invariants are valid. To compare the relative errors in the valid region, we calculated the element-wise ratio ϱ of the relative errors; see Fig. 1(c). The vast majority of indices (p, q) yield $\varrho > 1$, which means the relative error of the geometric invariants is higher than that of the GH invariants. The mean value of ϱ is 7.3 and the median equals 4.3, which clearly illustrates the higher stability of the GH invariants.

4.2. Template matching in a gradient field

We used the gradient of the picture of hair (Fig. 2(a)) in this experiment. We randomly selected 9 circular templates of the gradient field, rotated them by $5°$, and matched them against the original field. The matching was carried out by searching for the minimum ℓ_2-distance in the space of the GH invariants of orders $p + q \leq 4$ between the template and all field patches of the same size. Eight templates were found in their exact location, one was matched with a localization error of 1 pixel (see Fig. 2(b)). We repeated this experiment with template rotations $23°$, $41°$, $59°$, and $77°$, respectively. The results were always exactly the same as depicted in Fig. 2(b). The performance of the GH invariants in template matching is very good, regardless of the actual template content and of the template rotation.

4.3. Template matching in a fluid flow field

This experiment dealt with an important problem from fluid dynamics – vortex detection in a fluid flow vector field. We used the field showing the von Kármán vortex street, which is a repeating pattern of swirling vortices caused by the flow of a fluid around blunt bodies. In the von Kármán pattern, we can see several vortices arranged in two rows. The orientation of the "street" is given by the main flow direction and is generally not known a priori. A patch with a typical vortex was used as a template (see Fig. 3(a)). We rotated the template by $30°$. The task is to find all vortices of a similar shape regardless of their orientation. The search was performed in the space of the rotation invariants. We searched for all local minima of ℓ_2-distance below a user-defined threshold.

The results may be controlled by the number and the order of the invariants used. In Fig. 3(b), we can see the matching results when only the invariants up to the fourth order were employed. Almost all vortices in the field were detected, but there were also some false matches. The vortices in the bottom row are mirror reflections of that in the top row. If we want to detect them, we must use the absolute value of the imaginary parts of the invariants. As we increased the order

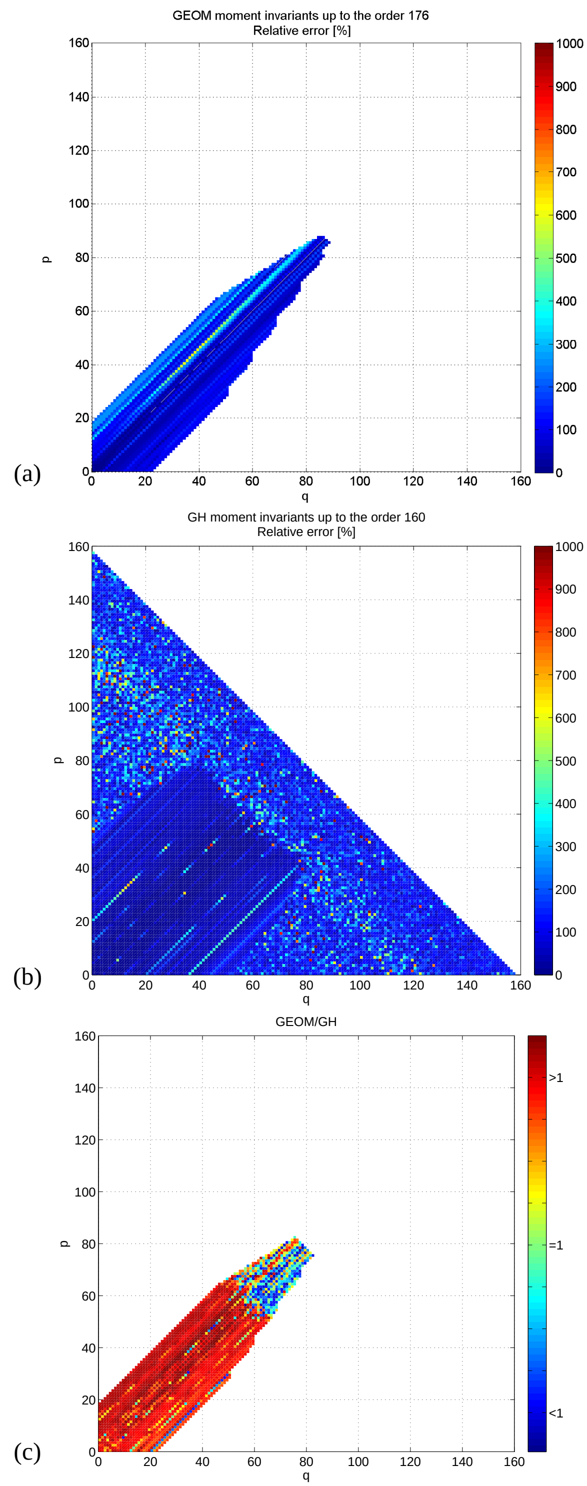
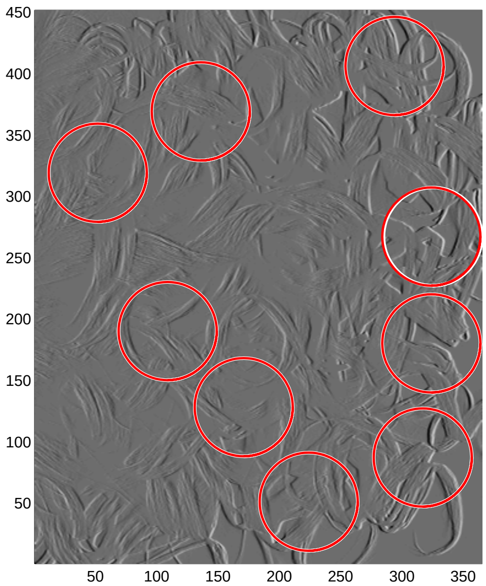


Fig. 1. Relative errors. (a) Geometric invariants. White area corresponds to NaN values of the invariants. (b) Gaussian-Hermite invariants. (c) Their ratio ϱ.

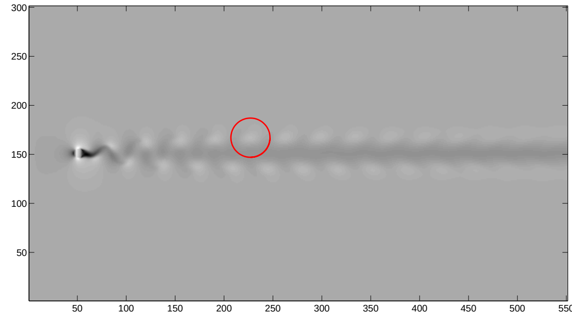
(a)



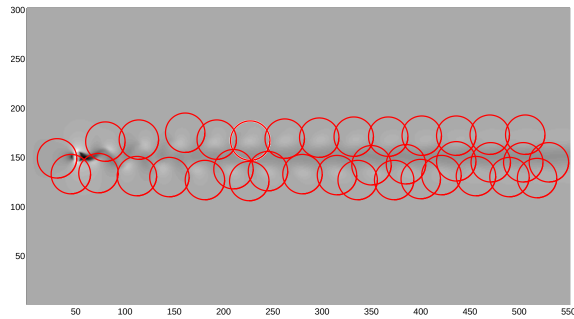
(b)

Fig. 2. Template matching. (a) Original picture of hair. (b) Gradient field (only the magnitudes are displayed). The ground-truth template positions are white and the localized positions are red. There is only one error of 1 pixel.

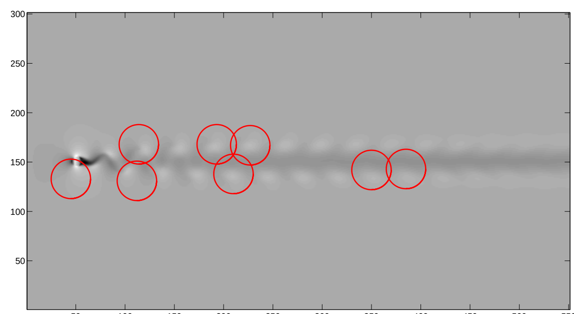
of the invariants, we identified only those vortices, which are more similar to the template (see Fig. 3(c) for the ninth order) and the number of the false matches was reduced. This process terminated at the order 14, where only a single vortex, the one identical with the template, was found. We performed this template matching on 300 consecutive frames of the Kármán



(a) The selected template.



(b) The matching vortices when only the GH invariants up to the fourth order have been employed.



(c) The matching vortices when the GH invariants up to the ninth order have been employed.

Fig. 3. The von Kármán vortex street.

street time sequence, which yielded an illustrative example of vortex tracking in a video.

5. CONCLUSION

The paper dealt with rotation invariants of vector fields, which are functions of orthogonal moments. We demonstrated that the use of orthogonal Gaussian-Hermite moments provides significantly higher numerical stability than the geometric/complex moment invariants. Further, we successfully applied them to real world pattern detection tasks.

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